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Unstable Interface Phenomena in a Micro Channel

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Abstract

The behavior of viscous fingerings caused by an external force is investigated by using a two-phase lattice Boltzmann method. The effects of the modified capillary number, the viscosity contrast, and the modified Darcy-Rayleigh number on the instability of interfaces are found. The calculated wave numbers are in good agreement with the theoretical ones in the range of wave numbers smaller than 10, but the calculated ones tend to become smaller than the theoretical ones in higher wave numbers.

Keyword: *lattice Boltzmann method, two-phase flow, viscous fingering, MEMS.*

1. Introduction

The behavior of two-phase flows in micro channels is of interest in relation to MEMS (Micro Electro Mechanical Systems) devices. Viscous fingering is one of interesting phenomena in such flows. So far, many investigations of viscous fingering have been performed by using experimental, numerical, and theoretical approaches [1]-[4]. However, the driving force of flows in the above researches was caused mainly by a pressure difference. Considering a capillary force in micro channels, the effect of an external force on the viscous fingering is important. In this paper, we apply a two-phase lattice Boltzmann method [5] to the investigation of interfacial instability of two-phase flows in a micro channel.

2. Numerical Method

Two-phase fluids flows in a narrow channel with length L and height h are considered (see Fig. 1). Assuming that the aspect ratio h/L is small and the flow velocity in the z-direction has a parabolic profile, we calculate two-phase flows only in the two-dimensional x- and y-directions. Hereinafter, non-dimensional variables, which defined by using a characteristic length h, a characteristic particle speed c, a characteristic time scale $t_0 = h/U$ where U is a characteristic flow speed, and a reference density ρ_0, are used as in Ref. [6]. The 2D9V model is used in the present paper. The physical space is divided into a square lattice, and the evolution of particle population at each lattice site is computed. Two particle velocity distribution functions, f_i and g_i, are used. The function f_i is used as an index function for the calculation of interface profiles, and the function g_i is used for the calculation of pressure and velocity of two-phase fluids with the same density. The evolution of the particle distribution functions $f_i(\boldsymbol{x},t)$ and $g_i(\boldsymbol{x},t)$ with velocity $\boldsymbol{c}_i$ at the point $\boldsymbol{x}$ and at time t is computed by the following equations:

$$f_i(\boldsymbol{x}+\boldsymbol{c}_i\Delta x, t+\Delta t) - f_i(\boldsymbol{x},t) = -\frac{1}{\tau_f}\left[f_i(\boldsymbol{x},t) - f_i^{\mathrm{eq}}(\boldsymbol{x},t)\right], \tag{1}$$

$$g_i(\boldsymbol{x}+\boldsymbol{c}_i\Delta x, t+\Delta t) - g_i(\boldsymbol{x},t) = -\frac{1}{\tau_g}\left[g_i(\boldsymbol{x},t) - g_i^{\mathrm{eq}}(\boldsymbol{x},t)\right] - 3E_i c_{i\alpha}\left[\frac{12\mu u_\alpha(\boldsymbol{x},t)}{h^2}\right]\Delta x + 3E_i c_{i\alpha} F_\alpha \Delta x, \tag{2}$$

where f_i^{eq} and g_i^{eq} are the equilibrium distribution functions given in Ref. [5], τ_f and τ_g are dimensionless single relaxation times, Δx is a spacing of the lattice, Δt is a time step, $\boldsymbol{c}_i$ is

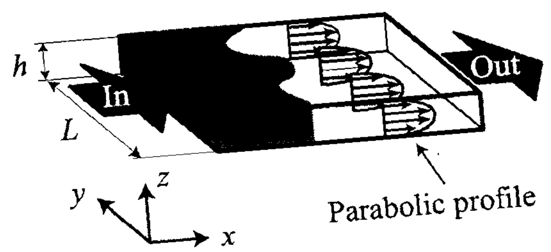

Fig. 1: Flow through a narrow channel.

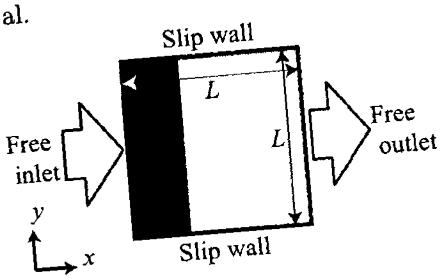

Fig. 2: Computational domain.

the particle velocity, and F_α is the external force. The second term of the right side of Eq.(2) represents the viscous force from the top and bottom walls, and the last term represents the external force applying to the fluids. The index function ϕ representing an interface and the macroscopic variables of the two-phase fluids (the pressure p and the velocity $\boldsymbol{u}$) are defined in terms of the two particle velocity distribution functions as follows:

$$\phi = \sum_{i=1}^{9} f_i, \quad p = \frac{1}{3}\sum_{i=1}^{9} g_i, \quad \boldsymbol{u} = \sum_{i=1}^{9} \boldsymbol{c}_i g_i. \tag{3}$$

Applying the asymptotic theory [6] to Eqs.(1)-(3), it is found that the macroscopic variables satisfy incompressible Navie-Stokes equations for two-phase fluids with relative errors of $O[(\Delta x)^2]$.

3. Results and Discussion

A computational domain is shown in Fig. 2. The aspect ratio is $h/L = 0.02$. A free slip boundary condition is used on the top and bottom walls and free inlet and outlet conditions are applied to the lateral sides of the domain. The domain is divided into a 400×400 square lattice. Two-phase fluids, black and white portions shown in Fig. 2, are filled in the channel. The external force is applied only to the black fluid after $t = 0$. The initial velocity is calculated by the Darcy's law $u_x = k/\mu_i(-dp/dx + F_i)$, where k is the permeability ($k = h^2/12$). The dimensionless parameters for the present problem are the modified capillary number $Ca' = 12\mu_1 U^* L^2/\sigma h^2$, the viscosity contrast $B = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$, and the modified Darcy-Rayleigh number $G = h^2(F_2\Delta x - F_1\Delta x)/12U^*(\mu_1 + \mu_2)$ [7], where σ is the interfacial tension, U^* is the average velocity of the interface at the time when the disturbance occurs, and the subscripts 1 and 2 represent the downstream and upstream fluids, respectively.

Figure 3 shows the calculated results of the time evolution of the interfacial shapes. In Fig. 3(a), the initial shape is given by a large sine-wave as an initial disturbance. After the flow starts, the large disturbance is settled down with the time. Therefore, in the case that the external force is applied only to the downstream fluid, the interface is stable. In Figs. 3(b)-(e), the external force is applied only to the upstream fluid, and a small and random disturbance is given to the initial velocity at the interface. Six different random disturbances are used. As the time passes, the small disturbance grows larger and larger, and finally we can observe viscous fingers. These results indicate that the case that the external force is applied only to the upstream fluid is unstable. Comparing Figs. 3(c), (d), and (e) with (b), the modified Darcy-Rayleigh number G of (c) is larger than that of (b), the viscosity contrast B of (d) is larger than that of (b), and the capillary number Ca' of (e) is smaller than that of (b). Thus, it is found that as Ca', B and G increase, viscous fingering grows up larger.

From the linear stability analysis, the theoretical wave number of fingers is given as follows [7]:

$$N_{\text{theo}} = \frac{1}{2\pi}\left[\frac{2(B+G)Ca'}{3(B+1)}\right]^{1/2} \tag{4}$$

The relation between the calculated wave numbers N_{obs} and the theoretical ones is shown in Fig. 4. The error bars represent the difference among the initial disturbances, and the circles

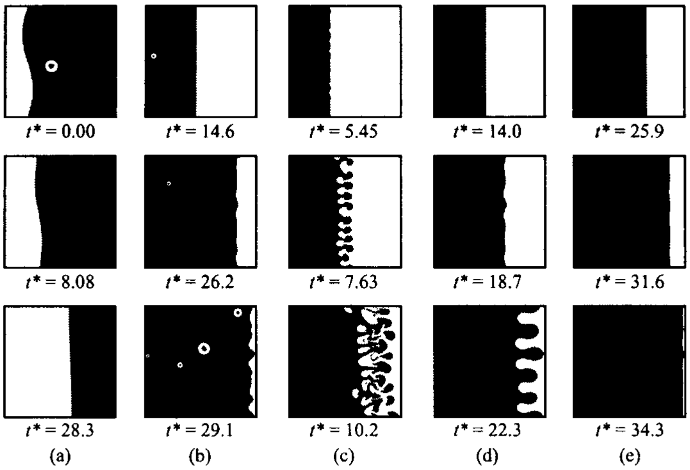

Fig. 3: The time evolution of the interfacial shapes ($t^* = tU^*/h$): (a) $Ca' = \infty, B = 0, G = -0.625$; (b) $Ca' = 3060, B = 0, G = 1.22$; (c) $Ca' = 3050, B = 0, G = 6.12$; (d) $Ca' = 3030, B = 0.6, G = 1.23$; (e) $Ca' = 1010, B = 0, G = 1.24$.

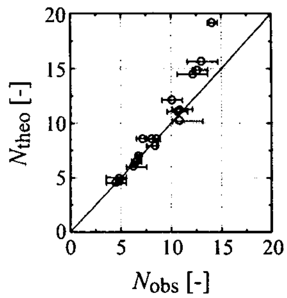

Fig. 4: Comparison of calculated wave numbers N_{obs} with theoretical ones N_{theo}.

stand for the averaged value. The wave numbers of the present results are in agreement with the theoretical ones in the range of wave numbers smaller than 10, but N_{obs} tends to become smaller than N_{theo} in higher wave numbers.

4. Conclusions

We have applied a two-phase lattice Boltzmann method to the simulations of viscous fingerings caused by an external force in a micro channel. From the computations, the following results are obtained.

- As the modified capillary number, the viscosity contrast, and the modified Darcy-Rayleigh number increase, viscous fingering grows up larger.
- The calculated wave numbers are in good agreement with the theoretical ones in the range of wave numbers smaller than 10, but the calculated ones tend to become smaller than the theoretical ones in higher wave numbers.

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