Mixed Heat and Fluid Flow in Annuli of Heated Rotating Eccentric Cylinders

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Abstract

A co-ordinate transformation approach coupled with implementation of a non-uniform mesh distribution method is used in the present work to study the mixed heat and fluid circulation in an annular region between two eccentric cylinders with the inner cylinder heated and rotating. A numerical "fluid leakage" prevention method is proposed in the present solution. In the present study, a net circulation around the inner cylinder defined by an inner cylinder streamline value, $\psi_i = f(\mathrm{Re})$, which is determined by using the criterion that the pressure distribution in the solution region is a single-valued function. Mathematically, this criterion implies that the line integral of the pressure gradient $\frac{\partial P}{\partial s}$ along any closed loop circumscribing the inner cylinder is zero i.e. $\oint\left(\frac{\partial P}{\partial s}\right)ds = 0.0$. Local values of $\left(\frac{\partial P}{\partial s}\right)_{\eta=0}$ were evaluated from the momentum conservation equations. Previously, the net circulation was obtained through a tedious process of expanding the global flow field through a GDQ expansion series [Hu, Lee & Shu (2000)].

The Prandtl number (Pr) considered here varies from 0.01 to 1.0 and Rayleigh number (Ra) is of the order of 10^5. Reynolds number Re in the range of 0 to 1120 was considered. Mono-thermal plume above the stationary inner was observed for higher Prandtl number fluids (Pr>>0.1) while bi-thermal plume above the stationary inner cylinder was observed for lower Prandtl number fluids (Pr<<0.1). However, when the inner cylinder is made to rotate, the thermal plume for higher and lower Prandtl number fluids were observed to move in different directions. The mechanism of the mono- and bi-thermal plumes movements were investigated through numerical flow visualization.

Modes of fluid circulation and Heat Transfer in the annuli of stationary concentric horizontal cylinders of different fluids were first investigated by Kuehn and Goldstein's (1980). For the present study, the inner cylinder is assumed heated and rotating at Reynolds number in the range of 0 to 1200. The Prandtl number considered here varies from 0.01 to 1.0 and Rayleigh number varies from 10^3 to 10^5. In the present study, natural convection is driven by vertical temperature gradient and vertical gravity force. The complex fluid flow resulted through the interaction with the effect of rotational rate of the inner eccentric cylinder is considered in this study.

With the inner cylinder rotating in the counter clockwise direction, the present study showed that, any cell on the right of the inner cylinder will be dragged upwards by virtue of the viscous drag. At the same time, any cell on the left of the inner cylinder is being dragged downward by the viscous action of the inner cylinder rotation. The hot fluid on the ascending side of the inner cylinder therefore rises even faster while on the descending side, the rising hot fluids are slowed down. Near the descending side, the induced viscous drag balanced off the thermal induced motion and a region of very low velocity is observed. The isotherms, as a result, are tilted in the direction of rotation from its corresponding stationary cases.

For fluid with Pr of the order of 0.02(Mercury), symmetrical bi-thermal plume was observed above the inner cylinder for the case of stationary inner cylinder. Two secondary cells were formed in addition to the main cells. Below these two cells, two small bubbles which correspond to the reattachment of the boundary layer are observed. The flow pattern in the annulus is thus observed to be multi-cellular. The heat transfer characteristics for these fluid flow with low Prandtl number were found to have points of maximum and minimum at the interior nodes, instead of the top and bottom nodes which is known for the higher Prandtl number fluids. The immediate effect of rotating the inner cylinder is to cause the two smaller bubbles to collapse. Due to the low viscosity (Pr=0.02), the viscous drag induced is not able to eliminate the two secondary cells. The disappearance of the two bubbles has marked effect on the flow field: a low pressure region is created in the vicinity of the location where the bubbles collapsed. Thus the two secondary cells, despite of the induced viscous drag, move in the direction to occupy this low pressure region. In this case, the multicellular flow with its bi-thermal plume on one side of the annulus for low Prandtl number fluid is suppressed. The net effect is that a single thermal plume is observed to move in the direction opposite to that of the inner rotating cylinder. This thermal plume movement is different and opposite to that observed for the higher Prandtl number (Pr>>0.10) fluid flow where the mono-thermal plume on top of the inner cylinder moves in the same direction as the rotation of the inner cylinder. The mean Nusselt number characteristics for various eccentric annular regions are also obtained in this study. Other related studies are given by Lee *et al* (1984-2002).

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