Dynamic Free-surface Deformations in Axisymmetric Liquid Bridges

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1. Abstract

Thermocapillary convection is a surface tension driven flow due to a temperature gradient along an interface. It occurs during a crystal-growth process and therefore understanding the convection is important to material processing in microgravity. Although modelling of the float-zone crystal-growth process has been of interest for a few decades, most studies of liquid bridges assumed non-deformable flat surfaces. In reality, the surface profile, g(t,z), is unknown and should be obtained as a solution to the coupled transport equations along with the surface force balance. Here we report on a numerical study of axisymmetric thermocapillary convection in liquid bridges with deformable surfaces. The interface is determined as part of the complete solution. The influence of the capillary number (Ca), Reynolds number (Re), Prandtl number (Pr) and aspect ratio(Ar) on the dynamics is explored.

2. Mathematical Model and Results

The physical system considered is a liquid bridge with a deformable free surface as shown in Fig. 1. The aspect ratio is R/H, where R and H are the radius and height of the liquid bridge. The upper and lower disks have nondimensional temperatures $T_{hot}=1$ and $T_{cold}=0$. The Biot number, Bi, which gives the free-surface heat loss is assumed to be zero.

Because no data with deforming liquid bridges are available in the literature, our numerical scheme is validated against known results for a flat surface in Table 1. Our results with zero capillary number (Ca=0, flat nondeformable surface) are in good agreement with those from other studies[1,2].

We have investigated convection up to Re=8000 with Ar=1, Pr=0.01 and various $Ca \leq 0.05$, and have found no oscillatory axisymmetric states. Assuming nondeformable flat interfaces, the critical linear theory Re for transition to steady 3D convection and unsteady oscillatory convection are, respectively, about 1900 and 5500[3]. We have also computed axisymmetric convection up to Re=5000 with Ar=1, Pr=1 and $Ca \leq 0.1$ and again no oscillatory convection was found. The critical Re for transition to oscillatory states with Ca=0 is about 2500 from linear theory[2,3] and three-dimensional numerical simulations[4]. Thus we conclude that only azimuthal waves can generate oscillations in a liquid bridge with either non-deformable or deformable surfaces. This is consistent with studies of convection in other cylindrical geometries[5].

The effect of Re and Ca on the surface deformations is shown in Figs. 2 and 3. The free surface is always convex near the cold wall and changes from concave to convex with increasing Re at a fixed Ca near the hot wall. As Re increases, the number of surface peaks increases from two to three. The surface shape is independent of Ca as shown in Fig. 3 with increasing surface elevations and depressions with Ca. Fig. 4 shows stream function minima, surface temperatures and velocities at various Ca. They are all independent of Ca. Thus, dynamic free-surface deformations do not influence the convection in the liquid bridge.

3. Conclusions

Thermocapillary convection in a liquid bridge including dynamic free surface deformations is investigated in a two-dimensional numerical study. Simulations with either non-deformable or deformable surfaces predict steady convection even at very high Re and Ca. Thus, dynamic free-surface deformations do not induce transitions to oscillatory axisymmetric convection. Surface deformations at low Re are determined by surface pressure variations. As Re increases, surface peaks increase from two to three. The surface shape is independent of Ca while surface deformations become larger with increasing Ca. Surface deformations decrease with increasing Ar or Pr at fixed other parameters. Thermocapillary convection inside the liquid bridge is insensitive to variations in Ca.

References

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Table 1: Comparison of stream function minima ($\psi_{\min} \times 10^2$) in a 2D liquid bridges (Ar=1, Bi=0.3 and Ca=0)

Re	Pr	Sumner et al.[1]	Wanschura et al.[2]	Present results
100	0.1		-1.02	-1.031
100	10	-0.4221	-0.425	-0.4216
10	100	-0.4205		-0.42

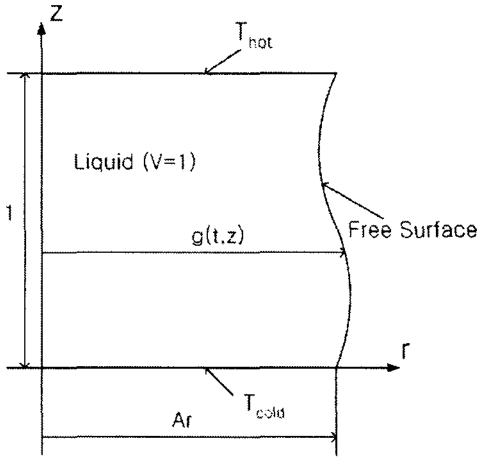

Fig. 1: Physical System.

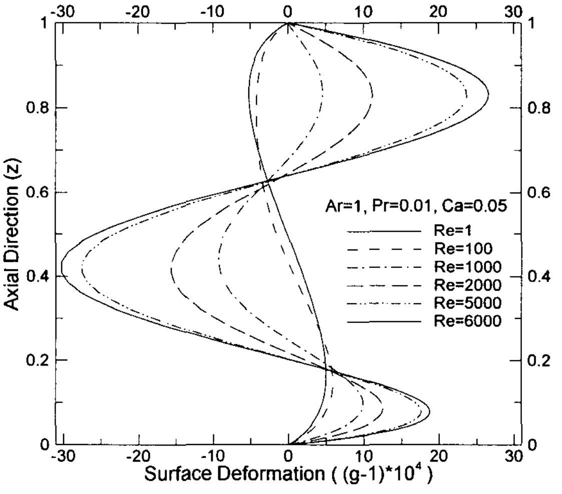

Fig. 2: Free Surface shapes with various Re.

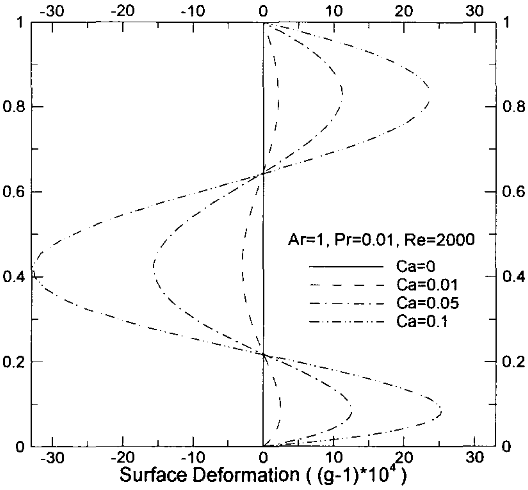

Fig. 3: Free surface shapes with various Ca.

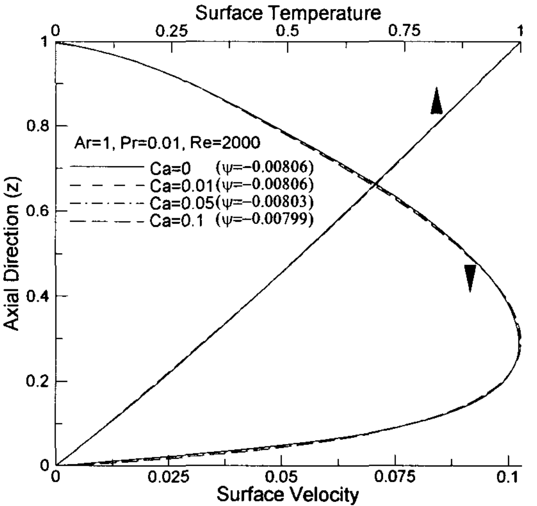

Fig. 4: Surface velocity and temperature distributions.