Simulation of Gravity Current in Two- and Three-Dimensions by Two Fluid Mixture Model of Lattice Boltzmann Method

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Abstract (11 pt Times Bold)

Gravity currents are horizontal convection due to density difference of two fluids in the gravity field. The heavy fluids runs below right fluids horizontally and/or the right fluid runs above the heavy fluid in reverse direction. There have been many reports on this subject.

On the other hand, the lattice Boltzmann method is now a powerful tool of computational fluid dynamics. In this paper, we adopted two-fluid missible model, because The gravitational force and the density difference are essential factors. This method is based on the particle motions, and calculate the distribution function of the particles by the following the lattice Bhatnager-Gross-Krook (BGK) equation

$$f_i(\mathbf{r}+\mathbf{c}\tau,t+\tau)=f_i(\mathbf{r},t)-\frac{1}{\phi}\{f_i(\mathbf{r},t)-f_i^{(0)}(\mathbf{r},t)\} \quad (1)$$

f_i is the particle distribution function, $f_i^{(0)}$ on the RHS refers to its local equilibrium state, $\mathbf{c}$ is the particle velocity, and ϕ is called the relaxation time factor. The term on RHS represents the collision. As mentioned above, two fluids (two kinds of particles, say red and blue ones) are employed Macroscopic variables are obtained from the distribution function,

$$\rho_b=\sum_i f_{bi}\ ,\quad \rho_r=\sum_i f_{ri} \quad (2)(3)$$

for the densities, and

$$(\rho_b+\rho_r)\mathbf{u}=\sum_i(f_{bi}\mathbf{c}_i+f_{ri}\mathbf{c}_i) \quad (4)$$

for the momentum, but not calculated from the differential equations, i.e., the Navier-Stokes equations. The version that employ stable scheme of finite difference method is called the finite difference lattice Boltzmann method. The basic equation is

$$\frac{\partial f_i}{\partial t}+c_{i\alpha}\frac{\partial f_i}{\partial x_\alpha}-ac_{i\alpha}\frac{\partial}{\partial x_\alpha}\frac{f_i-f_i^{(0)}}{\phi}=-\frac{1}{\phi}\left(f_i-f_i^{(0)}\right) \quad (5)$$

and, in three-dimensional case, this method (FDLBM) is used.

The local equilibrium distribution function has a polynomial form of fluid velocity up to the second order for the incompressible fluids as

$$f_a^{(0)}=F_p\rho\ [\ 1-2Bc_{\alpha\alpha}u_\alpha+2B^2c_{\alpha\alpha}c_{\alpha\beta}u_\alpha u_\beta+Bu^2-\frac{4}{3}B^3c_{\alpha\alpha}c_{\alpha\beta}c_{\alpha\gamma}u_\alpha u_\beta u_\gamma-2B^2c_{\alpha\alpha}u_\alpha u^2] \quad (6)$$

$$F_0=\frac{9-2D}{9},\quad F_1=\frac{D}{9b_1},\quad F_2=\frac{D}{9b_2},\quad B=-\frac{3}{2c^2} \quad (7a,b,c,d)$$

where D represents the dimension. The local equilibrium distribution function is re-defined for each particle. In (8) or (9) the density is ρ_r or ρ_b but the fluid velocity is common. In order to introduce the gravitational force, however, the fluid velocity $\mathbf{u}$ in the local equilibrium distribution function for blue particles should be changed to $\mathbf{u}-\mathbf{g}\phi$, then

$$f_{bi}^{(0)}=f_{bi}^{(0)}(t,\rho,\mathbf{u}-\mathbf{g}\phi) \quad (8)$$

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Ordinary Chapman-Enscog technique gives the continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_\alpha}{\partial x_\alpha}=0 \qquad (9)$$

and the equation of motion

$$\rho\left(\frac{\partial u_\alpha}{\partial t}+\frac{\partial u_\alpha u_\beta}{\partial x_\beta}\right)=-\frac{\partial P}{\partial x_\alpha}+\frac{\partial}{\partial x_\beta}\mu\left(\frac{\partial u_\alpha}{\partial x_\beta}+\frac{\partial u_\beta}{\partial x_\alpha}\right)+\frac{\partial}{\partial x_\alpha}\left(\lambda\frac{\partial u_\beta}{\partial x_\beta}\right)+2g\rho_b\delta_{\alpha 3} \qquad (10)$$

employed the Boussinesq approximation. The pressure and the viscosities are expressed as

$$P=\frac{1}{3}\rho, \qquad \mu=\frac{1}{3}\rho(\phi-a), \quad \lambda=-\frac{1}{3}\rho(\phi-a) \qquad (11a,b,c)$$

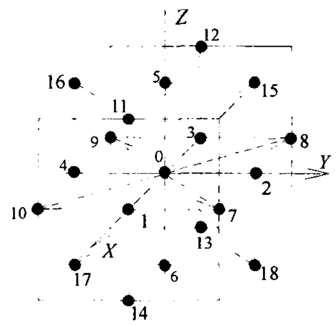

Fig 1: A cubic lattice and particles

The particle model used in this paper is 19 velocity model shown in Fig.1. A flow pattern of typical two-dimensional gravity current is shown in Fig.2, and a symmetric flow pattern is shown. Figure 3 shows three-dimensional flow pattern, and lobes and clefts are clearly detected.

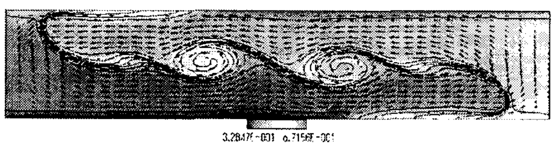
Fig.2 Two-dimensional flow pattern

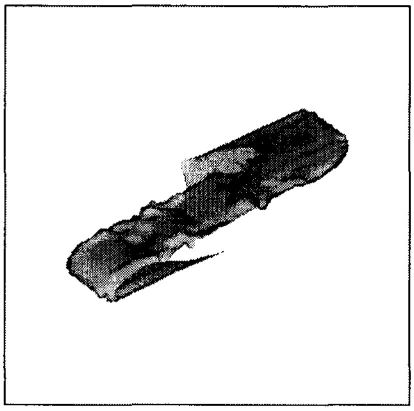
Fig.3 Three-dimensional flow pattern