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Modeling and Adaptive Motion Tracking Control of Two-Wheeled Welding Mobile Robot (WMR)

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Key Words : Welding mobile robot(WMR,), Adaptive control(), Tracking error(), Nonholonomic system(), Reference welding path(), Back-stepping control

Abstract

This paper proposes an adaptive control algorithm for nonholonomic mobile robots with unknown parameters and the proposed control method is used in numerical simulations for applying to a practical two-wheeled welding mobile robot(WMR). The proposed adaptive controller to track an arbitrary given welding path is designed by using back-stepping technique and is derived for a nonlinear model under the assumption such that the system parameters are partially known. Moreover, the proposed adaptive control system is stable in the sense of Lyapunov stability. Inertia moments of system are considered to be unknown parameters and their values can be estimated simply by using update laws proposed in an adaptive control scheme of this research. The simulation results are provided to show the effectiveness of the accurate tracking capability of the proposed controller for two-wheeled welding mobile robot with a smooth curved reference welding path.

1.

(automatic welding mobile robot, WMR) 가 Fierro, Sarkar, Fukao (welding torch)

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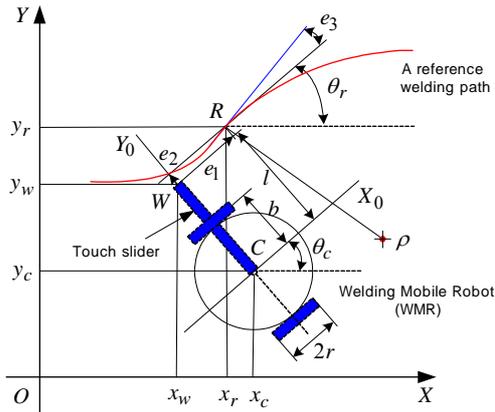


Fig. 1 Coordinates of two-wheeled WMR

Lyapunov

가 가

2.

2.1

Fig. 1

O - XY

C - X0Y0

가

$$q = [x_c \ y_c \ \theta_c \ \theta_r \ \theta_l]^T \quad (1)$$

, (x_c, y_c) C , θ_c
 , θ_r θ_l

i) pure rolling, ii) non-slipping
 가 가

$$\dot{x}_c \sin \theta_c - \dot{y}_c \cos \theta_c = 0 \quad (2)$$

$$\dot{x}_c \cos \theta_c + \dot{y}_c \sin \theta_c + b \dot{\theta}_c = r \dot{\theta}_r \quad (3)$$

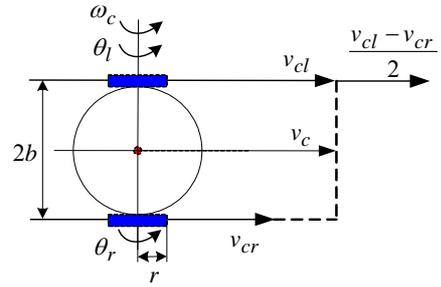


Fig. 2 The WMR with two actuated wheels

$$\dot{x}_c \cos \theta_c + \dot{y}_c \sin \theta_c - b \dot{\theta}_c = r \dot{\theta}_l \quad (4)$$

, (constraint)

$$A(q) \dot{q} = 0 \quad (5)$$

A(q)

$$A(q) = \begin{bmatrix} \sin \theta_c & -\cos \theta_c & 0 & 0 & 0 \\ \cos \theta_c & \sin \theta_c & b & -r & 0 \\ \cos \theta_c & \sin \theta_c & -b & 0 & -r \end{bmatrix} \quad (6)$$

$$\theta(t) = [\theta_r \ \theta_l]^T$$

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \\ \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} \frac{r}{2} \cos \theta_c & \frac{r}{2} \cos \theta_c \\ \frac{r}{2} \sin \theta_c & \frac{r}{2} \sin \theta_c \\ \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \quad (7)$$

$$\dot{q} = S(q) \dot{\theta}(t) \quad (8)$$

, $S(q)$ $A(q)S(q) = 0$
 full-rank

θ_r θ_l 3-state

, Fig. 2

$$\theta(t) = \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} \quad (9)$$

, v_c ω_c C

(1) 5

$$\eta = [v_c \ \omega_c]^T \tag{10}$$

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \cos \theta_c & 0 \\ \sin \theta_c & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} \tag{11}$$

가

Euler-Lagrangian

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = E(q)\tau - A^T(q)\lambda \tag{12}$$

$E(q)$ full-rank, $A(q)$ Jacobian matrix, and τ λ

$$\ddot{q} = S(q)\ddot{\theta} + \dot{S}(q)\dot{\theta} \tag{13}$$

$$S^T(MS\ddot{\theta} + M\dot{S}\dot{\theta} + CS\dot{\theta}) = S^T E \tau \tag{14}$$

$$\bar{M}(q)\ddot{\theta} + \bar{C}(q, \dot{q})\dot{\theta} = \tau \tag{15}$$

$$m = m_c + 2m_\omega$$

$$I = I_c + 2m_\omega(b^2 + d^2) + 2I_m$$

$$\bar{M} = \begin{bmatrix} \frac{r^2}{4b^2}(mb^2 + I) + I_\omega & \frac{r^2}{4b^2}(mb^2 - I) \\ \frac{r^2}{4b^2}(mb^2 - I) & \frac{r^2}{4b^2}(mb^2 + I) + I_\omega \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 0 & \frac{r^2}{2b}m_c d \dot{\theta}_c \\ -\frac{r^2}{2b}m_c d \dot{\theta}_c & 0 \end{bmatrix}$$

m_c , m_ω , I_c , I_ω , I_m

Fig. 1

b , d , r

C

2.2

Fig. 1, $W(x_\omega, y_\omega)$, C

$$\begin{aligned} x_w &= x_c - l \sin \theta_c \\ y_w &= y_c + l \cos \theta_c \\ \theta_w &= \theta_c \end{aligned} \tag{16}$$

$$\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{\theta}_w \end{bmatrix} = \begin{bmatrix} \cos \theta_c & -l \cos \theta_c \\ \sin \theta_c & -l \sin \theta_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} + \begin{bmatrix} -l \sin \theta_c \\ l \cos \theta_c \\ 0 \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{18}$$

$R(x_r, y_r)$, v_r , θ_r , v_r , ω_r

$R(x_r, y_r)$, $W(x_\omega, y_\omega)$

$$e = [e_1 \ e_2 \ e_3]^T$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_\omega \\ y_r - y_\omega \\ \theta_r - \theta_\omega \end{bmatrix} \tag{19}$$

e_1 , e_2 , e_3

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - l \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} \tag{20}$$

$$\dot{\zeta} = f(\zeta) + g(\zeta)\eta \tag{21}$$

(9) (15)

$$\begin{bmatrix} \frac{r}{m}m + \frac{1}{r}I_w & \frac{r}{2b}I + \frac{b}{r}I_w \\ \frac{r}{2}m + \frac{1}{r}I_w & -\frac{r}{2b}I - \frac{b}{r}I_w \end{bmatrix} \begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix} + \begin{bmatrix} \frac{r}{2b}m_c d\omega_c & -\frac{r}{2}m_c d\omega_c \\ -\frac{r}{2b}m_c d\omega_c & -\frac{r}{2}m_c d\omega_c \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \quad (22)$$

(21) (22)

3.

3.1

subsystem

$$\dot{\zeta} = f(\zeta) + g(\zeta)\eta \quad (23)$$

$$\Delta_1 \dot{\eta} = \Delta_2 h(\eta)\eta + k(\eta)u \quad (24)$$

$\zeta \in \mathfrak{R}^n, \eta, u \in \mathfrak{R}^m, f \in \mathfrak{R}^n$

$$\begin{matrix} \theta_{1i} & \theta_{2i} \\ \Delta_1 & \Delta_2 \end{matrix} \quad (23) \quad (24)$$

subsystem η -subsystem

가

1: update law

$$\dot{\hat{\theta}}_{1i} = r_{1i}(\eta_i - \alpha_i)\dot{\alpha}_i \quad (25)$$

$$\dot{\hat{\theta}}_{2i} = -r_{2i}(\eta_i - \alpha_i) \sum_{j=1}^m h_{ij}(\eta)\eta_j \quad (26)$$

$$u = k^{-1}(\eta)[-K_2(\eta - \alpha) - g^T(\zeta)\zeta + \hat{\Delta}_1 \dot{\alpha} - \hat{\Delta}_2 h(\eta)\eta] \quad (27)$$

(23) (24)

$t \rightarrow \infty \quad \zeta \rightarrow 0$

$K_1 \in \mathfrak{R}^{n \times n} \quad K_2 \in \mathfrak{R}^{m \times m}$

$\gamma_{1i}, \gamma_{2i} \ (i = 1, 2, \dots, m)$ (adaptive gain)
 $\Delta_1 \quad \Delta_2$, $\hat{\Delta}_1 \quad \hat{\Delta}_2$ stabilizing α

$$g(\zeta)\alpha = -k_1\zeta - f(\zeta) \quad (28)$$

(Proof) [12]

1 : $\Delta_i \ (i=1,2)$ 가 $\Delta_i \rightarrow \theta_i$

(25) (26) update law

$$\dot{\hat{\theta}}_1 = \gamma_1 \sum_{i=1}^m (\eta_i - \alpha_i)\dot{\alpha}_i \quad (29)$$

$$\dot{\hat{\theta}}_2 = -\gamma_2 \sum_{i=1}^m \sum_{j=1}^m (\eta_i - \alpha_i)h_{ij}(\eta)\eta_j \quad (30)$$

3.2

$$\alpha = [\alpha_1 \quad \alpha_2]^T \quad (21)$$

$v_c \quad \omega_c$

$$\zeta = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \quad f(\zeta) = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - \dot{l} \\ \omega_r \end{bmatrix}$$

$$g(\zeta) = \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix}, \quad \eta = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} \quad (31)$$

1 (28)

$$\begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = -K_1 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} - \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - \dot{l} \\ \omega_r \end{bmatrix} \quad (32)$$

$K_1 = \text{diag}(k_{11}, k_{12}, k_{13})$, $\eta = \alpha$

$$\dot{l} = v_r \sin e_3 + k_{12}e_2 \quad (33)$$

(control law)

(stabilizing function) 가

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} l(\omega_r + k_{13}e_3 + v_r \cos e_3 + k_{11}e_1) \\ \omega_r + k_{13}e_3 \end{bmatrix} \quad (34)$$

$$\tau = [\tau_r \quad \tau_l]^T$$

$$(22) \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

$$\begin{bmatrix} rm + I\omega & 0 \\ 0 & \frac{b}{r}I + \frac{2b}{r}I\omega \end{bmatrix} \begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix}$$

$$\frac{r}{b}m_c d \begin{bmatrix} 0 & b\omega_c \\ -\omega_c & 0 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \quad (35)$$

$$\Delta_1 \dot{\eta} = \Delta_2 h(\eta) + k(\eta)u \quad (36)$$

$$\theta_{ij}$$

$$u = [\tau_r \quad \tau_l]^T$$

$$\tau_r = \frac{1}{2} \{-k_{21}(v_c - \alpha_1) - k_{22}(\omega_c - \alpha_2) - (1+l)e_1 + e_3 + \hat{\theta}_{11}\dot{\alpha}_1 + \hat{\theta}_{12}\dot{\alpha}_2 + \hat{\theta}_2 \omega_c (b\omega_c - v_c)\} \quad (37)$$

$$\tau_l = \frac{1}{2} \{-k_{21}(v_c - \alpha_1) - k_{22}(\omega_c - \alpha_2) - (1-l)e_1 + e_3 + \hat{\theta}_{11}\dot{\alpha}_1 + \hat{\theta}_{12}\dot{\alpha}_2 + \hat{\theta}_2 \omega_c (b\omega_c - v_c)\} \quad (34)$$

$$K_2$$

$$K_2 = \text{diag}(k_{21}, k_{22}) \quad (38)$$

, update law

$$\dot{\hat{\theta}}_{11} = \gamma_{11}(\eta_1 - \alpha_1)\alpha_1 \quad (39)$$

$$\dot{\hat{\theta}}_{12} = \gamma_{12}(\eta_2 - \alpha_2)\alpha_2 \quad (40)$$

$$\dot{\hat{\theta}}_{11} = -\gamma_{11}(\eta_1 - \alpha_1) \sum_{j=1}^2 h_{1j}(\eta)\eta_j \quad (41)$$

$$\dot{\hat{\theta}}_{12} = -\gamma_{12}(\eta_2 - \alpha_2) \sum_{j=1}^2 h_{2j}(\eta)\eta_j \quad (42)$$

4.

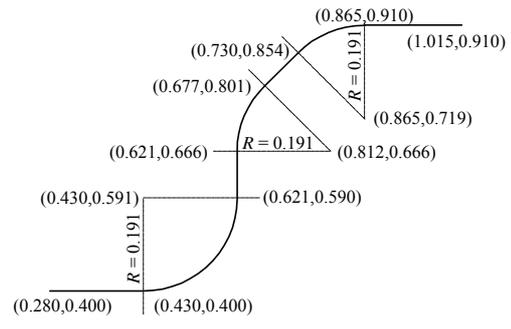


Fig. 3 The reference welding path

[???].

Fig. 3
7.5mm/sec

$$k_{11} = 4.2, \quad k_{12} = 8, \quad k_{13} = 3.4, \quad k_{21} = k_{22} = 10.$$

(adaptive gain)

$$\gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = 1$$

Fig. 4

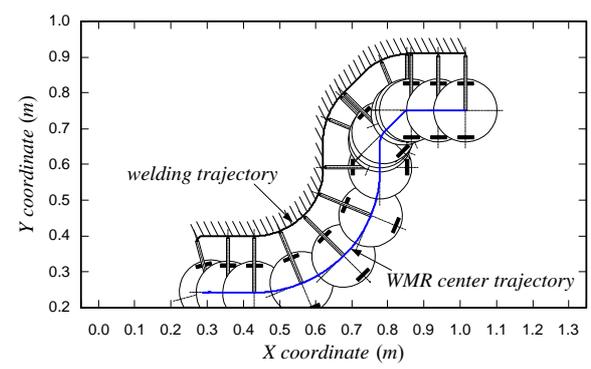


Fig. 4 The moving path of WMR

Fig. 5 Fig. 6

가

Fig. 7

가

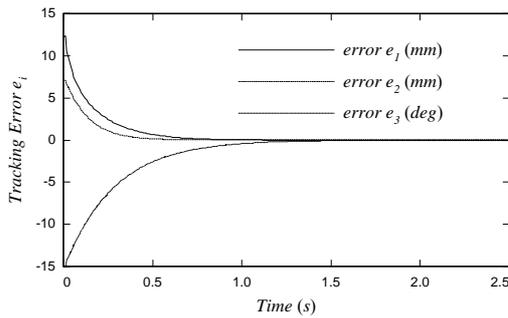


Fig. 5 Tracking errors at beginning

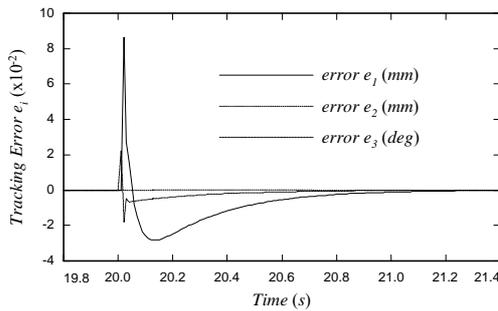
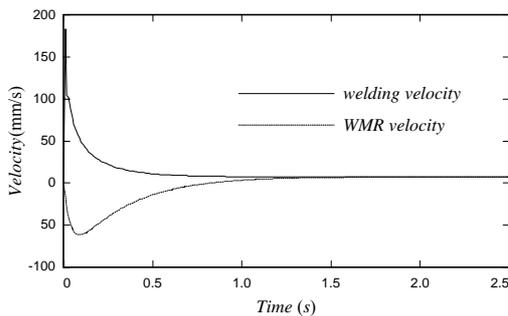
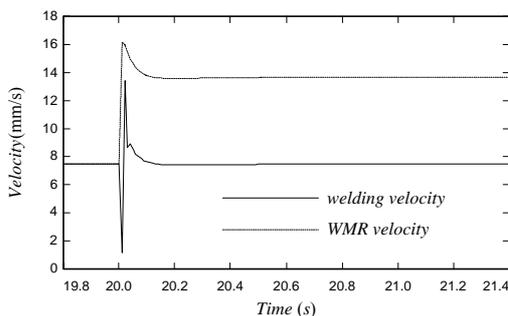


Fig. 6 Tracking error at corner



(a) At beginning



(b) At corner

Fig. 7 The velocities of welding point and WMR

5.

Lyapunov

Lyapunov

(stable)

update law

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