Simulation of Three Dimensional Micro Channel Flow by Lattice Boltzmann Method

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Abstract

In micro scale flow disparities exist between experimental results and predictions based on conventional theory. Beskok *et al.*[1] indicated that there might be four major effects; rarefaction, compressibility, thermal creeping and viscous heating. A molecular approach is more appropriate than a continuous one in incorporating the first two effects in the present isothermal analysis. It is generally agreed that the assumption of continuum fluid holds until the Knudsen number (Kn) equal to about 0.01. Flow enters the slip-flow regime in the range of Kn between 0.01 and 0.1. Arkilic et al. [2] obtained an analytical solution for the Navier-Stokes equation at a low Kn with a first-order slip model in 2-D micro channel flow. In this paper pressure driven flow is analyzed in a three dimensional micro T-channel by the multi-component LB method with the characteristics of rarefied gas diffusion taken into account. The results from the present study are compared with respect to the parameters such as pressure gradient and velocity profiles against former experimental measurements and numerical results in 2-D [3].

Here we present a brief description of the 3-D LB method, D3Q15, with multi components. Equation (1) describes evolution of the distribution function $f_i(\mathbf{x}, t)$ in space $\mathbf{x}$ and time t,

$$f_i(x+\delta_t e_i, t+\delta_t) - f_i(x,t) = -\frac{1}{\tau}\left(f_i(x,t) - f_i^{eq}(x,t)\right). \quad (1.1)$$

Macroscopic number density and velocity are obtained as follows.

$$\rho = \sum_i f_i, \quad \rho \boldsymbol{u} = \sum_i f_i \cdot \boldsymbol{e}_i \quad (1.2)$$

A functional form of the equilibrium function depends on the lattice structure [4] and it is well known that the equilibrium function yields the Navier-Stokes equation through the Chapman-Enskog procedure. According to Lim's assumption [3] that particles travel a distance λ while relaxing to their equilibrium state in a collision interval ω. The relaxation time scale τ is related to the Knudsen number as,

$$\tau = Kn(N_y - 1). \quad (1.3)$$

N_y is the number of lattice points in a characteristic direction and the local Kn is computed as $Kn_o/P^*(X)$ which is described in Arkilic et al. [2].
The multi component LBE model with interparticle interaction [5] simulates diffusion by various driving mechanisms. The resulting flow has a common velocity field in the absence of interaction forces. Conservation of momentum at each collision requires that

$$\boldsymbol{u}' = \sum_{\sigma=1}^{2} \frac{m_\sigma f_\sigma \boldsymbol{u}_\sigma}{\tau_\sigma} \Bigg/ \sum_{\sigma=1}^{2} \frac{m_\sigma \boldsymbol{u}_\sigma}{\tau_\sigma} \quad (1.4)$$

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where m_σ, the molecular mass of each component, is 1.0 in this study. The density distribution of the second component is calculated independently according to its own relaxation time and the same evolution equation as the first component. Momentum should be incremented accordingly if any external force is applied to the component σ. In the LB method this is simply achieved by replacing $\boldsymbol{u}$ in each density equilibrium function with $\boldsymbol{u}'+\tau_\sigma \boldsymbol{F}_\sigma/\rho_\sigma$. A general form of the interaction force is

$$\boldsymbol{F}_\sigma = -\psi_\sigma(\boldsymbol{x})\sum_i\sum_{\bar{\sigma}} G_{\sigma\bar{\sigma}}(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{e}_i)\psi_{\bar{\sigma}}(\boldsymbol{x}+\boldsymbol{e}_i)\boldsymbol{e}_i \tag{1.5}$$

where $G_{\sigma\bar{\sigma}}$ represents the strength of interaction and ψ_σ is an arbitrary function of the density distribution, f_i.

Keyword: *channel flow, Knudsen Number, LBM, micro scale flow, slip boundary condition*

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