위상복잡도 조절을 위한 설계 해상도 계층적 제어 기법

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Hierarchical design resolution control scheme for the systematic generation of optimal candidate designs having various topological complexities

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Key Words : Topological Complexity, Topology Optimization, Multiscale, Multiresolution, M-band Wavelet

Abstract

In many practical engineering design problems, there are some design and manufacturing considerations that are difficult or infeasible to express in terms of an objective function or a constraint. In this situation, a set of optimal candidate designs having different topological complexities, not just a single optimal design, is preferred. To generate systematically such design candidates, we propose a hierarchical multiscale design resolution control scheme. In order to adjust its topological complexity by choosing a different starting resolution level in the hierarchical design space, we propose to employ a general M-band wavelet transform in transforming the original design space into the multiscale design space.

1. Introduction

In topology optimization (see, e.g., Bensøe and Sigmund, 2003), the dependence of the topological complexity on the finite element mesh density is well known (Cheng and Olhoff, 1981). In typical compliance minimization problems, the use of finer meshes results in optimal designs with smaller-sized members with more complex connectivity.

Some investigations on the minimum length control and the mesh-dependency of the final topology are well summarized by Bensøe and Sigmund (2003). Recently, to suppress checkerboard and hinge patterns, a wavelet-based method, called the translation-invariant differentiable wavelet shrinkage (Yoon *et al.*, 2003), was also developed. Yoon and Kim (2003) extended the shrinkage technique to control feature size.

The motivations of the present investigation on topology complexity are the followings. In real engineering design practice, it is seldom possible to specify the desired degree of topological complexity as numerical values and often undesirable to preset the minimum member size before knowing various candidate designs. Besides, a single optimal solution maximizing the structural performance may not be the best design as it often causes manufacturing difficulties or yields some reliability problems. In this situation, designers look for several candidate designs having various levels of topological complexity while allowing some sacrifice in the structural performance. Motivated by this need, we aim at developing a new efficient topology design method to yield systematically a set of candidate designs having different topological complexities.

The main idea of this work is to generate the hierarchical multiscale design space having various refinement levels through the *M*-band Haar wavelet transform ($M \geq 2$). We employ 4-band, 8-band and 16-band wavelet transforms to transform the single-scale design space to the multiscale design space. As a means to quantify the topological complexity, the perimeter

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measure suggested by Haber *et al*. (1996) will be used.

For case studies, we consider two classes of topology design optimization problems: the compliance minimization problem and the compliant mechanism design problem. We use the conventional performance measures, but employ the non-conforming finite element (Jang *et al.*, 2003) for checkerboard-free results.

2. The Multiscale Design Space by the *M*-band Wavelet Transform

2.1 Overview

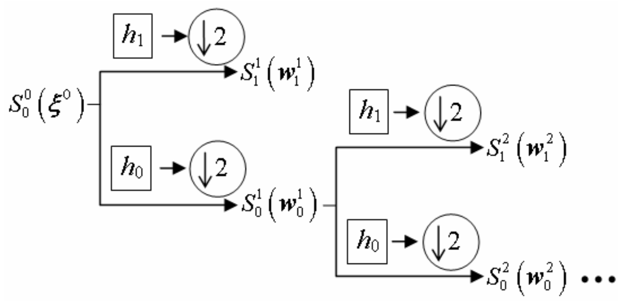
In standard topology optimization based on the density method, the design variables are the relative element densities $r(i)$ (i=0, $\cdots$, n_d= number of design variables) varying between 0 and 1. In the multiscale design method proposed by Kim and Yoon (2000) (also see Poulsen (2002)), we carry out design optimization using the multiscale design variables $w(i)$ $(i=0,\cdots,n_d)$ in the multiscale design space. If the single-scale density design space is directly transformed into the multiscale wavelet design space, the side constraints $0 \le r(i) \le 1$ $(i=1,\cdots,n_d)$ become complicated constraints. By introducing the auxiliary variables $x(i)$ ($-\infty \le x(i) \le \infty$), the original side constraints are automatically satisfied. To transform the auxiliary design space into the multiscale wavelet design space, we will consider the multiscale transform based on the Haar wavelet as it has the shortest support.

2.2 Multiscale Space Refinement by the *M*-band Wavelet Transform ($M \geq 2$)

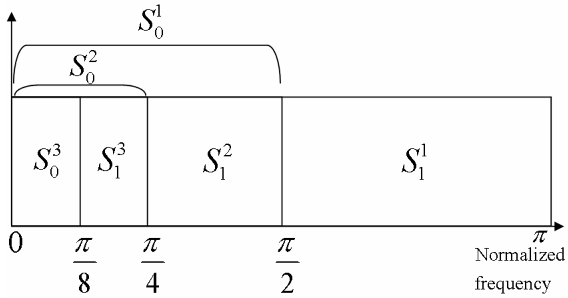
The main idea to refine the multiscale design space is to use the *M*-band wavelet transform ($M \geq 2$). We will first compare the characteristics of the 2-band and 4-band wavelet transforms. We will denote the single-scale design space by $S^0 = S_0^0$ generated by x^0.

$$x^0 = \{x(1), x(2), \cdots, x(n_d)\}^T \quad (-\infty \le x(i) \le \infty) \tag{1}$$

Now consider how the original single-scale space S_0^0 can be divided into an average space S_0^1 and a difference space S_1^1 by the 2-band wavelet transform. As depicted in Fig. 1(a), the standard single-scale space is divided into two-spaces and then the average space is re-divided into two spaces. The superscript *i* in $S_j^i(w_j^i)$ denotes the decomposition level and the subscript *j* implies "average" if *j*=0 or "difference" if *j*=1.



(a) Design space decomposition by the 2-band wavelet transform (The symbol "$\downarrow M$" stands for the downsampling by a factor of *M*)



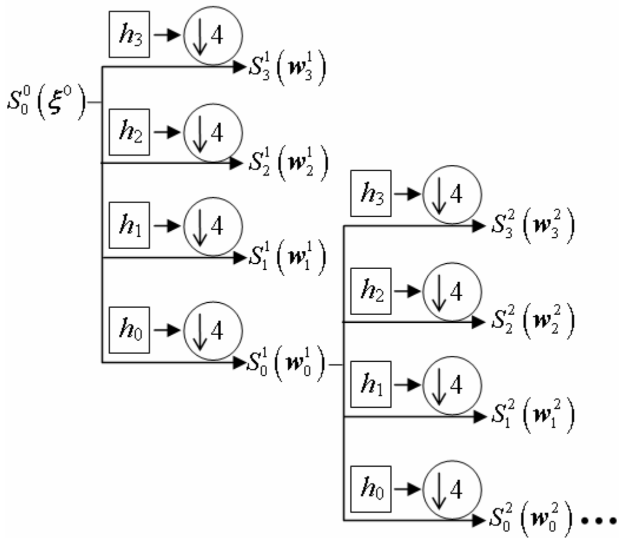
(b) Energy concentration of each space S_j^i in the frequency domain

Fig. 1 Application of the two-band wavelet transform in the one-dimensional case

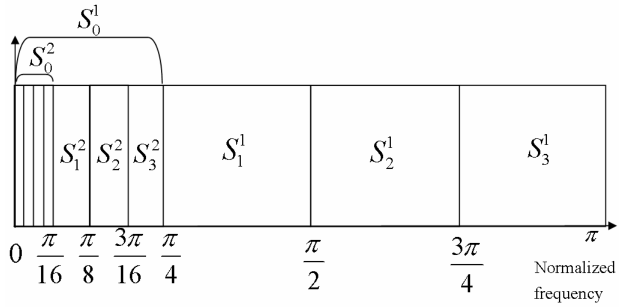
From the viewpoint of resolution, the resolution level increases from S_0^3 towards $S_0^0 = S^0$. In the multiresolution multiscale design strategy, one can start the initial design optimization at any space $S_0^{i_m}$ and proceed to the highest-resolution space $S_0^{i_0}$ $(i_m \ge i_0)$.

As illustrated in Fig. 1(b), these subspaces S_j^i have the frequency localization property within their frequency band. Thus, it is obvious that the topological complexity of optimized designs is affected by the maximum spatial frequency that a selected subspace S_0^i (i=0,1,2,$\cdots$) can represent. Therefore, the complexity will be affected by the choice of the subspaces.

In order to have more design candidates for a given highest-resolution design space, S_0^0 must be decomposed into more subspaces. To achieve this goal, we propose to use the *M*-band wavelet transform with *M*=2,4,8,16,$\cdots$ (see, e.g., Mallat (1998) for its definition) in transforming the auxiliary space x to the multiscale design space $\boldsymbol{W}$.



(a) Design space decomposition by the 4-band wavelet transform



(b) Energy concentration of each subspace S_j^i in the frequency domain

Fig. 2 Application of the four-band wavelet transform in one-dimensional cases

For the multiscale space decomposition illustrated in Fig. 2, one can now have the following multiresolution design scenarios (The symbol $\oplus$ stands for the direct sum.):

Scenario 1 (I_R=0.015625): $S_0^3\ (\to\cdots)\to S_0^0$

Scenario 2 (I_R=0.03125): $S_0^3\oplus S_1^3\ (\to\cdots)\to S_0^0$

Scenario 3 (I_R=0.046875):

$$S_0^3\oplus S_1^3\oplus S_2^3\ (\to\cdots)\to S_0^0$$

$$\vdots \tag{2}$$

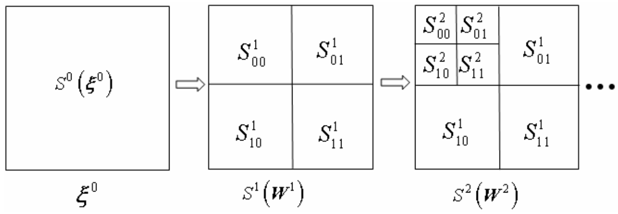
Scenario 9 (I_R=0.75):

$$S_0^1\oplus S_1^1\oplus S_2^1\ (\to\cdots)\to S_0^0$$

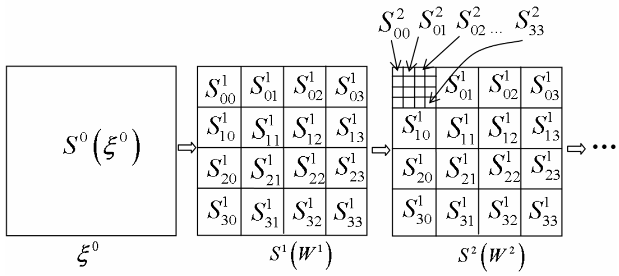
Scenario 10 (I_R=1.0): S_0^0

Because of the design space refinement by the 4-band transform, there are more multiresolution design scenarios possible than the 2-band case.

In the case of the two-dimensional design space, the original single-scale space S_0^0 consisting of the auxiliary design variables x can be decomposed into several multiscale subspaces as suggested in Fig. 3. To transform the auxiliary space x to the multiscale design space $\boldsymbol{W}$, one can use either the standard or the nonstandard wavelet transforms (see e.g., Stollnitz *et al.*, 1996). In this work, we will mainly use the nonstandard wavelet transform.



(a) By the 2-band wavelet transform



(b) By the 4-band wavelet transform

Fig. 3 Multiscale design space decomposition for the two-dimensional design space

3. Topology Complexity Control

3.1 Complexity Measure

Since we are concerned with topology complexity, we need some measure to quantify it. As such a measure, we employ the perimeter measure P introduced by Haber *et al*. (1996). The definition of the perimeter P is

$$P=\sum_{k=1}^{K}\ell_k\left(\sqrt{<r>_k^2+e^2}-e\right) \tag{3}$$

In Eq. (3), $<r>_k$ represents the density jump between an element interface k of length ℓ_k and the total number of the interfaces is denoted by K. A small positive number e is used to guarantee the differentiability of the perimeter, but it is set to zero as no differentiation of P is needed in this work.

3.2 Hierarchical Design Resolution Control

To control the topological complexity of the optimized design, we will choose the starting design spaces having different resolution levels, i.e. different values of I_R in the multiresolution multiscale setting. The multiresolution design process is completed with two resolution levels, the starting level and the highest resolution level. Here, we will consider two typical classes of topology optimization problems: the compliance minimization problem and the compliant mechanism design problem.

As the optimization algorithm, we use the method of moving asymptotes (Svanberg, 1987).

Compliance Minimization Problem

As a specific numerical example, we consider the MBB beam optimization problem depicted in Fig. 4.

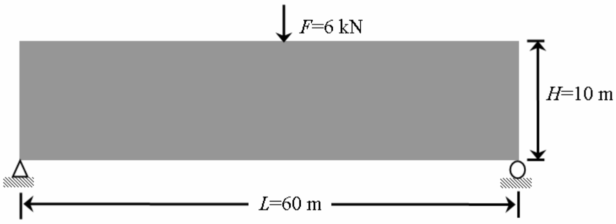


Fig. 4 The MBB beam design problem with the mass constraint ratio of 50%. (The design domain is descretized by 192×64 nonconforming finite elements.)

In Fig. 5, we present the final optimized designs by 4 different multiresolution design scenarios. Since non-conforming elements give checkerboard-free results, no filtering or similar image processing is used. As the relative initial design space resolution (I_R) increases, the value of perimeter P increases; see Fig. 5. On the other hand, the optimal value of the objective function L_{opt} at the final stage also tends to decrease as I_R becomes larger. Therefore, the simpler topological complexity usually comes at the sacrifice of the system performance.

(a) Scenario with I_R=0.25 (P=174.7984, L_{opt}=46.1118)

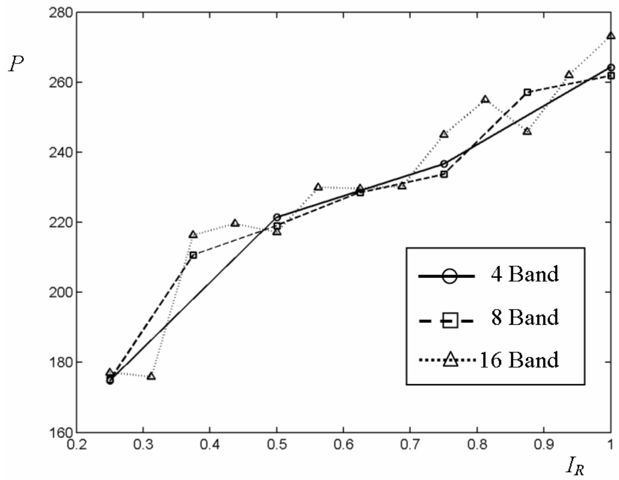
(b) Scenario with I_R=0.5 (P=221.5415, L_{opt}=45.5861)

(c) Scenario with I_R=0.75 (P=236.7185, L_{opt}=44.7226)

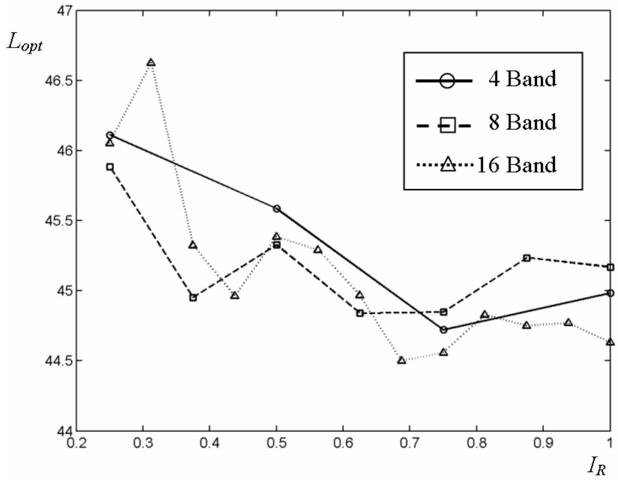
(d) Scenario with I_R=1.0 (P=264.2628, L_{opt}=44.9855)

Fig. 5 The optimized designs for the MBB problem by 4 different multiresolution design scenarios in the multiscale design space generated by the 4-band Haar wavelet transform

Now we consider the values of P and L_{opt} for various multiresolution design scenarios with 4-, 8- and 16-band wavelet transforms. Figure 6(a) shows that the topological complexity of the final optimized design is almost determined by the starting design resolution level. Furthermore, by using the M-band wavelet transforms with M>2, candidate optimal designs having various degrees of topological complexity can be quite systematically found.



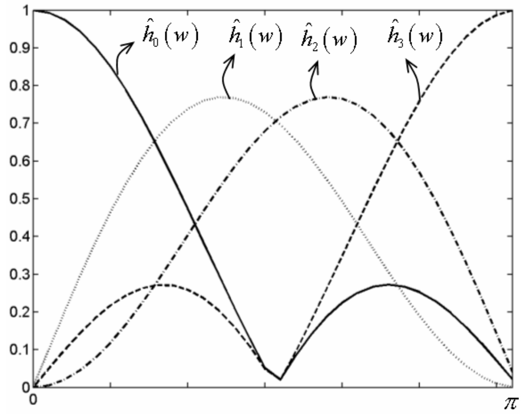
(a) The value of the perimeter P



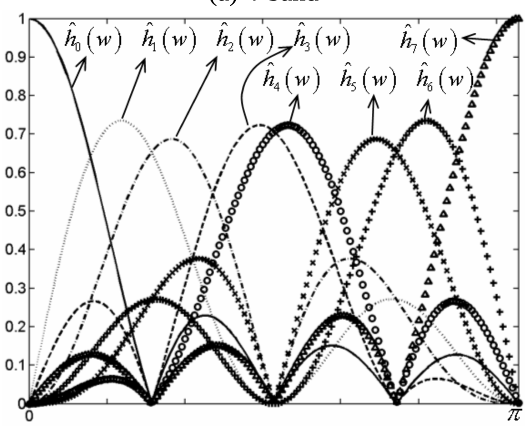
(b) The value of the objective function L_{opt}

Fig. 6 The values of the perimeter P and the objective function L_{opt} for various multiresolution design scenarios

Even for the same value of I_R, the use of different multiscale decomposition yields somewhat different values of P though the difference is not so significant. To see why this happens, we examine, in Fig. 7, the (spatial) frequency characteristics of the filters $\boldsymbol{h}$ used to generate S_0^{i+1}, S_1^{i+1}, etc. from S_0^i. Figure 7 compares the frequency characteristics of the filters used for 4-, and 8-band one-dimensional wavelet transforms. As illustrated by Fig. 7, the differences depending on the number of the wavelet bands used in the filter characteristics affect the frequency characteristics of the initial starting design space in the multiresolution setting. The use of the higher-band wavelets gives better frequency localizations, so it is easier to control the topological complexity of the final design. However, the use of higher-bands requires the wavelets having longer spatial supports. Therefore, it is not possible to control the topological complexity exactly while minimizing the objective function. Consequently, the P-I_R relation becomes somewhat less monotonic when the 16-band wavelet transform is used: see Fig. 6(a). However, there is a very good correlation between P and I_R especially when the band number is moderate.



(a) 4-band



(b) 8-band

Fig. 7 The spatial frequency characteristics of the filters ***h*** for the one-dimensional 4-, 8-band wavelet transforms

At this stage, we have not yet established an one-to-one correspondence between the frequency characteristics of ***h*** and the topology complexity. If we use other wavelets, this may be possible, but this requires further investigation. Nonetheless, the present strategy yields several design candidates having different topological complexity quite systematically.

Compliant Thermal Actuator Design

As the second typical class of topology optimization problems, we consider the design of a compliant mechanism, in particular, the design of a micro-sized compliant thermal actuator (see Fig. 8).

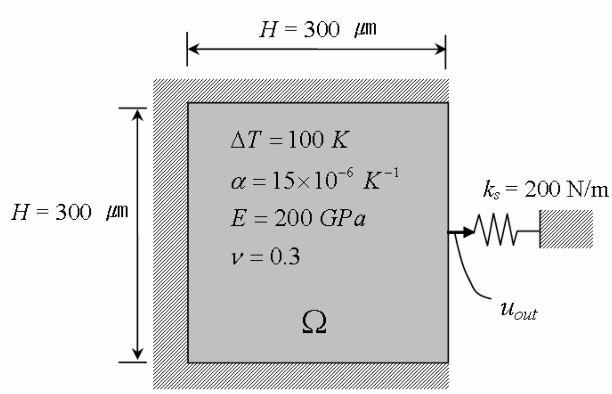
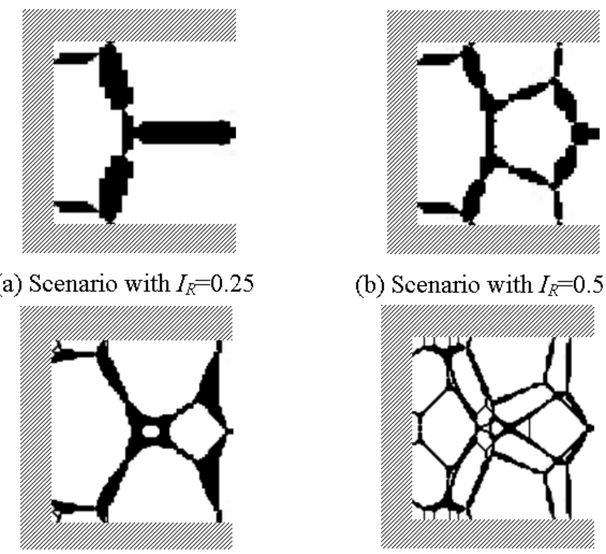


Fig. 8 The compliant thermal micro actuator design problem with the mass constraint ratio of 20%. (Half of the design domain is discretized by 128×64 nonconforming finite elements.)

We follow the formulation by Sigmund (2001) and state the design problem briefly.

$$\begin{aligned} &\text{Maximize } u_{out}(r) \\ &\text{Subject to } H(r) = \sum_{i=1}^{n_d} r_i v_i - M_0 \le 0 \\ &\qquad 0 < r_{\min} \le r_i \le 1 \quad (i = 1, 2, \cdots, n_d) \qquad (4) \\ &\qquad \boldsymbol{K}(r)\boldsymbol{U}(r) = \boldsymbol{R}(r) \\ &\qquad \boldsymbol{R} = \sum_{e=1}^{n_d} \int_{\Omega_e} \boldsymbol{B}_i^T \boldsymbol{D}(r_i) a \Delta T d\Omega \end{aligned}$$

where $\boldsymbol{B}_i$ is the strain-displacement relating matrix commonly used in the finite element and the constitutive matrix $\boldsymbol{D}(r_i)$. The symbol ΔT denotes the temperature change and $\boldsymbol{a}$ is the thermal coefficient vector.



(a) Scenario with I_R=0.25 (b) Scenario with I_R=0.5

(c) Scenario with I_R=0.75 (d) Scenario with I_R=1.0

Fig. 9 The optimized design for the compliant thermal micro actuator design problem by 4 different multiresolution design scenarios in the multiscale design space generated by the 16-band Haar wavelet transform

Figure 9 shows the final optimized results by 4-different multiresolution design scenarios with I_R=0.25, 0.5, 0.75, 1.0 where the multiscale design space is generated by the 16-band nonstandard Haar wavelet transforms. The correlation between the topological complexity and the relative resolution level I_R of the starting design space is plotted in Fig. 10. As in the compliance minimization problem, we can see that the topological complexity tends to increase as I_R increases.

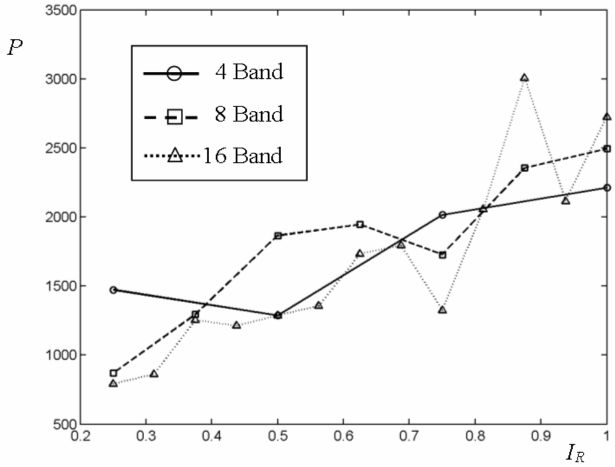


Fig. 10 The correlation between the value of the perimeter (P) and the relative resolution of the starting design space (I_R) for the problem described in Fig. 8

4. Conclusions

The topological complexity issue itself is not a new issue, but we investigate it from the viewpoint of the systematic generation of a set of several candidate designs having different complexities. Our approach was to vary the resolution level of the initial design space using the multiresolution multiscale design scenarios. To refine the multiscale design space and thus to allow various resolution levels to start with, we developed a method using M-band wavelet transforms (with M=4,8,16,$\cdots$). The followings are the observations we have made through the present investigation:

1. The relative resolution level I_R of the starting design space in the multiresolution multiscale setting governs the topological complexity of the highest-resolution optimized design.
2. If the number of the wavelet band is not so large, the perimeter P tends to increase monotonically as the starting resolution level increases; there is a good correlation between I_R and P.

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