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Numerical Investigation of the Stability of Flows induced by a Surface Acoustic Wave along a Slab

W. Kwang-Hua Chu
Department of Physics, Northwest Normal University, Gansu
Lanzhou 730070, PR China. (E-mail : chukh01@nwnu.edu.cn)

Abstract
The stability of flows induced by a surface acoustic wave (SAW) propagating along the deformable walls in a confined parallel-plane microchannel or slab in the laminar flow regime is investigated. The governing equation which was derived by considering the nonlinear coupling between the deformable or waving interface and viscous fluids is linearized and then the problem is solved by a verified code based on the spectral method together with the associated interface and boundary conditions. The value of the critical Reynolds number was found to be near 1439 which is much smaller than the rigid-wall case : 5772 for conventional pressure-driven flows.

Keywords : spectral method, precondition, deformable wall.

1. Introduction

The application of linear and/or nonlinear surface acoustic waves (SAW) and their relevant studies have been found in diverse (researches) fields like condense matter physics, materials science or surface chemistry/physics, environmental, communication and sensor technologies, etc [1]. Fundamental or theoretical and experimental studies of interphase nonlocal transport phenomena in a gas-solid system which appear as a result of a different type of non-equilibrium representing propagation of a surface acoustic wave (SAW) in a solid-wall had been performed since late 1980s. Meanwhile, the need in current applications of MEMS (MicroElectroMechanical System) [2] and especially microfluidics which require handling fluids has stimulated some new areas of research : invention of flexible components from which to assemble functionally complex fluidic devices, and examination of the fundamental behavior of fluids in deformable microchannels [2]. The challenges are how to overcome or control macroscopically ambient or environmental noises which are of significance in microdomains [1-2].

We have investigated the transport within a deformable microslab which will be common in microdomains of bio-MEMS applications and found certain interesting physical behavior due to the weakly nonlinear coupling between the surface wave and the velocity-slip along the wall [2]. To further study the stability issues for flows of Newtonian fluids in microdomains, we shall solve the problem by a verified code which was based on the spectral method [3-4] to obtain the neutral stability boundary curves. In this study, we shall assume that the Mach number $Ma \ll 1$, and the governing equations are the incompressible Navier-Stokes equations which are associated with the no-slip boundary conditions along the deformable walls.

2. Formulations

We consider a 2D channel of uniform thickness filled with a homogeneous Newtonian viscous fluid. The flat-plane walls of the channel are rather flexible, on which are imposed traveling sinusoidal waves of small amplitude a (due to SAW). The vertical displacements of the upper and lower walls ($y = d$ & $-d$) are thus presumed to be η and $-\eta$, respectively, where $\eta = a \cos \frac{2\pi}{\lambda}(x - ct)$, λ is the wave length, and c the wave speed. x and y are Cartesian coordinates, with x measured in the direction of wave propagation and y measured in the direction

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normal to the mean position of the walls.

We have a characteristic velocity c and three characteristic lengths a, λ, and d. It would be expedient to simplify these equations by introducing dimensionless variables (w.r.t c and d) [2]. The amplitude ratio ϵ, the wave number α, and the Reynolds number Re are defined by $\epsilon = a/d$, $\alpha = (2\pi d)/\lambda$, $Re = (c\,d)/\nu$. The 2D (x- and y-) momentum equations and the equation of continuity could be in terms of the stream function ψ if the pressure (p) term is eliminated. We shall seek a solution in the form of a series in the parameter ϵ : $\psi = \psi_0 + \epsilon\psi_1 + \epsilon^2\psi_2 + \cdots$, $(\partial p/\partial x) = (\partial p/\partial x)_0 + \epsilon(\partial p/\partial x)_1 + \epsilon^2(\partial p/\partial x)_2 + \cdots$, with $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$. The final governing equation is

$$\frac{\partial}{\partial t}\nabla^2\psi + \psi_y\nabla^2\psi_x - \psi_x\nabla^2\psi_y = \frac{1}{Re}\nabla^4\psi, \qquad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{1}$$

and subscripts indicate the partial differentiation. Thus, we have

$$\frac{\partial}{\partial t}\nabla^2\psi_0 + \psi_{0y}\nabla^2\psi_{0x} - \psi_{0x}\nabla^2\psi_{0y} = \frac{1}{Re}\nabla^4\psi_0, \tag{2}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_1 + \psi_{0y}\nabla^2\psi_{1x} + \psi_{1y}\nabla^2\psi_{0x} - \psi_{0x}\nabla^2\psi_{1y} - \psi_{1x}\nabla^2\psi_{0y} = \frac{1}{Re}\nabla^4\psi_1, \tag{3}$$

and other higher order terms. The gas is subjected to boundary conditions imposed by the symmetric motion of the walls and the zero velocity-slip : $u = 0$, $v = \pm\partial\eta/\partial t$ at $y = \pm(1+\eta)$. The boundary conditions may also be expanded in powers of η and then ϵ [2]. Equations above, together with the condition of symmetry and a uniform constant pressure-gradient in the x-direction, yield : $\psi_0 = K_0(y - y^3/3)$, $K_0 = Re\,(-\partial p/\partial x)_0/2$, $\psi_1 = \{\phi(y)e^{i\alpha(x-t)} + \phi^*(y)e^{-i\alpha(x-t)}\}/2$, where the asterisk denotes the complex conjugate. A substitution of ψ_1 into Eqn. (3) yields

$$\{\frac{d^2}{dy^2} - \alpha^2 + i\alpha Re[1 - K_0(1-y^2)]\}(\frac{d^2}{dy^2} - \alpha^2)\phi - 2i\alpha K_0 Re\phi = 0. \tag{4}$$

The associated boundary conditions are $\phi_y(\pm1) = 2K_0$, $\phi(\pm1) = \pm1$. To obtain the stability characteristics for SAW driven flows by using verified codes developed before [3] for calculating the Orr-Sommerfeld spectra, we transform equation (4) into the Orr-Sommerfeld form by rescaling and redimensionalization of physical parameters and variables mentioned before (e.g., the careful selection of K_0 and c). The matrices thus formed are of poor condition because they are not diagonal, symmetric. We precondition these complex matrices to get less errors [3]. Here we adapt Osborne's algorithm [5] to precondition these complex matrices via rescaling, i.e., by certain diagonal similarity transformations of the matrix (errors are in terms of the Euclidean norm of the matrix) designed to reduce its norm [3]. The reduced matrix is of upper Hessenberg form. We then perform the stabilized LR transformations for these matrices to get the eigenvalues.

3. Preliminary Results

We obtain those spectra for the SAW driven flow with the associated dynamic and/or kinematic boundary (interface) conditions by carefully adjusting the Reynolds number (Re) and the wave number (α). After intensive calculations, we finally obtain the neutral boundary curves for specific Re and α and plot them into Fig. 1. We have roughly $Re_{cr} = 1439$ for SAW driven flows. The critical Reynolds number for conventional flows (without SAW driven) is around 5772 [3]. Acknowledgements. This porject is supported by the National Natural Science Foundation of China under the grant No. : 10274061.

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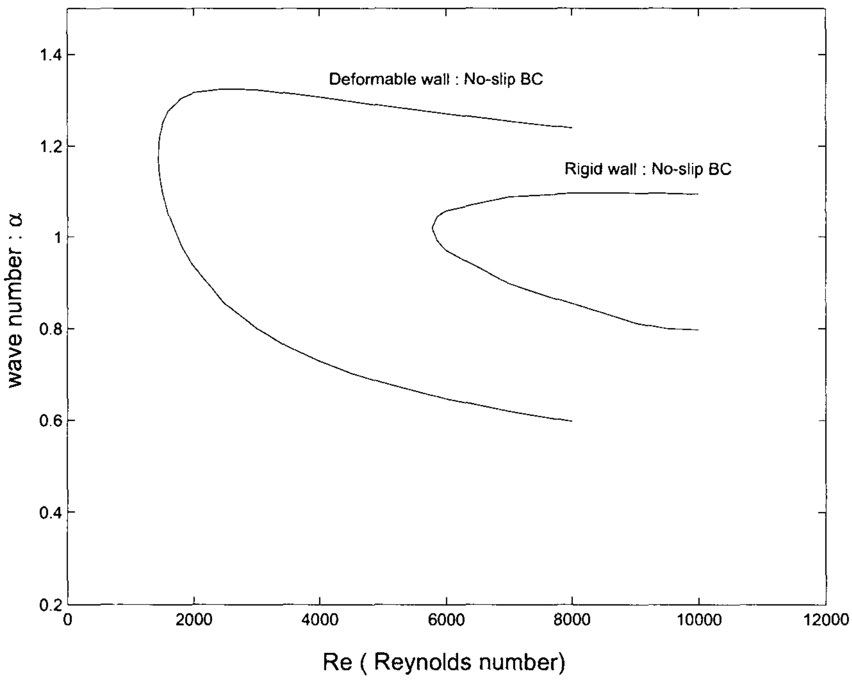

Fig. 1 Wall effects on the neutral stability boundary of the plane Poiseuille flow. The critical Reynolds number of the flow for the deformable-wall, rigid-wall cases (Re_{cr}) are $\sim 1439, 5772$, respectively.