The Application of Generalized Characteristic Coordinate System

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Abstract

In the generalized characteristic coordinate system (GCCS) proposed by Wu and Shi [1], the frame moves at a speed which is a linear combination of the convective speed and the sound speed, thus unifying the classical Eulerian approach, Lagrangian approach, and the unified coordinate system (UCS) of Hui and his co-workers [2]. Here some properties of Euler equations in the GCCS are studied and the advantages of GCCS in capturing expansion fans and shock waves are demonstrated by the results of numerical tests.

Keyword: *Generalized characteristic coordinate system, Euler equations, Riemann problem, Expansion fan, Shock wave.*

1. Introduction (Extended Abstract)

There are several coordinate systems used to describe fluid flows. The most classical ones are the Eulerian approach and Lagrangian approach. In the Eulerian approach, one considers what happens at every fixed point in space as a function of time. The velocities and other properties of fluid elements are considered to be functions of time and fixed space coordinates. In the Lagrangian approach, one looks for the dynamic history of each selected fluid element. The positions of fluid particles and other properties are considered to be functions of the time and their initial positions. Both approaches have some advantages in classical fluid mechanics. They are regarded as equivalent to each other except that the Lagrangian system gives more information: it tells each fluid particle's history.

But for applications in Computational Fluid Dynamics, they may behave quite differently. For the Eulerian approach, fluid particles move across cell interfaces. It is this convective flux that causes excessive numerical diffusion in the numerical solutions. As a result, discontinuities such as shock waves and slip lines (contact discontinuities) are smeared out. The Lagrangian approach needs the use of a moving frame and uses fluid particles as computational cells. Consequently, there is no convective flux across cell boundaries and the numerical diffusion can be minimized. But the grid may deform indefinitely in certain cases, which leads to decrease in accuracy and breakdown of computation.

Recently, Hui and his co-workers [2] proposed a unified coordinate system (UCS) that unifies both approaches. This unified approach combines the advantages of the Eulerian approach and the Lagrangian approach. It involves a free parameter h. The traditional Eulerian approach and Lagrangian approach correspond to the particular cases $h=0$ and $h=1$, respectively. However, Wu [3] showed that across a physical shock, the UCS could fail to work by making the transformation between the two coordinate systems non-invertible.

Recently, Wu and Shi [1] proposed a more general coordinate system moving at a speed, which is a linear combination of the convective speed and the sound speed:

$$B = \alpha \cdot u + \beta \cdot a \qquad (1)$$

Where u is the convective speed of fluid particles, and a the sound speed. α and β are two constants.

The coordinate system that moves at the characteristic speed given by equation (1) is called *generalized characteristic coordinate system* (GCCS). From the generalized characteristic coordinate system we recover the classical Eulerian approach for $\alpha = \beta = 0$, the classical Lagrangian approach for $\alpha = 1$ and $\beta = 0$, and the unified coordinate system (UCS) of Hui et al [2] for $\alpha = h$ and $\beta = 0$.

Since the GCCS includes all the classical coordinate systems as particular cases, it provides a general framework of coordinate transformation. This aspect alone has its fundamental meaning in increasing the knowledge. In the Eulerian system, it is well known that the solution of a Riemann problem for gas dynamics is composed of a left-going expansion fan (or shock wave), a contact discontinuity (in the middle), and a right-going shock wave (or expansion fan). In the GCCS, we have another simple wave: the motionless A-discontinuity, which has a speed zero. So the solution of the Riemann problem in the GCCS and the equivalence of Riemann problems on both the GCCS and the original Eulerian coordinate system should be studied.

Furthermore, compressible flows involve many special structures, which move at some characteristic speed. Using GCCS allows for easy tracking of these special structures. For example, shear lines and contact discontinuities move at the characteristic speed u, expansion fans move at the characteristic speed $u + a$ or $u - a$. If we take $B = u \pm a$ inside an expansion fan, then the expansion fan is tracked exactly. It could be expected that expansion fans can be more exactly computed by using a GCCS than the classical coordinate systems.

2. Main contents of this paper

The current study will be restricted to the following subjects:

(1), the derivation of GCCS and conservation form of the compressible flow equations in the GCCS.

(2), the solution of the Riemann problem in the GCCS and the equivalence of Riemann problems on both the GCCS and the original Eulerian coordinate system. The equivalence of weak solutions between the original Eulerian coordinate system and the GCCS is studied and the results are validated by numerical tests. It is shown that under some conditions, an expansion fan in the physical space could remain to be an expansive wave, degenerate to a linear wave, or be transformed into a compression wave in the GCCS.

(3) the ability of the GCCS in CFD to resolve shock waves, contact discontinuities, and notably the expansion fans for which classical methods do not work very well. Numerical tests have shown the significance of the use of GCCS in capturing expansion fans and shock waves. Expansion fans and shock waves can be more exactly computed by using a GCCS than the classical coordinate systems because they are tracked exactly by the GCCS moving at the characteristic speed of them.

References:

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