2-D Robust Design Optimization on Unstructured Meshes

Sang Wook Lee [1], *Oh Joon Kwon* [2]
1. *Graduate Research Assistant, KAIST, princelee@kaist.ac.kr*
2. *Associate Professor, KAIST, ojkwon@mail.kaist.ac.kr*

Corresponding author ***Oh Joon Kwon***

Abstract

A method for performing two-dimensional lift-constraint drag minimization in inviscid compressible flows on unstructured meshes is developed. Sensitivities of objective function with respect to the design variables are efficiently obtained by using a continuous adjoint method. In addition, parallel algorithm is used in multi-point design optimization to enhance the computational efficiency. The characteristics of single-point and multi-point optimization are examined, and the comparison of these two method is presented.

Keyword: *Shape Optimization, Unstructured Mesh, Continuous Adjoint Method, Multi-point design.*

1. Introduction

The requirement of high performance with low cost is the main motivation of optimal design. In the early days of aerodynamics, time, cost, and labor intensive wind tunnel test is the main method of design. In the 1980s, Computational Fluid Dynamics(CFD) is recognized as a powerful analysis tool, so CFD begins to used in shape design. Currently, CFD is widely used not only to understand and simulate the complicated flow physics, but also to find optimal shape providing maximum performance in the desired operating condition[1]. Various shape optimization method has been investigated, but the main problem of current optimization is that the best performance is only obtained on the narrow range of operating condition and there are no guarantee on the off-design performance[2].

In the present study, multi-point shape optimization is presented to overcome the localized optimization. The sensitivity of objective function is obtained by continuous adjoint method. In addition, parallel algorithm is used to enhance the numerical efficiency in the case of multi-point design.

2. Numerical Method

2.1 Euler Equations

The governing Euler equations can be written in an integral form over a control volume V:

$$\frac{\partial}{\partial t}\int_V Q dV + \oint_S F(Q,\vec{n})dS = 0 \qquad (1)$$

The inviscid flux, $F(Q,\vec{n})$, across each cell face is discretized by using the second-order accurate Roe's flux difference splitting formular. An implicit time integration algorithm based on the linearized Euler backward differencing is used to derive the solution to steady state. The linear system of equation is solved at each iteration by using a point Gauss-Seidel method.

2.2 Adjoint Equations and Sensitivity Analysis

The objective function for aerodynamic shape optimization is expressed as an integral form about the pressure over the surface :

$$I_C = \int_B g(Q(D)) \cdot k(D) ds \tag{2}$$

where $g(Q(D))$ and $k(D)$ are the functions of flow variables and geometry, respectively. In the adjoint approach, a new cost function is defined as

$$\begin{aligned} I_C &= I_C(Q,D) + \int_\Omega (\Psi, R) d\Omega \\ &= I_C(Q,D) + I_R(R,\Psi) \end{aligned} \tag{3}$$

Where R represents the steady-state flow equations, Ψ is the adjoint variables (Lagrangian multiplier), and D is the vector of design variables. The adjoint equations are obtained by taking the variation of equation(3) and regrouping terms[3]:

$$\frac{\partial \Psi}{\partial t} - A^T \frac{\partial \Psi}{\partial x} - B^T \frac{\partial \Psi}{\partial y} = 0 \tag{4}$$

The vector of adjoint variables at the far field is set to zero and the solid wall boundary condition can be obtained as :

$$k_x \Psi_2 + k_y \Psi_3 + k \frac{\partial I}{\partial p} = 0 \tag{5}$$

Finally, the adjoint sensitivity is expressed as :

$$\delta I = \int_B R(\Psi_1 + u\Psi_2 + v\Psi_3 + H_t \Psi_4) dS + \int_B \tilde{k}(D) g(Q) dS \tag{6}$$

where

$$R = -\tilde{x}\left(\frac{\partial Q_2}{\partial x} k_x + \frac{\partial Q_3}{\partial x} k_y\right) - \tilde{y}\left(\frac{\partial Q_2}{\partial y} k_x + \frac{\partial Q_3}{\partial y} k_y\right) - \left(Q_2 \tilde{k}_x + Q_3 \tilde{k}_y\right) \tag{7}$$

where $\tilde{k}_x, \tilde{k}_y, \tilde{x}, \tilde{y}$ are the variation of the x-, y-component of the surface normal vector, x-, y-coordinate of surface mesh point, respectively.

2.3 Parallel Implementation

In multi-point design case, the gradient and objective function must be computed at each design point. In the present method, each cpu computes gradient and objective function at each design condition, and finally communicate and add each other. This is not only simple, but also very efficient parallel algorithm because the communication time is negligible. Message Passing Interface(MPI) is used to communicate each other. All calculations were made on a PC-based linux-cluster having 2.4GHz CPUs.

3. Results and Discussion

The present method has been applied to a NACA0012 airfoil section at a free stream Mach number of 0.75 and an angle-of-attack of 1.747 degrees. The wave drag is to be minimized by removing the shock wave while maintaining the desired lift. The objective function for this problem is written as :

$$I = \frac{1}{2}\left(C_L - C_{L,0}\right)^2 + \frac{10}{2}\left(C_D\right)^2 \tag{8}$$

Figure 1 shows partial view of the computational mesh composed of 6,375 triangular cells and 3,286 nodes. Ten design variables are used on the upper surface of the airfoil. Sequential optimization algorithm such as BFGS with SQP is used. Figure 2 shows the comparison of pressure distributions. The drag is decreased by reducing the shock strength and increasing the leading-edge suction as

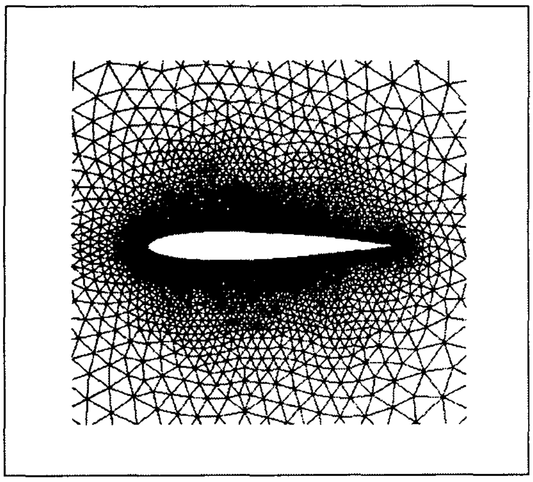

Figure 1. Partial view of computational mesh.

shown in figure 2. Though this newly designed airfoil shows excellent performance in the design point, this advantage is obtained only narrow range of design point. The off-design performance of designed airfoil is very poor as shown in figure 3, due to the absence of the consideration of the off-design performance in the design process.

A straightforward approach to avoid this localized optimization is to consider different Mach numbers and to generalize the objective function to a linear combination of flight condition[4]. Figure 3 shows the comparison of drag profiles. By using a 4-point optimization, much better drag reduction is obtained in the range of Mach number of [0.7, 0.8]. But it is still localized minimization because there is no consideration of flight condition except the 4-design points.

In the final paper, results of the two dimensional robust design optimization in inviscid compressible flows will be given. Detail description of the formulation will also be included in the final paper. The method will be applied to NACA0012 and RAE2822 airfoils and the comparison of single-point, multi-point, and robust design will be included.

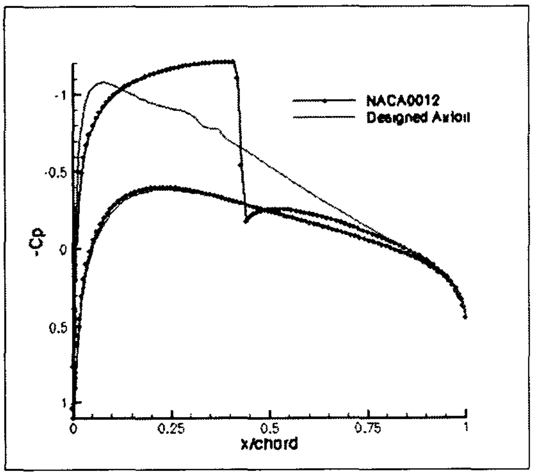

Figure 2. Comparison of pressure distributions.

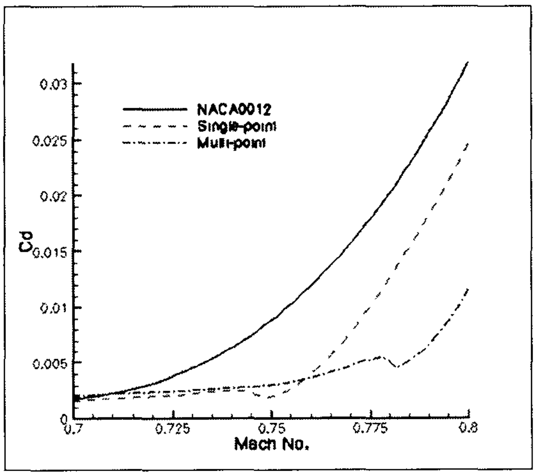

Figure 3. Comparison of drag profiles.

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