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Method for Accurate Calculation of Multi Dimensional Flow
(Variable Interpolation Method for Roe's FDS)

Hyung Mo Koo [1], *Kyu Hong Kim* [2], *Chongam Kim* [3], *Oh-Hyun Rho* [4]
1.Department of Aerospace Engineering Seoul National University,151-742, Korea.Hmkoo_75@hotmail.com
2. Seoul National University.Aerocfd1@hotmail.com
3. Seoul National University. Chongam@plaza.snu.ac.kr
4. Seoul National University. Aero@snu.ac.kr

Corresponding author ***Chongam Kim***

Abstract

In order to reduce numerical dissipation in a multi-dimensional flow, a new variable interpolation method for Roe's FDS is proposed. By introducing the Mach number weighting function w, the properties at the cell-interface are interpolated and in a non flow-aligned grid system, it can give more accurate solution because of less numerical dissipation. Various test cases including oblique contact discontinuity are simulated and compared with the results of original Roe's FDS.

Keyword: *Roe's FDS, Contact Discontinuity, MUSCL*

1. A new interpolation method for Roe's FDS

1.1 Multi dimensional flow problem

If a grid line were aligned with a discontinuity, the problem would be consistent with one-dimensional flow problem and the schemes such as Roe's FDS, AUSM+ or AUSMPW+ produce no numerical dissipation across a discontinuity.[1][2][4] So, it can be maintained. On the other hand, if a discontinuity is inclined to a grid line, numerical dissipation is certainly generated across a discontinuity and as a result, a discontinuity will be smeared diffusively. In this reason, to get satisfactory results, more accurate scheme needed. The present method is to model the cell-interface property to reduce numerical dissipation in the multi dimensional flow problem like oblique discontinuity.

1.2 Definition of Mach number weight function

The function w is a kind of indicator to inform where a shock is located. Shock location is the important information for the interpolation because whether the flow is supersonic or subsonic can cause the numerical instability. This means the direction of information should be considered in the interpolation for the accurate solution as well as robustness. The Mach number weighting function is defined as follows.

$$w = \left[\min\left(1, \max\left(|M_L|, |M_R|\right)\right)\right]^2 \tag{1}$$

1.3 Definition of property at a cell-interface

The property at the cell-interface is defined as follows in reflecting the different phenomena of supersonic or subsonic flow according to Mach number weighting function defined in the section 1.2.

$$\Phi_{L,\frac{1}{2}} = 0.5(1+w)\Phi_L + 0.5(1-w)\Phi_R \tag{2.a}$$

$$\Phi_{R,\frac{1}{2}} = 0.5(1+w)\Phi_R + 0.5(1-w)\Phi_L \tag{2.b}$$

where $\Phi = (\rho \;\; \rho u \;\; \rho v \;\; \rho H)^T$ and L,R means the left and right of the cell-interface.

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But the interpolated values in Eq.(2)can violate the monotonicity, another condition needed for the method to be monotone. That condition comes from MUSCL, which can provide limiting values of newly defined properties.

$$\Phi_{L,\lim} = \Phi_i + \frac{1}{4}\left[(1-\kappa)\phi\left(\frac{\Phi_{i+1}-\Phi_i}{\Phi_i-\Phi_{i-1}}\right)(\Phi_i-\Phi_{i-1}) + (1+\kappa)\phi\left(\frac{\Phi_i-\Phi_{i-1}}{\Phi_{i+1}-\Phi_i}\right)(\Phi_{i+1}-\Phi_i)\right]_{i\,or\,j} \quad (3.a)$$

$$\Phi_{R,\lim} = \Phi_{i+1} - \frac{1}{4}\left[(1+\kappa)\phi\left(\frac{\Phi_{i+2}-\Phi_{i+1}}{\Phi_{i+1}-\Phi_i}\right)(\Phi_{i+1}-\Phi_i) + (1-\kappa)\phi\left(\frac{\Phi_{i+1}-\Phi_i}{\Phi_{i+2}-\Phi_{i+1}}\right)(\Phi_{i+2}-\Phi_{i+1})\right]_{i\,or\,j} \quad (3.b)$$

superbee limiter:

$$\phi(r) = \max(0, \min(2r,1), \min(r,2)) \quad (4)$$

where $\Phi_{L,\lim}$, $\Phi_{R,\lim}$ are the limiting value of the left and right side of the cell-interface. Finally the properties at the cell-interface follow.

If $(\Phi_{L,R} - \Phi_{L,R\frac{1}{2}})(\Phi_{L,R\frac{1}{2}} - \Phi_{L,R\,\lim}) \le 0$

$$\Phi_{L,R\frac{1}{2}} = \Phi_{L,R\,\lim} \quad (5.a)$$

elsewhere

$$\Phi_{L,\frac{1}{2}} = 0.5(1+w)\Phi_L + 0.5(1-w)\Phi_R \quad (5.b)$$

$$\Phi_{R,\frac{1}{2}} = 0.5(1+w)\Phi_R + 0.5(1-w)\Phi_L \quad (5.c)$$

1.3 Interpolation method for Roe's FDS

The numerical flux of Roe's FDS at the cell-interface is written as follows.

$$F_{i+\frac{1}{2}} = \frac{1}{2}\left[F_i + F_{i+1} - |\hat{A}|\Delta Q\right] \quad (6.a)$$

$$F_i = \begin{pmatrix} \rho U \\ \rho u U + n_x p \\ \rho v U + n_y p \\ \rho U H \end{pmatrix}_i \quad (6.b)$$

$$|\hat{A}|\Delta Q = |\Delta\hat{F}|_1 + |\Delta\hat{F}|_2 + |\Delta\hat{F}|_3 + |\Delta\hat{F}|_4 \quad (6.c)$$

where $|\hat{A}|\Delta Q$ is the matrix dissipation that characterize the Roe's FDS and decomposed as follows

$$|\Delta\hat{F}|_1 = |\hat{U}|\left(\Delta\rho - \frac{\Delta p}{\hat{c}^2}\right)\begin{pmatrix} 1 \\ \hat{u} \\ \hat{v} \\ \dfrac{\hat{u}^2+\hat{v}^2}{2} \end{pmatrix}, \quad |\Delta\hat{F}|_2 = |\hat{U}|\hat{\rho}\begin{pmatrix} 0 \\ \Delta u - n_x\Delta U \\ \Delta v - n_y\Delta U \\ \hat{u}\Delta u + \hat{v}\Delta v - \hat{U}\Delta U \end{pmatrix}$$

$$\left|\Delta\hat{F}\right|_{3,4} = \left|\hat{U} \pm \hat{c}\right| \left(\frac{\Delta p \pm \hat{\rho}\hat{c}\Delta U}{2\hat{c}^2} \right) \begin{pmatrix} 1 \\ \hat{u} \pm n_x \hat{c} \\ \hat{v} \pm n_y \hat{c} \\ \hat{H} \pm \hat{c}\hat{U} \end{pmatrix} \tag{7}$$

U indicates contravariant velocity and n_x, n_y are unit normal vector components at the cell interface.^ means Roe averaged values at the cell interface and c is the speed of sound. From section 1.3, the newly defined cell-interface properties are used to determine $\Delta\rho, \Delta u, \Delta v$ in the Eq.(7). This represents less numerical dissipation in the numerical flux at Eq.(6.a) for the oblique contact discontinuity problem.

2. Numerical Results for Oblique Contact Discontinuity (Inviscid)

Initial conditions which represents the contact discontinuity inclined about 45 degree is as follows.

$$(\rho_L, u_L, v_L, p_L) = (2.0, 0.1, 0.1, 0.714)$$
$$(\rho_R, u_R, v_R, p_R) = (1.0, 0.1, 0.1, 0.714)$$

Fig.1 and 2 show the results of present method and original Roe's FDS with MUSCL approach. As shown in the Fig.1, present method can capture the oblique contact discontinuity through 10 cells while the original Roe's FDS through 18 cells. It represents the present method can calculate the oblique contact discontinuity more accurately than the original Roe's FDS scheme.

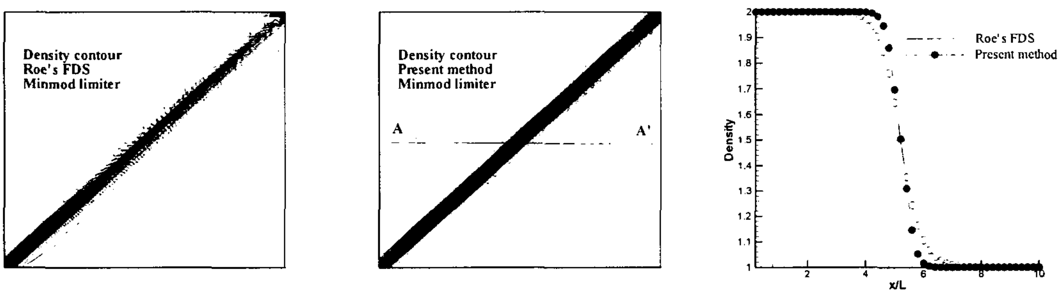

Fig. 1 Density distributions of original Roe's FDS and the present method (minmod limiter)

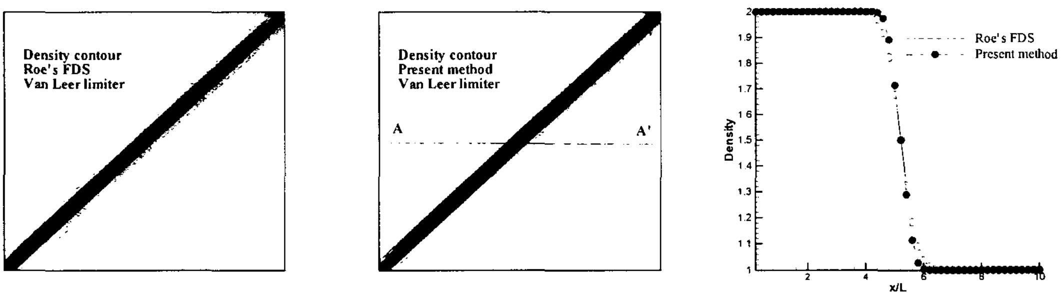

Fig. 2 Density distributions of original Roe's FDS and the present method (Van Leer limiter)

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