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A Preconditioning Method for Two-Phase Flows with Cavitation

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Abstract

A preconditioned numerical method for gas-liquid two-phase flow is applied to solve cavitating flow. The present method employs a density based finite-difference method of dual time-stepping integration procedure and Roe's flux difference splitting approximation with MUSCL-TVD scheme. A homogeneous equilibrium cavitation model is used. The method permits simple treatment of the whole gas-liquid two-phase flow field including wave propagation, large density changes and incompressible flow characteristics at low Mach number. By this method, two-dimensional internal flows through a venturi tube and decelerating cascades are computed and discussed.

Keyword : two-phase flow, cavitation, phase change, preconditioning method, dual time-step.

1. Introduction

When cavitation occurs unexpectedly accompanying the attachment and collapsing of cavitation bubbles near solid surfaces, it causes the noise, vibration and damage as well as changes the performance characteristics in hydraulic machine systems. As the results, it has an unfavorable effect on the performance and eventually brings the low efficiency. In the sense of reducing these, therefore, technology of accurate prediction and estimation of cavitation is very important in development of high-speed fluid devices.

Lately, the present author and co-workers have proposed numerical methods for cavity flow [1,2] with a homogeneous equilibrium model [3] taking account of the compressibility of the gas-liquid two-phase media.

The purpose of this paper is to verify an applicability of the preconditioning solution method by the author [1] to treat incompressible flow characteristic at low Mach number in cavity flows. As the numerical examples, two-dimensional (2-D) internal flows through a backward-facing step duct and a venturi tube are simulated. Detailed cavity flow behavior is investigated and velocity and pressure distributions obtained by the present preconditioned and non-preconditioned solution method are compared with experimental data.

2. Fundamental Equations

Based on the homogeneous cavitation model concept, the 2-D governing equations for the mixture mass, momentum, energy and the gas-phase mass conservation can be written in the curvilinear coordinates (ξ, η) as follows [1,4]:

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial \xi} + \frac{\partial \boldsymbol{F}}{\partial \eta} = \frac{\partial \boldsymbol{E}_v}{\partial \xi} + \frac{\partial \boldsymbol{F}_v}{\partial \eta} + \boldsymbol{S} \tag{1}$$

where $\boldsymbol{Q}$ is the unknown variable vector, $\boldsymbol{E}$, $\boldsymbol{F}$ are flux vectors and $\boldsymbol{E}_v$, $\boldsymbol{F}_v$ are viscous terms, and $\boldsymbol{S}$ is the source term.

$$\boldsymbol{Q} = J \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \\ \rho Y \end{pmatrix}, \quad \boldsymbol{S} = J \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ S_e - S_c \end{pmatrix}, \quad \boldsymbol{E} = J \begin{pmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ \rho U H \\ \rho U Y \end{pmatrix}, \quad \boldsymbol{E}_v = J \begin{pmatrix} 0 \\ \xi_x \tau_{xx} + \xi_y \tau_{xy} \\ \xi_x \tau_{yx} + \xi_y \tau_{yy} \\ \xi_x T_{11} + \xi_y T_{22} \\ \xi_x \Re Y_x + \xi_y \Re Y_y \end{pmatrix}$$

where, the Jacobian J of the transformation by $J = x_\xi y_\eta - x_\eta y_\xi$. $T_{11} = u\tau_{xx} + v\tau_{xy} + \kappa \partial T/\partial x$, $T_{22} = u\tau_{yx} + v\tau_{yy} + \kappa \partial T/\partial y$ and κ is the coefficient of thermal conductivity. Also, $\Re$ is effective exchange coefficient, Se and Sc are rate of evappration and condensation, respectively.

3. Preconditioning Formulation

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The hydraulic flow including cavitations can be characterized as non-linear, viscous flow with laminar and turbulent regions. Also this flow with hydraulic transients and hydroacoustics presents the compressible flow characteristic at low Mach number. For such flow, compressible flow model with preconditioning method [1,5,6] is advantageous.

Applying the preconditioning method to Eq.(1), we obtain a 2-D preconditioned governing equations with unknown variable vectors $\boldsymbol{W} = [p, u, v, T, Y]^T$ written in the curvilinear coordinates as follows:

$$\Gamma^{-1}\frac{\partial \boldsymbol{W}}{\partial \tau} + \Gamma_w^{-1}\frac{\partial \boldsymbol{W}}{\partial t} + \frac{\partial(\boldsymbol{E} - \boldsymbol{E}_v)}{\partial \xi} + \frac{\partial(\boldsymbol{F} - \boldsymbol{F}_v)}{\partial \eta} = \boldsymbol{S} \tag{2}$$

In this study, the preconditioning matrix Γ^{-1} is formed by the addition of the vector $\theta[1, u, v, H, Y]^T$ to the first column of the Jacobian matrix $\partial \boldsymbol{Q}/\partial \boldsymbol{W}$. τ is pseudo-time and Γ_w^{-1} is a transform matrix. In this paper, the preconditioned governing equations (2) are numerically integrated by using a finite-difference method of dual time-stepping integration procedure. And then the Roe's flux difference splitting method with the MUSCL-TVD scheme is applied to enhance the numerical stability, especially for the existence of steep gradient in density as well as pressure near the gas-liquid interface.

4. Numerical Results

As the numerical examples, 2-D internal flows through a venturi tube and decelerating cascades are simulated. And then, detailed cavity flow behavior including the growing and shedding of the cavity for the above flow fields are investigated. Velocity and pressure distributions obtained by the present preconditioned and non-preconditioned solution method are compared with experimental data. Figure 1 shows a time evolution of cavity flow (void fraction) through a convergent-divergent nozzle [7]. The sheet cavitation occurred near throat is developing and flowing toward downstream. Details are referred to the full paper.

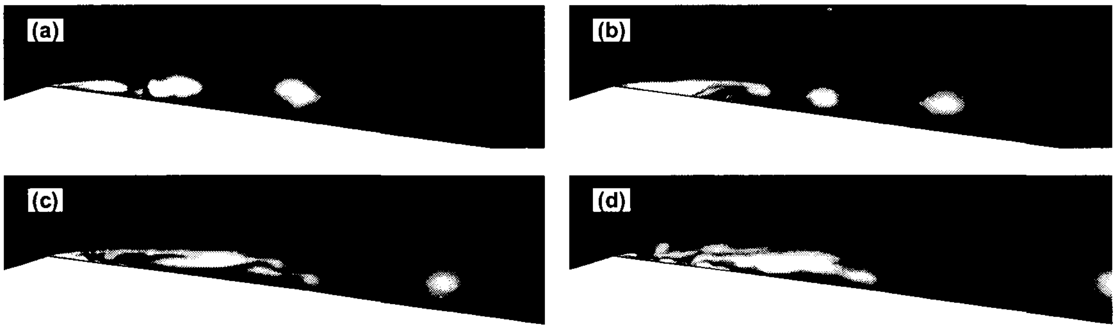

Fig.1. Time evolution of cavity flow (void fraction)

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