DEM 을 이용한 회전원통에서의 세탁물의 동역학적 해석

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Dynamic Analysis of Fabric in a Rotating Horizontal Drum Using the Discrete Element Method

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Key Words : Fabric, Horizontal Drum, Fabric Dynamics, Discrete Element Method.

Abstract

In order to provide a tool for designing more efficient methods of mixing fabric, a simplified discrete element computational model was developed for modeling fabric dynamics in a rotating horizontal drum. Because modeling the interactions between actual pieces of fabric is quite complex, a simplified model was developed where individual pieces of bundled fabric are represented by spherical particles. The simulations are used to investigate fabric bundle kinematics, the power required to drive the rotating drum, and the power dissipated through normal and tangential contacts. Parametric studies indicate only fill percentage, drum rotation speed, and friction coefficient play significant roles in the fabric bundle dynamics.

1. Introduction

In order to design more efficient methods for cleaning fabric in washing machines, a good understanding of fabric dynamics is desirable. Despite speculation that the mechanical action between fabric elements plays a significant role in the cleaning process, little is known regarding these mechanical forces. There exists little published analytical or computational data on how fabric behaves in washing machines and experimental data is typically proprietary. The purpose of this work is to present a computational model for studying fabric dynamics in rotating horizontal drums and present results from parametric studies utilizing this model. This work is the first published study to investigate this topic.

Ward (2000) investigated the dimensionless parameters involved in designing a horizontal washer. In this paper, the trajectory of a single fabric "plug" in a horizontal rotating drum was modeled. No comparisons with experimental data were made and the model did not consider multiple pieces of fabric or fabric flexibility. Recently there has been considerable fabric simulation work in the computer graphics industry (CGI). These simulations are used in a variety of applications including apparel CAD/CAM systems, virtual environments for movies and videogames, and WWW-based fashion models for clothing retailers. The emphasis of these simulations is to produce fast, visually realistic representations of how fabric behaves rather than mechanically accurate models. Although seemingly unrelated, the dynamics of particles in ball mills provide useful insights into better understanding fabric dynamics in rotating horizontal drums. Despite containing a significantly different material, the macroscopic behavior of ball mills and fabric in rotating drums is expected to be similar since the geometries are essentially identical. The dynamics of particles in ball mills have been widely studied (Acharya, 2000; Cleary, 2001).

In this paper the discrete element computational method is used to model the dynamics of individual fabric bundles in a horizontal drum washer. Although the model is simplistic, it nevertheless provides some insight into the behavior and interaction of fabric. The model is used to perform parametric studies to better understand the trends occurring in a rotating horizontal drum.

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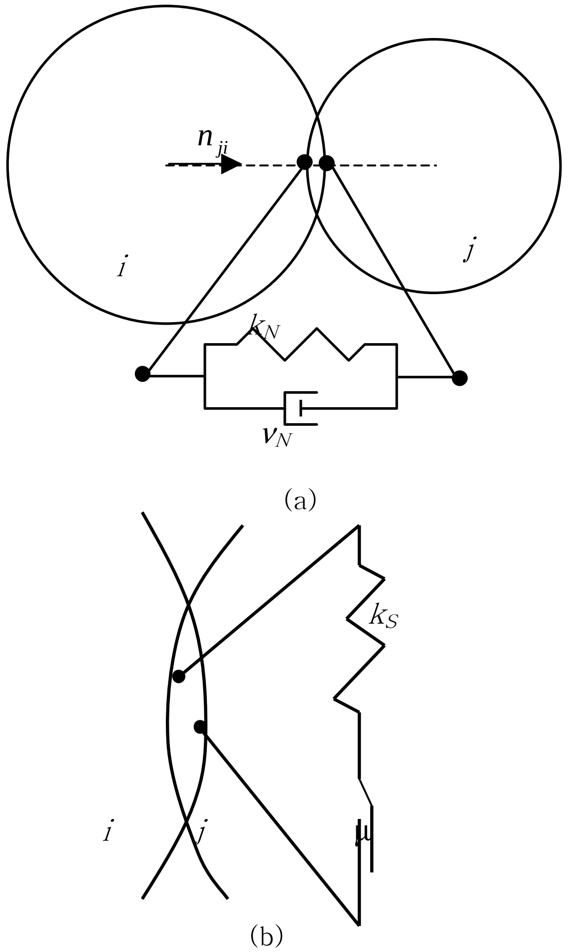


Fig. 1 Schematics of the (a) damped linear spring normal contact model and the (b) linear spring in series with a sliding friction element tangential contact model.

2. Computational model

In the model developed for this study utilizes the soft-particle discrete element (DE) method. In the soft-particle DE method, the dynamics of individual elements, or in this case, fabric bundles, are modeled using Newton's Laws. At every moment in time, the forces acting on each fabric bundle are calculated, Newton's Second Law is used to determine each bundle's acceleration, and the resulting equations of motion are integrated in time to determine new bundle states. The process is repeated until some ending condition, a maximum time for example, is reached. A more detailed description of the DE method is given by Cundall and Strack (1979).

The DE approach is very similar to the force-based, physical models currently used in the CGI industry (Breen et al., 1994). In the CGI simulations, cloth is typically modeled as an array of interacting plates or point masses in which spring, dashpot, and gravitational forces act on the intra-cloth elements. This technique has been used to simulate hanging cloth, cloth draped over objects, waving flags, and clothes on virtual human models.

Because modeling the interactions between pieces of three-dimensional, flexible fabric is quite complex, a simplified model is utilized here. Individual pieces of fabric are assumed to form "bundles" that are represented in the DE model as two-dimensional, circular elements. Observations of real fabric behavior in horizontal drums indicate that smaller fabric pieces do indeed tend to ball up to form bundles. When represented as circular particles, the horizontal rotating drum fabric bundle model is essentially a ball mill geometry albeit with different drum diameters, rotation speeds, fill levels, and material properties.

The forces acting on a fabric bundle are assumed to consist of a gravity body force and mechanical contact forces due to contact with other bundles and the washer drum and baffles. Hydrodynamic forces are not included in the model since the amount of water typically found in horizontal drum washers is small and the fabric is essentially just wetted rather than being fully immersed.

The gravitational body force acting on bundle i, $\mathbf{F}_{G,i}$, is given by:

$$F_{G,i} = m_i \, \mathrm{g}$$

where m_i is the mass of bundle i and $\mathbf{g}$ is the acceleration due to gravity. The gravity body force acts at the bundle's center of gravity.

Modeling the contact force between bundles is more complex. In the soft-particle DE approach the contact force is determined by allowing the bundles to slightly overlap when in contact, typically less than 1% of the bundle radius. This overlap represents the surface deflections occurring when real materials come into contact. The magnitude of the overlap, and often the relative speed of the impact, are used to determine the resulting contact force. One of the most simple and common contact force models is the damped, linear spring normal contact model and the linear spring in series with a sliding friction element tangential contact model, both of which were originally proposed by Cundall and Strack (1979) (refer to figure 1). In these models, the springs provide an elastic, repulsive force while the dashpot and sliding friction coefficient provide

The normal contact force that bundle j exerts on bundle i, $\mathbf{F}_{N,ji}$ is:

$$F_{N,ji} = [-k_N \alpha_{ji} + \nu_N (\Delta x_{c,ji} \cdot n_{ji})] \, n_{ji}$$

where k_N and $_N$ are the normal spring constant and dashpot coefficient, respectively, $_{ji}$ is the overlap between the two bundles, $\Delta\dot{\mathbf{x}}_{c,ji}$ is the velocity of bundle j relative to bundle i at the contact point, and $\hat{\mathbf{n}}_{ji}$ is the unit vector pointing from the center of bundle i to the center of bundle j. The force on bundle i due to contact with bundle j acting in the direction tangent to the contact, $\mathbf{F}_{S,ji}$, is given by:

Table 1 Baseline parameter used in the simulations

Parameter	Value
Drum diameter, *D*	47.5 (cm)
Drum rotation speed,	54 (rpm)
Bundle diameter, *d*	2.375 (cm)
Bundle density,	1000 (kg/m^3)
Number of bundles, *N*	100 (25%)
Bundle/bundle and bundle/drum	0.5
Bundle/bundle and bundle/drum	0.1
Bundle/bundle k_N	9.25*10^9 (N/m)
Bundle/bundle $_N$	5.35*10^3 (N*m/s)
Bundle/bundle k_S/k_N	1
Bundle/drum k_N	9.70*10^9 (N/m)
Bundle/drum $_N$	7.74*10^3 (N*m/s)
Bundle/drum k_S/k_N	1
Simulation time step, *t*	3.70*10^{-6} (sec)

$$F_{S,ji} = \begin{cases} -\int k_S \Delta\dot{x}\cdot\hat{s}_{ji}\,dt & if \quad \mu\left|F_N\right| > \left|\int k_S \Delta\dot{x}\cdot\hat{s}_{ji}\,dt\right| \\ -\mu\left|F_N\right|\hat{s}_{ji} & if \quad \mu\left|F_N\right| \le \left|\int k_S \Delta\dot{x}\cdot\hat{s}_{ji}\,dt\right| \end{cases}$$

where k_S and are the tangential spring stiffness and friction coefficient, respectively, t_0 is the time at which the contact was initiated, and $\hat{\mathbf{s}}_{ji}$ is the unit vector normal to $\hat{\mathbf{n}}_{ji}$ pointing in the direction of the relative tangential velocity. The tangential spring is active when the spring force is a smaller than the frictional tangential force. However, when the friction force is smaller, the spring extension is adjusted so that it provides a force equivalent to the sliding friction force.

The normal spring stiffness and dashpot coefficient are determined from an effective coefficient of restitution for the collision between two fabric bundles, and by limiting the amount of overlap allowed during the collision (Wassgren, 1996). The properties used in this study are given in Table 1.

The simulated drum geometry is based on measurements of an actual horizontal axis washer. The circular drum has a diameter, D = 47.5 cm with three baffles located every 120° having the dimensions. The drum rotates at 54 rpm corresponding to a typical wash cycle speed.

Several measurements are made in the simulations. In addition to investigating the bundle kinematics, measurements were also made of the power required to drive the drum and the power dissipation and rates of energy storage in the bundle contacts and motion.

The power required to drive the drum is equal to the torque acting on the drum due to contact with the fabric bundles multiplied by the rotational speed of the drum (a constant in the current simulations). Since the locations and magnitudes of all of the drum/bundle contacts are known, the overall drum torque, power, and horizontal and vertical forces can be easily determined.

The simulations began with fabric bundles randomly

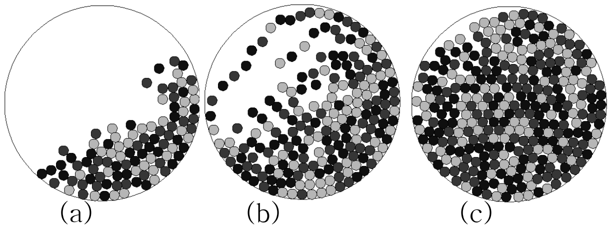
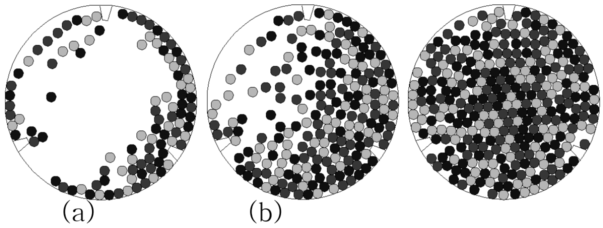


Fig. 2 Snapshots from without-baffle simulations for



fill percentages of (a) 25%, (b) 50%, and (c) 75%.

Fig. 3 Snapshots from with-baffle simulations for fill percentages of (a) 25%, (b) 50%, and (c) 75%.

placed within an already rotating drum. The simulations were allowed to reach a steady state, which typically occurred after approximately 5 seconds, before measurements were made. All of the results reported here were made over the 10 second period between t = 20 - 30 seconds.

3. Results and discussion

A number of parametric studies were performed in order to better understand what parameters most affect fabric bundle dynamics. Specifically, the effects of drum fill percentage, drum rotation speed, bundle size, coefficient of restitution, and friction coefficient were investigated.

3.1 Effects of fill percentage

Simulation snapshots are presented in figure 2 (without baffles) and figure 3 (with baffles) for fill percentages of 25%, 50%, and 75%. Considerable bundle mixing occurs during cascading. For a fill percentage of 50% and no baffles, the load begins to cataract where fabric bundles move part way up the drum surface and fall back onto the bed surface. At a fill percentage of 75% and no baffles, the drum is nearly full and little relative bundle motion occurs except in a small region at the load free surface.

The bundle load kinematics for the with-baffle cases are nearly the same as the without-baffle cases for the 50% and 75% fill percentages. At these fill percentages, most of the bundle load moves in solid body rotation so that the baffles have little effect on the bundle trajectories. However, the 25% fill percentage case with baffles is significantly different than the without-baffle case. At this fill percentage, the baffles carry

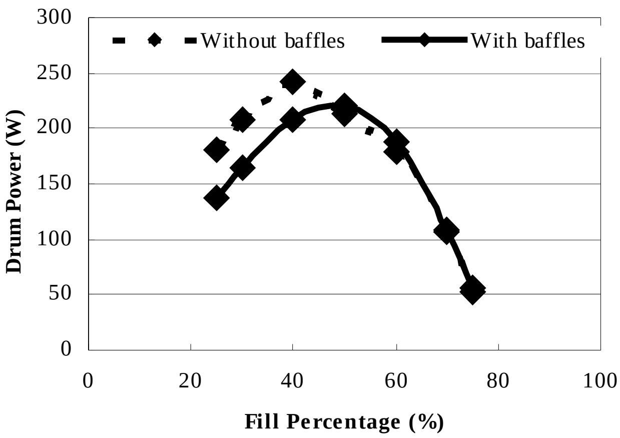


Fig. 4 The drum power plotted as a function of fill percentage for drums with and without baffles.

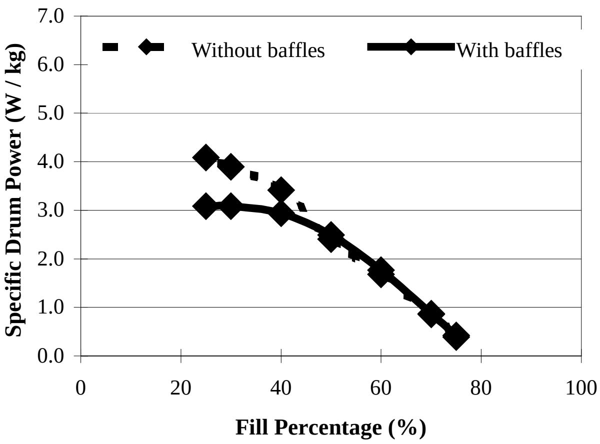


Fig. 5 The specific drum power plotted as a function of fill percentage for drums with and without baffles.

bundles against the drum circumference for most of the drum rotation cycle giving trajectories that appear similar to what would be expected if the drum speed was sufficient to cause centrifuging. As a result, less bundle mixing occurs for the 25% with-baffle case compared to the 25% without-baffle case.

In addition to qualitative kinematic investigations, the simulations were also used to investigate power dissipation characteristics as a function of drum fill percentage. Figure 4 plots this steady state drum power as a function of fill percentage for the with-baffle and without-baffle cases. A maximum in the power occurs at a fill percentage between 40% and 50% for both cases. Metcalfe et al. (1995) observed that when a drum of particulates is more than 50% filled, there remains a region of material near the axis of rotation that remains unmixed. Since there is little motion between particles, the power dissipated in particle collisions is reduced. Hence, a 50% fill percentage is expected to produce the greatest power dissipation since it is the maximum fill

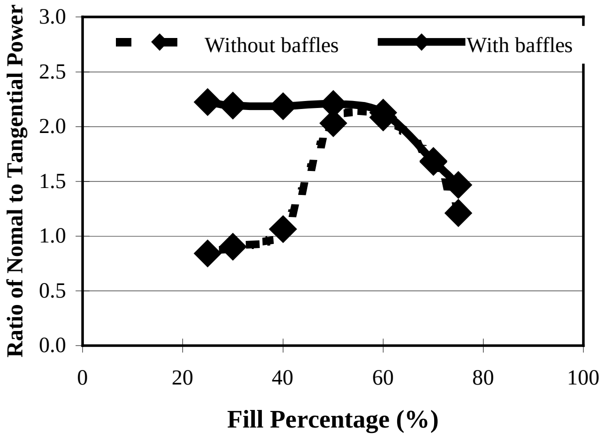


Fig. 6 The ratio of power dissipated and stored in normal contacts to the power dissipated and stored in tangential contacts plotted as a function of fill percentage.

percentage at which a significant degree of mixing occurs. Indeed, experimental measurements of the power required to drive a ball mill show a maximum at a fill percentage between 40% and 50% (Acharya, 2000; Cleary, 2001). The with-baffle and without-baffle cases have similar drum powers at the larger fill percentages, consistent with the observation that the bundle kinematics are similar in this regime. At smaller fill percentages the with-baffle case has a smaller drum power since less relative bundle motion (corresponding to less power dissipation) occurs as described previously.

A more useful quantity to investigate than the drum power is the specific drum power, defined as the power divided by the total load mass. Larger values of the specific drum power correspond to larger rates of energy dissipation (or mechanical action) between the fabric bundles. The data shown in figure 4 are plotted in figure 5 in terms of the specific power. The specific power decreases with increasing fill percentage indicating that more mechanical action occurs on a fabric bundle as the load size decreases.

In order to determine what percentage of the mechanical action acting fabric bundles is due to normal and tangential interactions, the ratio of the normal-to-tangential power (includes both spring and dissipative elements) is plotted in figure 6 as a function of the fill percentage. The ratio ranges between just less than one to just greater than two indicating that the normal contact interactions generally dissipate more power than the tangential interactions. Despite using significantly different parameters and a different baffle geometry, the ball mill simulations by Acharya (2000) gave a normal-to-tangential power ratio of approximately 3.5, close to the values determined here.

Of particular note in the plot are the significant differences between the with- and without-baffle cases at small fill percentages. The simulations including baffles give a nearly constant ratio of approximately 2.2

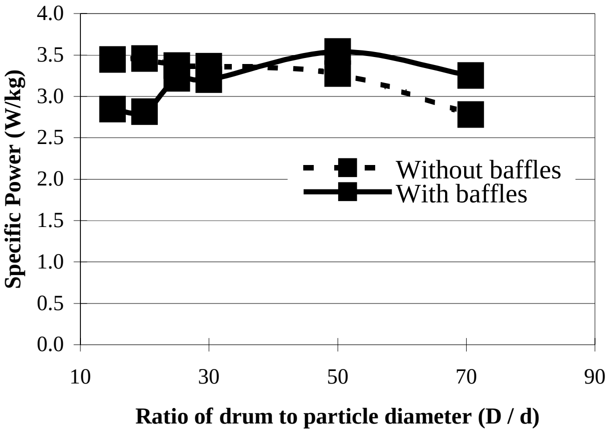


Fig. 7 The specific drum power plotted against dimensionless bundle size, *D*/*d*, for a drum with and without baffles and a 40% fill percentage. The drum diameter of *D* = 47.5 cm remains constant in the simulations.

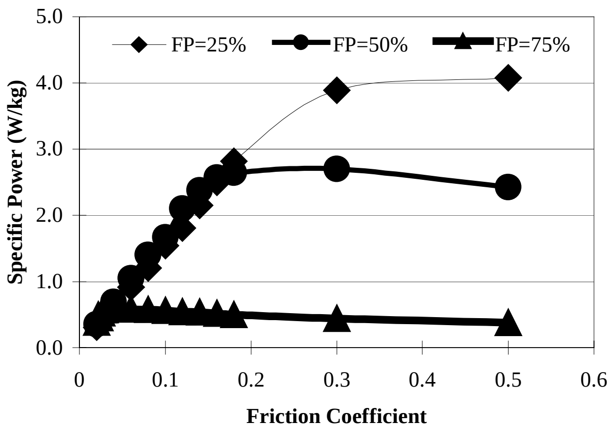


Fig. 8 The specific drum power plotted against friction coefficient for a drum without baffles and fill percentages of 25%, 50%, and 75%. The friction coefficient between bundles and between bundles and the drum are equal.

for fill percentages less than 60%. For larger fill percentages the ratio decreases monotonically. However, when the baffles are not included, the ratio is approximately one for fill percentages less than 40% with the value rapidly increasing to a value near two at a fill percentage of approximately 50%. For larger values of the fill percentage the without-baffle case is similar to the with baffle case.

The similarity at large fill percentages appears to be due to the similarity in bundle trajectories as discussed previously. At smaller fill percentages without baffles, the fabric bundles avalanche down the load free surface. The resulting relative tangential motion produces significant tangential interactions. When baffles are included, however, there is little relative tangential motion between the bundles since they are carried along the drum circumference by the baffles. Hence, the normal-to-tangential power ratio for the with-baffle case at small fill percentages is expected to be larger than the corresponding without-baffle case.

3.2 Effects of bundle diameter

A parametric study of the bundle size was also conducted. Figure 7 plots the specific drum power as a function of the drum-to-bundle diameter ratio, *D*/*d*, for a 40% fill percentage. Note that the baseline bundle diameter of *d* = 2.375 cm corresponds to *D*/*d* = 20. The specific drum power for the without-baffles case decreases by approximately 20% over a corresponding decrease of nearly 370% in the bundle diameter. The specific drum power for the with-baffles case changes by approximately the same amount over the same range of *D*/*d*; however, a maximum occurs at a fill percentage near 50%. Hence, the specific drum power is a weak function of the fabric bundle diameter.

3.3 Effects of friction coefficient

The effects of friction on the specific power are shown in figure 8 for a drum with no baffles and fill percentages of 25%, 50%, and 75%. Note that the friction coefficient between the fabric bundles and between the bundles and the drum rim are equal in the all of the investigations. The specific power generally increases from zero at = 0 to an approximately constant value for > 0.3. Zero specific power at = 0 is expected for the no-baffle case since if friction is not present, then the bundles simply slide along the surface of the drum.

Plotting the normal-to-tangential power ratio as a function of the friction coefficient reveals additional complex behavior. At points (a) and (b) in the figure 9, the friction coefficient is small enough so that bundles do not circulate and instead the fabric load moves as a rigid body that rocks from side to side at the bottom of the drum. For these conditions the normal contact power is zero since the relative normal velocity between bundles is zero. There is still, however, a small, but non-zero, tangential component due to the rocking motion of the load. With increasing friction coefficient (points (c) and (d)), the bundles abruptly begin to convect within the load and cascade down the free surface of the bed. This relative bundle movement results in both normal and tangential power dissipation.

The normal-to-tangential power ratio increases with increasing friction coefficient for the 25% and 50% fill percentages but decreases for the 75% fill percentage. At the 25% and 50% fill percentages increasing friction results in a transition from cascading to cataracting (points (e) and (f)) and, as a result, larger normal power dissipation due to collisions of the falling bundles against the remainder of the fabric load. At the 75% percentage, however, cataracting cannot occur in the limited free

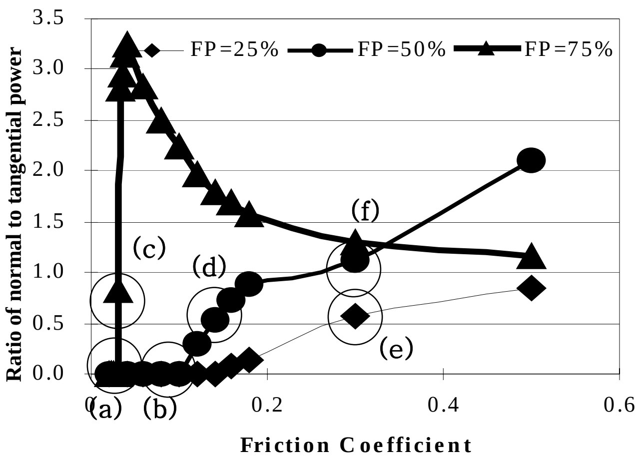


Fig. 9 The normal-to-tangential power ratio plotted against friction coefficient for a drum without baffles and fill percentages of 25%, 50%, and 75%. The friction coefficient between bundles and between bundles and the drum are equal.

volume and instead cascading appears. Few large normal contact dissipation events occur and the normal-to-tangential power ratio decreases with increasing friction coefficient.

4. Conclusion

The current work indicates that fill percentage, drum rotation speed, and friction coefficient play significant roles in the dynamics of fabric bundles in horizontal rotating drums. Drum baffles have only a minor effect on the power dissipation and kinematics for fill percentages greater than 50%. Bundle size has a relatively weak influence on the bundle dynamics.

In order to maximize the specific power dissipated in bundle contacts (*i.e.* the mechanical action on the fabric), the fill percentage should be decreased Increasing the friction coefficient increases the specific power in addition to significantly affecting the ratio of normal to tangential power, especially at large fill percentages. Baffles decrease the specific power and normal-to-tangential power ratio at small fill percentages due to the centrifuging-style trajectories of the bundles. These investigations should provide designers of fabric mixing devices additional tools and insights into the dynamics of fabric in rotating horizontal drums.

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