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Optimal Vibration Control of Rigid Plate Elastically Supported at the Edges

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Key Words : Optimal vibration control(), Rigid plate(), Output feedback(), Multi-degree of freedom(), Relative weighting parameter(가), Design constraint(), Active vibration control()

Abstract

Rigid plate elastically supported at the edges is modeled and the performance of the optimal vibration control under sinusoidal excitation is tested. The controller based on the linear quadratic regulator with output feedback is designed to control the multi-degree of freedom vibration. Relative weighting parameters are considered as design constraints to determine the limitation of maximum control force and state parameters. Control force calculated by proportional output feedback of the displacement and velocity is used to suppress the vibration induced by the sinusoidal external force. The active vibration control of vibrating plate by the LQR controller is examined through the numerical simulations that show the effectiveness of optimal control scheme on the three degrees of freedom structure.

m :
 θ, φ : C.G y, x
 k_1, k_2, k_3, k_4 : (spring constant)
 l_1, l_2, l_3, l_4 : C.G
 a, b : C.G x, y 1972 Yao⁽¹⁾
 J_1, J_2 : C.G y, x (optimal control)⁽²⁻³⁾
 f_e :
 $F_{cz}, M_{c\theta}, M_{c\varphi}$: C.G z $y,$
 x

1.

3

4

†

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*

(4-5)

**

가

가 가 (6-7)
 LQR(linear quadratic
 regulator)⁽⁷⁻⁹⁾ , 가

LQR (control
 force) 가 (control

LQR

2.

2.1

Fig.1

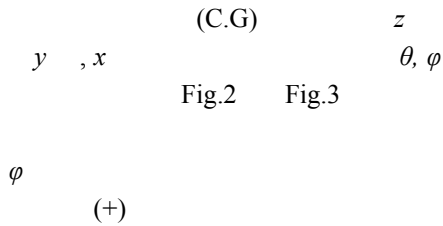


Fig.2 Fig.3

(1)

$$\begin{aligned}
 m\ddot{z} &= -k_1(z-l_1\theta-l_3\varphi) - k_2(z-l_1\theta+l_4\varphi) \\
 &\quad - k_3(z+l_2\theta-l_3\varphi) - k_4(z+l_2\theta+l_4\varphi) + f_e + F_{cz} \\
 J_1\ddot{\theta} &= k_1(z-l_1\theta-l_3\varphi)l_1 + k_2(z-l_1\theta+l_4\varphi)l_1 \\
 &\quad - k_3(z+l_2\theta-l_3\varphi)l_2 - k_4(z+l_2\theta+l_4\varphi)l_2 + af_e + M_{c\theta} \\
 J_2\ddot{\varphi} &= k_1(z-l_1\theta-l_3\varphi)l_3 - k_2(z-l_1\theta+l_4\varphi)l_4 \\
 &\quad + k_3(z+l_2\theta-l_3\varphi)l_3 - k_4(z+l_2\theta+l_4\varphi)l_4 + bf_e + M_{c\varphi}
 \end{aligned} \tag{1}$$

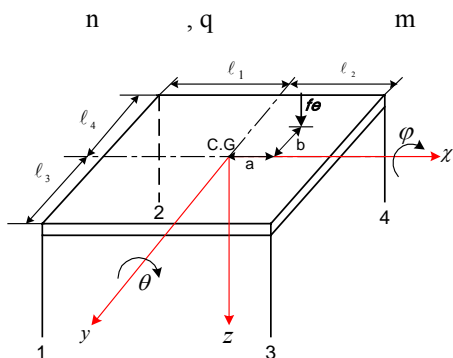


Fig.1 Schematic diagram of rigid rectangular plate

$$[M]\ddot{p}(t) + [K]p(t) = [E_1]f_e(t) + [E_2]u(t) \tag{2}$$

$[M]$, $n \times n$ $[E_1]$, $n \times n$
 $[K]$, $n \times q$ $[E_2]$, $n \times m$

$$[M] = \begin{bmatrix} m & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_2 \end{bmatrix}, [E_1] = \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}, [E_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_1+k_2 & -k_1l_1-k_2l_1 & -k_1l_3+k_2l_4 \\ +k_3+k_4 & +k_3l_2+k_4l_2 & -k_3l_3+k_4l_4 \\ -k_1l_1-k_2l_1 & k_1l_1^2+k_2l_1^2 & k_1l_1l_3-k_2l_1l_4 \\ +k_3l_2+k_4l_2 & +k_3l_2^2+k_4l_2^2 & -k_3l_2l_3+k_4l_2l_4 \\ -k_1l_3+k_2l_4 & k_1l_1l_3-k_2l_1l_4 & k_1l_3^2+k_2l_4^2 \\ -k_3l_3+k_4l_4 & -k_3l_2l_3+k_4l_2l_4 & +k_3l_3^2+k_4l_4^2 \end{bmatrix}$$

$q \times 1$ $f_e(t)$ (single input)
 가 $n \times 1$ $p(t)$ $m \times 1$
 $u(t)$

$$p(t) = [z(t) \ \theta(t) \ \varphi(t)]^T, u(t) = [F_{cz} \ M_{c\theta} \ M_{c\varphi}]^T$$

2.2

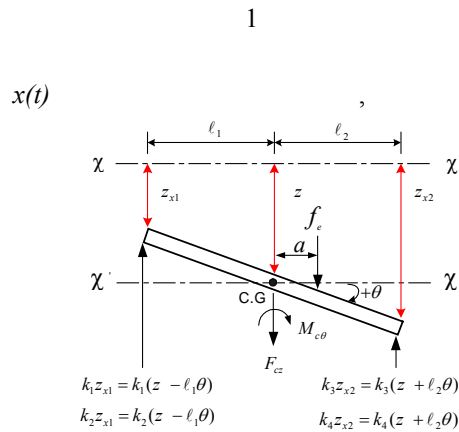


Fig.2 F.B.D of plate viewed on y-axis

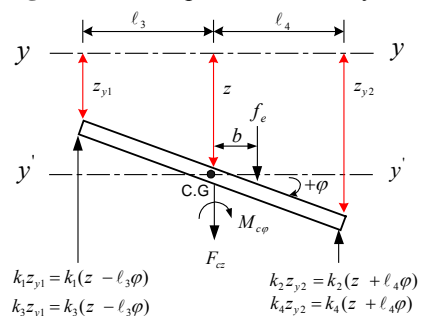


Fig.3 F.B.D of plate viewed on x-axis

$$x(t) = [x_1(t) \ x_2(t)]^T = [p(t) \ \dot{p}(t)]^T \quad (3)$$

(2)

$$\dot{x}(t) = [A]x(t) + [B]u(t) + [E]f_e(t) \quad (4)$$

$$[A] = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}, \quad [B] = \begin{bmatrix} 0 \\ M^{-1}E_2 \end{bmatrix}$$

$$[E] = \begin{bmatrix} 0 \\ M^{-1}E_1 \end{bmatrix}$$

$$\begin{matrix} & 2n \times 2n & & [A], 2n \times m \\ & [B], 2n \times q & & [E], 2n \times 1 \\ x(t), m \times 1 & & u(t) & \\ & & q \times 1 & f_e(t) \end{matrix}$$

$$x(t) = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [z \ \theta \ \varphi \ \dot{z} \ \dot{\theta} \ \dot{\varphi}]^T$$

$$u(t) = [F_{cz} \ M_{c\theta} \ M_{c\varphi}]^T$$

$$[A] = \begin{bmatrix} 0(3 \times 3) & I(3 \times 3) \\ -M^{-1}K & 0(3 \times 3) \end{bmatrix} \quad (5)$$

$$[B] = \begin{bmatrix} 0(3 \times 3) \\ \hline 1/m & 0 & 0 \\ 0 & 1/J_1 & 0 \\ 0 & 0 & 1/J_2 \end{bmatrix}, \quad [E] = \begin{bmatrix} 0(3 \times 1) \\ \hline 1/m \\ a/J_1 \\ b/J_2 \end{bmatrix}$$

[A] $-M^{-1}K$

$$\begin{bmatrix} -1/m(k_1+k_2) & -1/m(-k_1l_1-k_2l_1) & -1/m(-k_1l_3+k_2l_4) \\ +k_3+k_4 & +k_3l_2+k_4l_2 & -k_3l_3+k_4l_4 \\ \hline -1/J_1(-k_1l_1-k_2l_1) & -1/J_1(k_1l_1^2+k_2l_1^2) & -1/J_1(k_1l_1l_3-k_2l_1l_4) \\ +k_3l_2+k_4l_2 & +k_3l_2^2+k_4l_2^2 & -k_3l_2l_3+k_4l_2l_4 \\ \hline -1/J_2(-k_1l_3+k_2l_4) & -1/J_2(k_1l_1l_3-k_2l_1l_4) & -1/J_2(k_1l_3^2+k_2l_4^2) \\ -k_3l_3+k_4l_4 & -k_3l_2l_3+k_4l_2l_4 & +k_3l_3^2+k_4l_4^2 \end{bmatrix}$$

$$r \times 1 \quad y(t) \quad (6)$$

$$y(t) = [C]x(t) + [D]u(t) \quad (6)$$

[C] $r \times 2n$

[D] $r \times m$

$$0 \quad (x(t)) \quad \text{가}$$

$(x, \theta, \varphi, \dot{x}, \dot{\theta}, \dot{\varphi})$ C.G

가 ()

Fig.1

1, 2, 3, 4

z_1, z_2, z_3, z_4 z, θ, φ

$$\begin{aligned} z_1 &= z - l_1\theta - l_3\varphi, & z_2 &= z - l_1\theta + l_4\varphi \\ z_3 &= z + l_2\theta - l_3\varphi, & z_4 &= z + l_2\theta + l_4\varphi \end{aligned} \quad (7)$$

[C] (7)

$$[C] = \begin{bmatrix} CC & 0(4 \times 3) \\ 0(4 \times 3) & CC \end{bmatrix}, \quad [CC] = \begin{bmatrix} 1 & -l_1 & -l_3 \\ 1 & -l_1 & l_4 \\ 1 & l_2 & -l_3 \\ 1 & l_2 & l_4 \end{bmatrix}, \quad (8)$$

$y(t)$ (6)

1,2,3,4

8 x 1

3.

3.1 가 가

(controllability) 가 (observability) 가

(4) (6)

$$\dot{x}(t) = [A]x(t) + [B]u(t)$$

$$y(t) = [C]x(t) + [D]u(t) \quad (9)$$

가

$t_0 \ t \ t_1$

$u(t)$

가 가

가

(9)

가

2n x

2n-m

$$[B \mid AB \mid A^2B \mid \dots \mid A^{2n-1}B]$$

(rank)가 2n

가 (7)

가 $r \times (2n+1)m$

$$[CB \mid CAB \mid CA^2B \mid \dots \mid CA^{2n-1}B \mid D]$$

(rank)가 r

가

$x(t_0)$ 가

$t_0 \ t \ t_1$

$y(t)$

가 2n x 2n-r LQR Matlab (10-11)
 가 8 x 21 가 6 x 48
 8, 6 가

3.2 Linear Quadratic Regulator

(4) (6)
 $\dot{x}(t) = [A]x(t) + [B]u(t) + [E]f_e(t)$
 $y(t) = [C]x(t) + [D]u(t)$ (10)

(A, B)가 가 (A, C)가 가
 가 u(t)

$u(t) = -Ky(t)$ (11)

$\dot{x}(t) = [A - BKC]x(t) + [E]f_e(t)$ (11)

$x(t) = -Kx(t)$ (9)

u(t) = -Kx(t), LQR
 (12) 2 (quadratic objective function)

$J = \frac{1}{2} \int_0^T (x^T Q x + u^T R u) dt$ (12)

Q 2n x 2n (symmetric, positive semi-definite), R m x m (symmetric, positive definite)

가 Q, R 가 x(t)
 u(t) LQR

$u(t) = -Kx(t) = -R^{-1} B^T P x(t)$

, P Riccati
 $A^T P + PA + Q - PBR^{-1} B^T P = 0$

3.3 (, , ,)
 가

(13)
 $-x_{i,max} < x_i < x_{i,max} \quad i = 1, \dots, n$
 $-u_{j,max} < u_j < u_{j,max} \quad j = 1, \dots, r$ (13)

(13)
 Q, R
 가 Q, R
 가 , R

Q
 0 , Q

R
 가

(11) K (14)
 Q, R

$[Q] = diag(n_1, n_2, \dots, n_8) \quad n_1, n_2, \dots, n_8 = 1$
 $[R] = \cdot diag(m_1, m_2, m_3, m_4) \quad m_1, m_2, m_3, m_4 = 1$ (14)

가 ρ

4.

4.1 , LQR

Fig.1 Table 1
 가 10N, 가
 (1 : 18.8Hz, 2
 : 32.7Hz) 15Hz, 25Hz, 40Hz

5N 10N 50%

, Riccati
 15% 가

$$[M] = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.0017 & 0 \\ 0 & 0 & 0.0008 \end{bmatrix}, [E_1] = \begin{bmatrix} 1 \\ 0.03 \\ 0.03 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 3520 & -26.4 & -17.6 \\ -26.4 & 71.7 & 0.1 \\ -17.6 & 0.1 & 31.9 \end{bmatrix}, [E_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matlab Program Simulink
Fig.4

Table 2 Maximum control force for each frequency

Freq. (Hz)	log	Max. control force(N)			
		f ₁	f ₂	f ₃	f ₄
15	-3	0.96	2.74	2.23	4.03
25		0.97	2.74	2.22	4.03
40		0.93	2.58	2.08	3.83

Table 3 Maximum state responses(z, θ, φ) for each frequency

Freq. (Hz)	log	Max. displacement & angle on C.G of plate(m, radian)			Control ratio(%)		
		z	θ	φ	z	θ	φ
15	-3	0.0009	0.0015	0.0032	94	92	90
25		0.0005	0.0009	0.0019	94	96	94
40		0.0003	0.0006	0.0012	81	97	96

Table 4 Output control gains

log	Output control gains(8 x 4 matrix)							
-3	0.43	0.14	0.14	-0.14	23.72	7.91	7.91	-7.90
	0.14	0.43	-0.14	0.14	7.91	23.72	-7.90	7.91
	0.14	-0.14	0.43	0.14	7.90	-7.90	23.72	7.90
	-0.14	0.14	0.14	0.43	-7.90	7.91	7.91	23.72

4.2

Fig.5, 6 Table 2, 3, 4

Fig.5 가 ρ

15Hz, 25Hz, 40Hz

가 (z, θ, φ)

Fig.6 가 log ρ = -3

15%)

Fig.6 가 log ρ = -3

가 15%

Table 2

Table 3

Table 4

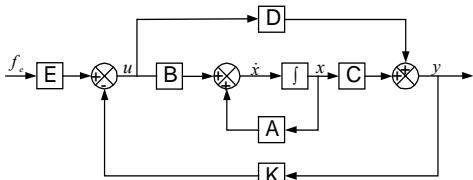
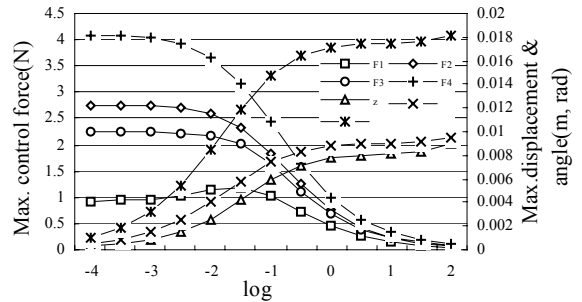


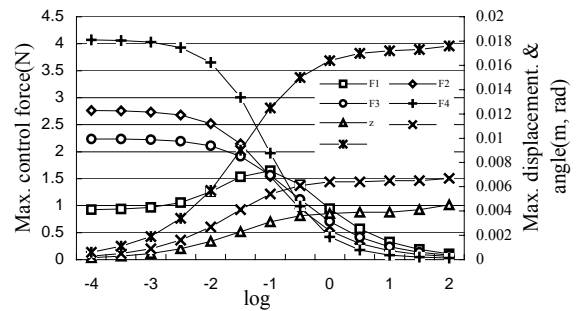
Fig.4 Block diagram of output feedback control

Table 1 Properties of vibrating plate

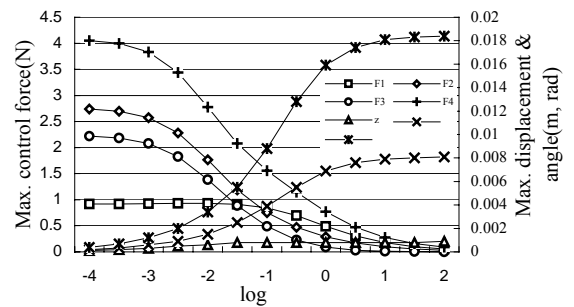
m	0.25	Kg
J ₁ , J ₂	0.0017, 0.00075	Kgm ²
k ₁ , k ₂ , k ₃ , k ₄	880	N/m
Dimensions	285(L) x 190(W) x 1.5(H)	mm
l ₁ , l ₂ , l ₃ , l ₄	150, 135, 100, 90	mm
a, b	30, 30	mm



(a) Frequency 15Hz

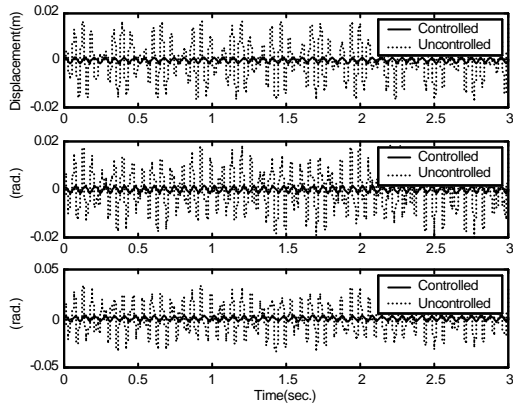


(b) Frequency 25Hz

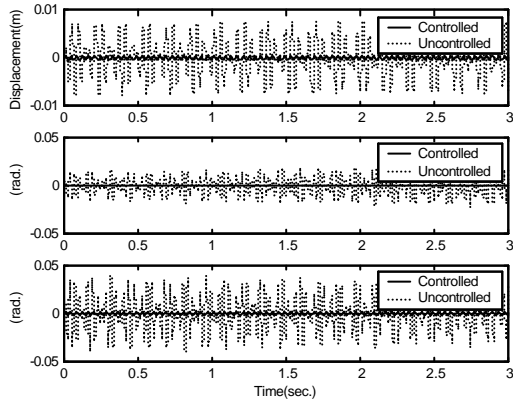


(c) Frequency 40Hz

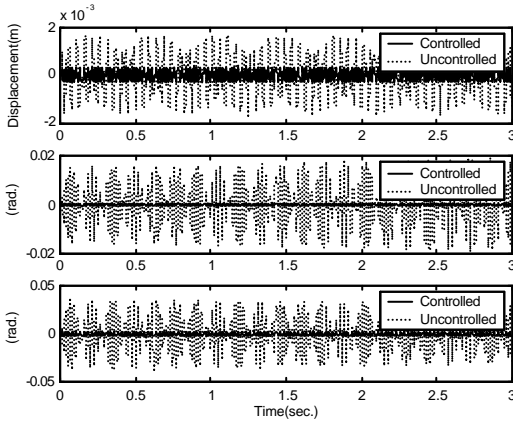
Fig.5 The comparison between max. control force and max. state responses(z, θ, φ)



(a) Frequency 15Hz



(b) Frequency 25Hz



(c) Frequency 40Hz

Fig.6 Comparison of displacement and angle responses on $r = -3$ during controlled and uncontrolled

5.

LQR
가

3

3

)
가

가 가

가

- (1) Hyun Moo Koh, Kwan Soon Park, Won Suk Park, 1995, "Active Control of Structural Vibration by Optimal Output Feedback Controller Considering Design Constraints", KSEC report, Vol 15, pp. 1535~1544.
- (2) James R. Rowland, 1986, "Linear Control Systems : Modeling, Analysis and Design", John Wiley & Sons, pp. 469~484.
- (3) Sergey Edward Lyshevski, 2001, "Control Systems Theory with Engineering Applications", Birkhauser, pp. 142~148 & 204~206.
- (4) Charles L. Phillips, Royce D. Harbor, 1988, "Feedback Control Systems", Prentice-Hall, pp. 73~76 & 536~538.
- (5) Gene F. Franklin, J. David Powell, 1991, Feedback Control of Dynamic Systems", 2nd ed., Addison-Wesley, pp. 361~364.
- (6) Richard C. Dorf, Robert H. Bishop, 1998, "Modern Control Systems", 8th ed., Addison-Wesley, pp. 641~645.
- (7) Katsuhiko Ogata, 1990, "Modern Control Engineering", 2nd ed., Prentice-Hall, Vol III, pp. 682~695 & 817~826.
- (8) Robert E. Skelton, 1988, "Dynamic Systems Control : Linear Systems Analysis and Synthesis", John Wiley & Sons, pp. 344~351.
- (9) Donald E. Kirk, 1970, "Optimal Control Theory_An Introduction", Prentice-Hall, pp. 209~227.
- (10) Hadi Saadat, 1993, "Computational Aids in Control System using Matlab", Mc Graw-Hill, pp. 126~133.
- (11) Bahram Shahiam, Michael Hassul, 1993, "Control System Design using Matlab", Prentice-Hall, pp. 366~371.