한국경영과학회/대한산업공학회 2003 춘계공동학술대회
2003 년 5 월 16 일-17 일 한동대학교(포항)

The Fundamental Understanding Of The Real Options Value Through Several Different Methods

Kim, Gyutai [1] **and Choi, Sungho** [2]

1) Department of Industrial Engineering, Chosun University
Tel.: 062-230-7122
e-mail: gtkim@mail.chosun.ac.kr

2) Department of Industrial and System Engineering, Kangnung National University
Tel.: 033-640-2374
e-mail: shchoi@kangnung.ac.kr

ABSTRCT

The real option pricing theory has emerged as the new investment decision-making techniques superceding the traditional discounted cash flow techniques and thus has greatly received much attention from academics and practitioners in these days. The theory has been widely applied to a variety of corporate strategic projects such as a new drug R&D, an internet start-up, an advanced manufacturing system, and so on.

A lot of people who are interested in the real option pricing theory complain that it is difficult to understand the true meaning of the real option value, though. One of the most conspicuous reasons for the complaint may be due to the fact that there exit many different ways to calculate the real options value. In this paper, we will present a replicating portfolio method, a risk-neutral probability method, a risk-adjusted discount rate method (quasi capital asset pricing method), and an opportunity cost concept-based method under the conditions of a binomial lattice option pricing theory.

INTRODUCTION

In a real option pricing theory, a various kind of methodologies and approaches are used to calculate an option value. They may be lattice models (ex. binomial, trinomial, quadranomial, and multinomial trees), closed-form models like the Black-Scholes model, partial differential equations path dependent simulation methods like Monte Carlo simulation, and so on. Figure 1 shows that each of them possesses its own advantages over the others.

In this paper, among the methods we will concentrate on the binomial lattice option pricing (BLOP) method that is recognized as one of the methods most widely used to calculate the real option values. As described in Figure 1, binomial lattice option pricing methods are easy to implement and easy to explain. They are also highly flexible but require significant computing power and time-steps to obtain good approximations. It is important to note, however, that in the limit, results obtained through the use of binomial lattice option pricing methods tend to approach those derived from closed-form solutions, and hence, it is always

	Discrete-Time	Continuous-Time		
Method	Lattice Model	Closed-Form	Differential Equations	Simulation
Advantage	♦ Intuitively Appealing ♦ Flexible	♦ Simplified Calculations ♦ Straightforward	♦ Model Flexibility ♦ Mathematically Accurate	♦ Adaptable ♦ Broad Applicability
Disadvantage	♦ Cumbersome ♦ Labor Intensive	♦ Limiting Assumption ♦ Limited Applicability	♦ Approximate Solutions ♦ Complicated	♦ Required Special Skills ♦ Potential Misuse

Figure 1. A various real option value calculation models

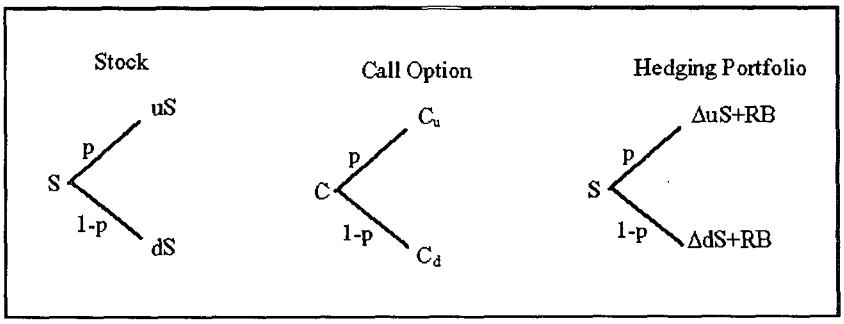

Figure 2. One time period BLOP model

recommended that both approaches be used to verify the results.

In the binomial lattice world, several basic similarities are worth mentioning. No matter the types of real options problems you are trying to solve, if the binomial lattice option pricing method is used, the solution can be obtained in one of the following four ways : i) a replicating portfolio method, ii) a risk-neutral method, iii) a risk-adjusted discount rate method, and iv) an opportunity cost concept-based method. The first two methods have been well known to us, while the last two ones have not yet almost been introduced. We will present each of them focusing on the implemental aspect rather than the theoretical one with a numerical example.

THE BLOP METHOD

The BLOP method for valuing stock options is a discrete-time period model and explains the fundamental economic principle of option valuation by the arbitrage method quite clearly. Sharp W. first suggested the binomial approach and Cox, Ross, and Rubinstein (1979) developed the BLOP. They showed that the Black-Scholes model could be obtained through limiting the BLOP. The option value provided by the BLOP method is a good approximation to one provided by the Black-Scholes model.

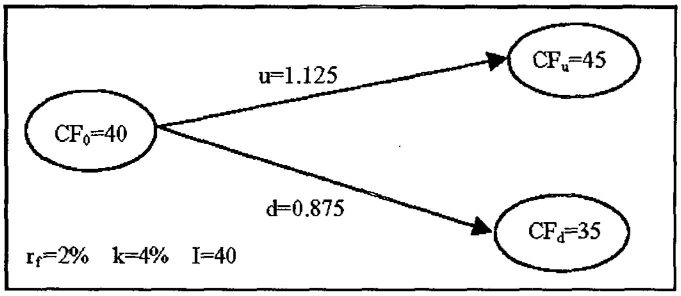

Figure 3. A binomial lattice model for the numerical example

It is assumed in the BLOP model that a binomial process is multiplicative over discrete time period and there exist only two discrete events at each time period. One happens at which the rate of return on stock is assumed to be u-1 with probability of "p", the other does at which it is assumed to be d-1 with probability of "1-p". If the current stock price is S at the end of one period, the stock price would either go up to uS with probability of "p" or go down to dS with probability of "1-p". This process is repeated over the following time period. In general at the end of "i" periods the stock price would have "i+1" values given by "$u^j d^{i-j} S$" for j=0,1,2,….,i. Figure 2 shows one time period BLOP model.

FOUR DIFFERENT REAL OPTION VALUE CALCULATION METHODS

The four methods (a replicating portfolio method, a risk neutral probability method, a risk-adjusted discount rate method, and an opportunity cost concept-based method) will be explained referring to the numerical example depicted in Figure 3. For convenience, we will first introduce the definition of the notations used throughout the paper.

u: a fixed ratio that a cash flow moves up.
d: a fixed ratio that a cash flow moves down.
p: an objective probability.
q: a risk-neutral probability.
r_f: a risk-free discount rate.
k: a risk-adjusted discounted rate for a project.
r: a discount rate for a call option.
Δ: a perfect hedging ratio.
I : the initial investment.
B : the amount of money borrowed.
V : the present value of the investment future cash flows

Consider the real investment project requiring the initial investment of $40 million. And its present value of the investment project is also $40 million and its planning horizon is only one year. Depending on the economic condition of the next year, its value will be either $45 million or $35 million. If the economic condition of the next year turns out to be favorable, the project value will increase by 12.5% resulting in $45 million. Otherwise, its value will decrease by 12.5% resulting in $35 million. The risk-adjusted discounted rate for the project and the risk-free discounted rate are given 4% and 2%, respectively. The question that arises now is whether or not to delay executing the investment projects assuming that the project can be delayed by one year only.

To answer this question, we need to calculate the traditional net present value (NPV) and the strategic net present value (SNPV). The SNPV represents the true value of the investment opportunity. If SNPV> NPV holds, then it can be said that that it is desirable

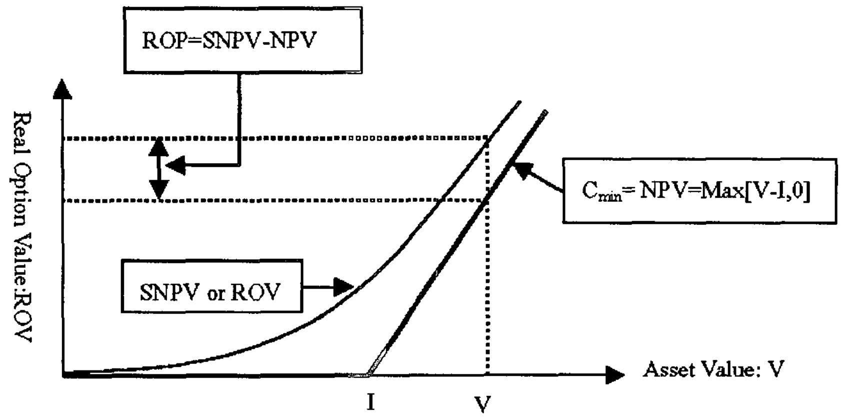

Figure 4. Real option value diagram

to delay executing the investment project. The difference between the SNPV and the traditional NPV is called " the real option premium (ROP)" or " the value of flexibility". The relationship among the SNPV, the traditional NPV, the present value of the investment future cash flows (V), and the ROP is well documented in Figure 4 (for more detail, refer to Herath).

i) Replicating Portfolio Method

This section is based on Hull. The predominant assumptions behind replicating portfolio methods are that there are no arbitrage opportunities and that there exist a number of traded assets in the market that can be obtained to replicate the existing asset's payout profile.

To implement the replicating portfolio methods, we need to set up a portfolio of the traded-stock and the option in such a way that there is no uncertainty about the value of the portfolio at the end of maturity. Then, it can be argued that the return earned on the portfolio must equal the risk-free discount rate because it has no risk. This enables us to work out the cost of setting up the portfolio and, therefore, the option's price. Because there are two securities (the stock and option) and only two possible outcomes, it is always possible to set up the riskless portfolio.

Consider a portfolio consisting of a long position in Δ shares of the stock (the underlying investment project) and a short position in one call option. We calculate the value of Δ that makes the portfolio riskless. If the value of the investment project goes from \$40 million up to \$45 million, the value of the investment project is 45Δ and the value of the option is 5, so that the total value of the portfolio is 45Δ-5. If its value goes down to \$35 million, the total value of the portfolio is 35Δ. The portfolio is riskless if the value of Δ is chosen so that the final value of the portfolio is the same for both alternatives. It means

$$45\Delta - 5 = 35\Delta,$$

$$\Delta = 0.5$$

A riskless portfolio is, therefore

Long : 0.5 shares

Short: 1 option

If the value of the investment project moves up to \$45 million, the value of the portfolio is

$$45 \times 0.5 - 5 = 17.5$$

If the value of the investment project moves down to \$35 million, the value of the portfolio is

$35 \times 0.5 = 17.5$

Regardless of whether the value of the investment project moves up or down, the value of the portfolio is always \$17.5 million at the end of the option's maturity.

Riskless portfolio must, in the absence of arbitrage opportunities, earn the risk-free interest rate. Since the risk-free interest rate was given assumingly to be 2%, the value of the portfolio today must be the present value of \$17.5 million.

$$\frac{17.5}{1.02} = 17.157$$

Since the value of the stock price today is known to be \$40, the option value, denoted by "C", is

$40 \times 0.5 - C = 17.157$

$C = 2.84$

Equivalently, we can obtain the corresponding option value solving the following two equations.

$$45\Delta + 1.02B = 5 \quad (1)$$

$$35\Delta + 1.02B = 0 \quad (2)$$

Solving Equation (1) and (2) with respect to Δ and B, we can obtain the followings:

Δ=0.5 B=-17.15686

Plugging these values into the following equation, we can get the option value of 2.84

$C = \Delta CF_0 + B = (0.5)(40) - 17.15686 = 2.84$

Based on the option value of 2.84 obtained, we can infer the implied discount rate for the option under consideration. To find out it, we need to first calculate the objective probability of "p". It can be found solving the following equation:

$$p = \frac{(1+k)-d}{(u-d)} = \frac{(1.04)-0.875}{(1.125-0.875)} = 0.66$$

$$C = \frac{C_u \cdot p + C_d \cdot (1-p)}{(1+k)}$$

$$= \frac{(5)\cdot(0.66)+(0)\cdot(0.34)}{(1+k)} = 2.843$$

$k = 16.075\%$

The value of the implied discount rate of 16.075% was indirectly found in this method, but it will be directly found in the third method. It should be noted that the objective probability of "p" has to be compatible with the risk-adjusted discount rate of "k".

ii) Risk-Neutral Probability Method

In a risk-neutral world all individuals are indifferent to risk. They require no compensation for risk, and the expected return on all securities is the risk-free interest rate. Equation (3) shows that the value of the option is its expected payoff in a risk-neutral world discounted at the risk-free discount rate.

$$C = \frac{C_u \cdot q + C_d \cdot (1-q)}{(1+r_f)} \quad (3)$$

The value of a risk neutral probability in Equation (3) is found out solving the following equation with respect to "q".

$$r_f = q \cdot (u-1) + (1-q)\cdot(d-1)$$

$$q = \frac{(1+r_f)-d}{u-d}$$

The risk-neutral probability is not true probability associated with the asset price movements. Rather a risk adjustment has been implicitly made to the probabilities (rather than the discount rate) such that the discounted risk-neutral expected value produces the correct option value.

$$q = \frac{(1.02)-0.875}{1.125-0.875} = 0.58$$

$$C = \frac{5 \cdot 0.58}{1.02} = 2.84$$

Here, we used the risk-free discount rate compatible for the risk-neutral probability. If we followed the same logic, we might use the risk-adjusted discount rate for the project as a pair for the objective probability and would have the distorted real option value as following;

$$C = \frac{5 \cdot 0.66}{1.04} = 3.17$$

As we note, this is different from those we obtained before. The main reason for this distortion is due to the fact that we have the different risk profile for the project cash flows, which is caused depending on whether or not to take the option into consideration.

It should be noted that the replicating portfolio method discounts the expected cash flows at a risk-adjusted discount rate for an option. Conversely, the risk-neutral probability method discounts a certain-equivalent cash flow adjusted by a risk-neutral probability at a risk-free discount rate.

iii) Risk-Adjusted Discount Rate Method

In the previous method, we calculated the option value using a risk-neutral probability with its corresponding risk-free discount rate. The question that arises now is whether an approach based on a risk-adjusted discount rate can be made to work when valuing options. With a replicating portfolio method we found out the risk-adjusted discount rate implied in the option value after we calculated the option value.

Here, the risk-adjusted discount rate is not for a stock, but for an option. In this paper, we differentiate the risk-adjusted discount rate for a stock (investment project), denoted by "k", from the option's one, denoted by "r". Kim, Gyutai (2001), and Hodder, Mello and Sick (2001) proposed the mathematical equation to find out the risk-adjusted discount rate for an option. One of them is Equation (4) developed by Kim, Gyutai. Using Equation (4), we obtained the risk-adjusted discount rate for the option of 16.069% and derived the option value with this discount rate.

$$r = r_f + (k - r_f)\frac{(C_u - C_d)/(u-d)}{q \cdot C_u + (1-q) \cdot C_d} \cdot (1 + r_f) \qquad (4)$$

$$r = 0.02 + (0.04 - 0.02)\left\{\frac{(5-0)/(1.125-0.875)}{(0.58)\cdot(5) + (0.42)\cdot(0)}\right\} \cdot (1.02)$$

$$= 16.069\%$$

$$C = \frac{5 \cdot 0.66}{1.16069} = 2.84$$

It should be noted that the compatible probability with this discount rate is the objective probability instead of the risk-neutral probability, and the objective probability does not work with the risk-adjusted discount rate for a stock(investment project). Fore more detail discussion on this topic, refer to Hodder, Mellor, and Sick (2001), and Kim (2001). Hodder, et. al. and Kim derived the same equation in a different approach.

In general, a risk-adjusted discount rate method requires more information than the previous tow methods. . Since an option's riskiness depends on the current stock price and the time left until the option matures, it differs at each node. Therefore, a risk-adjusted discount rate method requires a risk-adjusted discount rate for each node, while a risk-neutral probability method does not ask for a risk-neutral probability for each node.

iv) The Opportunity Cost Concept Method

It is usually said that understanding the option pricing theory as non-financing people is difficult mainly because it was originated in financing and most related research have been done by financing people. Kim, GT, et. al. Developed another way to calculate an option value using a familiar concept to non-financing people like an opportunity cost one.

To the end, they decomposed the option value into three opportunity costs : an interest earning opportunity (IEO), an opportunity loss (OL), and an expected opportunity gain (EOG).

When the company decides to delay making the investment by one year, three things are expected to happen. First, the company does not have to spend the investment cost, I, today. Therefore, this investment cost can be diverted to alternative projects or interest-bearing accounts, until a decision is made on the project at hand. Let's assume that the delayed I earns the risk-free interest rate, r_f, for one year. This interest earned will be called the interest earning opportunity. Secondly, the company will not be providing its products or services during this delayed first year and thus loses its revenue opportunity. The amount of this loss will be termed the opportunity loss. The OL is calculated as the present value of the expected future cash flows. Thirdly, the deferral option allows the company to avoid making a loss on their investment if the project value is unfavorable at the end of year one. The savings from not investing in an unfavorable project will be termed the expected opportunity gain. Therefore, the ROP for a deferral option can be expressed as:

$$ROP = IEO - OL + EOG$$

$$C = NPV + \left\{ \frac{ACF_u \cdot q + ACF_d \cdot (1-q)}{(1+r_f} \right\} + \frac{I \cdot rf}{(1+rf)}$$

$$- \frac{ACF_u \cdot q + ACF_d \cdot (1-q)}{(1+r_f)}$$

$$+ \frac{(I - \frac{ACF_d}{r_f}) \cdot (1-q)}{(1+r_f)}$$

$$= 0 + \left\{ \frac{0.9 \cdot 0.58 + 0.7 \cdot 0.42}{1.02} \right\} + \frac{40 \cdot 0.02}{1.02}$$

$$- \frac{(0.9) \cdot (0.58) + (0.7) \cdot (0.40)}{(1.02)}$$

$$+ \frac{\left(40 - \frac{0.7}{0.02}\right)\square(0.42)}{(1.02)}$$

=0+0.8+0.784-0.8+2.058=2.84

where

$ACF_u = CF_u \bullet r_f$

$ACF_d = CF_d \bullet r_f$

In general, the opportunity cost concept-based model is divided into three groups according to the relationship among the initial investment, the upward cash flow, and the downward cash flow. For the demonstration purpose only, we showed only one of these three models.

CONCLUDING REMARKS

The real option pricing theory has received much attention from academics and practitioners for valuing the real investment project. However, mainly due to the fact that the real option pricing theory originated from the financial option pricing theory, most of people interested in the real option pricing theory have difficulty in understanding what the ROV really means.

To ease the difficulty, we presented four different methods to calculate the ROV under the binomial lattice environment. Each method has its own characteristics over the others. Specially, the opportunity cost concept-based method was presented with the terminology very familiar to most of people who have the basic knowledge on the investment appraisal. As the related future research, we need to investigate the relationship among four methods more deeply.

REFERENCE

Copeland, Tom and Antikarov, Vladimir (2001), Real Options: A Practitioner's Guide, Texere, NY

Cox, John C., Rubinstein, Mark (1985), Options Markets, Prentice-Hall, NJ.

Herath, H.S.B., (1998), On Application of Option Pricing Theory to Engineering Economic Decision under Uncertainty, Auburn University.

Hodder, James E, Mello, Antonio S., and Sick, Gordon (2001), Valuing Real Options: Can Risk-Adjusted Discounting Be Made to Work?, Journal of Applied Corporate Finance, 14(2), 90-101.

Hull, John C, Options (2000), Futures, and Other Derivatives, Prentice-Hall, NJ.

Kim, Gyutai (2001), The Relationship Between A Real Option Value and Discount Rate, Working Paper, Auburn University.

Kim, Gyutai, Miller Luke, and Kim, Yoonbai (2002), Calculating The Strategic Net Present Value (*SNPV*) For A Single Period Deferral Based On The Opportunity Cost Concept, Submitted to IEEE Transactions On Engineering Management(On Review).