# Calculation of Critical Radius of the Spherical Cavity in Presence of Surface Tension and Fluid Viscosity

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**Extended abstract**

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The problem about complete collapse of a bubble in an ideal incompressible fluid was solved by Rayleigh [1]. Analysis of viscous effects showed [2] that there is a critical value $R_*$ of the bubble radius $R$. The value $R_*$ divides two different types of the bubble evolution. If $R$ is less than $R_*$ we have collapse which is analogous to the Rayleigh example. If $R$ is greater than $R_*$ the time of collapse is infinite one.

The objective of the present work is to calculate $R_*$ exactly in incompressible fluid in presence of viscous effects on the boundary and the surface tension, and carry out numerical investigations different regimes of the bubble evolution.

Governing equation describing the dynamics of the boundary bubble has the following form:

$$\frac{3V^2}{2}+RV\frac{dV}{dR}+\frac{2\sigma}{\rho R}+\frac{P_\infty}{\rho}=-\frac{4\nu}{R}V. \tag{1}$$

Here $R$ is the radius of the bubble, $V=dR/dt$. Parameters $\nu$, $\sigma$, $\rho$, characterize the fluid properties: $\rho$ is the fluid density, $\nu$ is the kinematics viscosity, $\sigma$ is the surface tension. We suppose that there is no gas inside the bubble, i.e. effects of cool and heat are not accounted for. The equation (1) generalizes the Rayleigh equation [1] describing the collapse of a spherical cavity in the case of $\sigma=0,\ \nu=0$.

It is easy to obtain a solution of (1) in the case $\nu=0$:

$$V=\sqrt{-\frac{2P_\infty}{3\rho}-\frac{2\sigma}{\rho R}+\frac{C}{R^3}}. \tag{2}$$

If $\nu\neq 0$ we use the method of variation of an arbitrary constant. It means that the equation (1) is considered to be an inhomogeneous differential one and $C=C(R)$ is a function with respect to $R$. The differential equation for $C$ has the following form:

$$\left(\frac{dC}{dR}\right)^2=64\nu^2R^2\left(-\frac{2P_\infty}{3\rho}-\frac{2\sigma}{\rho R}+\frac{C}{R^3}\right). \tag{3}$$

Solution of the equation (3) is supposed to be a polynomial of $S=\sqrt{R}$ : $C=C(S)=\sum_{i=0}^{n}C_iS^i$ . We substitute this presentation into (3) and obtain the system of equations for $C_i$: The number of equations is equal to 7, and the number of unknown coefficient is equal to 5. Nevertheless it is possible to point out the set of $C_i$ and $P_\infty$ satisfying the system of equations (3):

$$C_0 = C_1 = C_3 = 0,\ C_2 = 64\nu^2,\ C_4 = -\frac{2\sigma}{3\rho},\ P_\infty = P_* = -\frac{\sigma^2}{24\nu^2\rho}. \tag{4}$$

From the physical point of view it means that the pressure is an extending one. Hence the exact solution for velocity $V$ has the following form:

$$V = \sqrt{-\frac{2P_*}{3\rho} - \frac{8\sigma}{3\rho R} + \frac{64\nu^2}{R^2}} = \frac{\sigma}{6\nu\rho} - \frac{8\nu}{R}. \tag{5}$$

Because of $V = dR/dt$ we obtain from here the time motion of the cavity boundary:

$$t = \sqrt{\frac{\rho}{2}} \int \frac{dR}{\sqrt{E - U(R)}},\quad E = \frac{\sigma^2}{72\nu^2\rho},\quad U(R) = \frac{4\sigma}{3R} - \frac{32\nu^2\rho}{R^2}. \tag{6}$$

This formula allows us to use the approach of theoretical mechanics for the further analysis. Presentation (6) coincides with the time motion of a particle in the field of potential $U(R)$. The energy of the particle is equal to $E$ . It is easy to show that the potential has the maximum at

$$R_* = \frac{48\nu^2\rho}{\sigma}, \tag{8}$$

and $E = U(R_*)$ . From (5), (8) it follows that $V(R_*) = 0$ and $V(R) > 0$ at $R > R_*$, $V(R) < 0$ at $R < R_*$. It is clear that the quantity $R_*$ divides two different types of motion. If $R$ is less than $R_*$ the motion of particle is finite and a cavity collapses down. If $R$ is greater than $R_*$ the particle goes to infinity and a cavity extends because of external positive pressure $(-P_\infty)$.

Hence the quantity $R_*$ is a critical radius for the cavity. Under the normal conditions the value of $R_*$ is equal to $6{,}58 \cdot 10^{-7}$ meter.

We investigated phase trajectories of the equation (1) numerically. It is shown that the point $(V, R) = (0, R_*)$ is a saddle one. There are four sectors dividing different regimes of bubble motion.

## References


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