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이상 유동 이론에서의 평면 변형 벤딩

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Plane-strain bending based on ideal flow theory

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Abstract

The major objective of this paper is to clarify the effect of constitutive laws on bulk forming design based on the ideal flow theory. The latter theory is in general applicable for perfectly/plastic materials. However, its kinematics equations constitute a closed-form system, which are valid for any incompressible materials, therefore enabling us to extend design solutions based on the perfectly/plastic constitutive law to more realistic laws with rate sensitive hardening behavior. In the present paper, several constitutive laws commonly accepted for the modeling of cold and hot metal forming processes are considered and the effect of these laws on one particular plane-strain design is demonstrated. The closed form solution obtained describes a non-trivial nonsteady ideal process. The design solutions based on the ideal flow theory are not unique. To achieve the uniqueness, the criterion that the plastic work required to deform the initial shape of a given class of shapes into a prescribed final shape attains its minimum is adopted. Comparison with a non-ideal process is also made.

Key Words : Constitutive laws, Bulk forming design, Ideal flow theory, Perfectly/plastic materials, Rate sensitive hardening behavior

1. Introduction

In the ideal plastic flow theory, all material elements follow minimum work paths in which one family of material lines is perpetually tangent to principal strain rate vectors[1]. The theory can be applied to find strain paths that lead to the minimum work of deformation for given initial and final shapes and/or to find an optimal initial shape assuming that the final shape is prescribed. A disadvantage of the theory is that, in principle, it is only applicable for rigid-perfectly plastic solids based on the Tresca yield criterion and its associated flow rule[2]. In the present paper, an effect of different constitutive laws on the ideal flow design is discussed.

2. Statement of the problem

Consider a class of initial shapes, which are rectangular with sides $2L$ and H as shown in Fig. 1(a). Its area $\Omega = 2LH$ is fixed, but L and H may vary to satisfy the criterion of optimality. The initial shape should be transformed into the final shape prescribed in advance in Fig. 1(b) under the plane-strain condition. The final shape is completely defined by the

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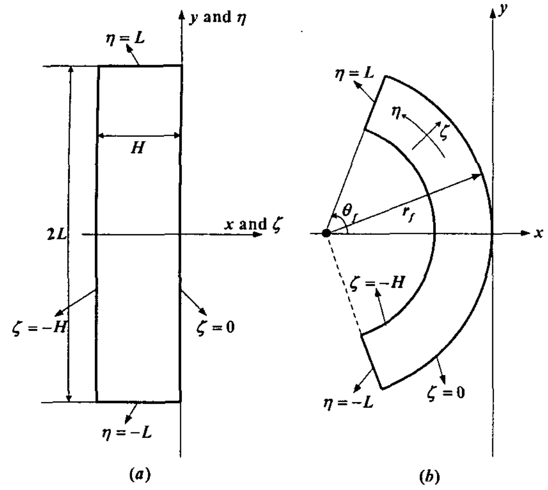

Fig. 1 Notations for (a) initial and (b) final shapes

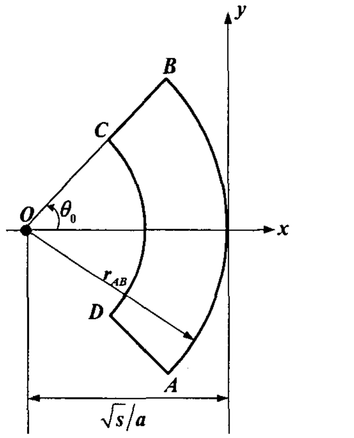

Fig. 2 Notations for an arbitrary intermediate shape

radius r_f and the angle θ_f. The surface of the body is assumed to be free of shear stress. The constitutive law for the material consists of the yield criterion and its associated flow rule. The yield stress is assumed to be dependent on the equivalent strain or/and the equivalent strain rate.

3. Kinematics of the process

Kinematics of the process is described by the incompressibility equation and the condition that one family of material lines is perpetually tangent to principal strain rate vectors. An additional requirement is that neither principal strain rate changes its sign at each material point throughout the process. It is convenient to introduce a Cartesian Eulerian coordinate system xy and a Lagrangian convective coordinate system $\zeta\eta$ defined by the conditions $x=\zeta$ and $y=\eta$ at the initial instant, $t=0$. Consider the following transformation equations

$$x=\sqrt{\frac{\zeta}{a}+\frac{s}{a^2}}\cos(2a\eta)-\frac{\sqrt{s}}{a}$$
$$y=\sqrt{\frac{\zeta}{a}+\frac{s}{a^2}}\sin(2a\eta) \qquad (1)$$

where a is a time-like variable such that $a=0$ at the initial instant and s is a function of a. It is possible to show that the initial conditions are satisfied if $s=1/4$ at $a=0$. Using Eq.(1) the components of the metric tensor of the Lagrangian coordinates are determined in the form

$$g_{\zeta\zeta}=1/4(\zeta a+s),\ g_{\eta\eta}=4(\zeta a+s),\ g_{\zeta\eta}=0. \qquad (2)$$

Since $g_{\zeta\eta}=0$, the coordinate curves of the Lagrangian coordinate system coincide with trajectories of the principal strain. Using Eq.(2) the principal strain rates can be found in the form

$$\xi_{\zeta\zeta}=-\xi_{\eta\eta}=-\frac{(\zeta+ds/da)}{2(\zeta a+s)}\frac{da}{dt}. \qquad (3)$$

The only condition that remains to be considered is that neither principal strain rate changes its sign at each material point throughout the process. The sign of each of these strain rates changes where $\zeta=-ds/da$. Therefore, ideal flow paths are obtained if $ds/da=-\zeta_0=const$. It follows from this condition and the initial condition for s that $s=1/4-\zeta_0 a$. Then, Eqs. (2) and (3) result in

$$\xi_{eq}=\pm\frac{1}{g_{\zeta\zeta}}\frac{\partial g_{\zeta\zeta}}{\partial t}=\mp\frac{4(\bar{\zeta}-\bar{\zeta}_0)}{4Ha(\bar{\zeta}-\bar{\zeta}_0)+1}\frac{d(Ha)}{dt} \qquad (4)$$

where the upper sign corresponds to $-1\le\bar{\zeta}\le\bar{\zeta}_0$ and the lower sign to $\bar{\zeta}_0\le\bar{\zeta}\le 0$ and the equivalent strain rate ξ_{eq} is defined as $2|\xi_{\zeta\zeta}|$. Also, $\bar{\zeta}=\zeta/H$ and $\bar{\zeta}_0=\zeta_0/H$. The equivalent strain is given by

$$\partial \varepsilon_{eq} / \partial t = \xi_{eq} . \quad (5)$$

Substituting Eq.(4) into Eq.(5) and integrating, with the initial condition $\varepsilon_{eq} = 0$ at $a = 0$, gives

$$\varepsilon_{eq} = \mp \ln\left[4Ha\left(\bar{\zeta} - \bar{\zeta}_0\right) + 1\right] . \quad (6)$$

The structure of Eqs.(1) shows that the boundary of any intermediate shape consists of two circular arcs and two straight lines (Fig.2). For later convenience, a cylindrical coordinate system $r\theta$ with its origin at $x = -\sqrt{s}/a$ and $y = 0$ is introduced by

$$x + \sqrt{s}/a = r\cos\theta \text{ and } y = r\sin\theta . \quad (7)$$

It follows from Eqs.(1) and (7) that

$$r = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}} \text{ and } \theta = 2a\eta . \quad (8)$$

4. Stress analysis

It follows from Eqs.(8) that the coordinate curves of the $\zeta\eta$ coordinate system coincide with the coordinate curves of the $r\theta$ coordinate system. Therefore, the isotropic plane-strain yield criterion becomes $\sigma_{rr} - \sigma_{\theta\theta} = \pm 2k$ where k is the shear yield stress. It follows form Eqs.(4) and (6) that neither ξ_{eq} nor ε_{eq} depends on η or, due to Eqs.(8), on θ. Hence, the only non-trivial equilibrium equation is

$$\frac{\partial \sigma_{rr}}{\partial \bar{\zeta}} = \mp \frac{4kHa}{4\left(\bar{\zeta} - \bar{\zeta}_0\right)Ha + 1} . \quad (9)$$

It is clear that the solution to this equation exists if the boundary conditions on σ_{rr} are $\sigma_{rr} = const$ at $\bar{\zeta} = 0$ and $\bar{\zeta} = -1$.

5. Process design for different constitutive laws

The solution described in the previous sections contains one free parameter, $\bar{\zeta}_0$. This parameter can be determined by means of the criterion of optimality (the plastic work attains its minimum among a class of initial shapes chosen). In Lagrangian coordinates, the plastic work per unit volume is defined by the equation

$$\frac{\partial w}{\partial (Ha)} = \mp \frac{4k\left(\bar{\zeta} - \bar{\zeta}_0\right)}{4Ha\left(\bar{\zeta} - \bar{\zeta}_0\right) + 1} . \quad (10)$$

The plastic work required to deform the initial shape into the final shape, W, can be found by integration w over the area. The value of W has been minimized numerically assuming that $k = \sigma_0 \Phi\left(\varepsilon_{eq}\right)\Lambda\left(\xi_{eq}\right)$ where $\Lambda\left(\xi_{eq}\right) = 1 + \left(\xi_{eq}/\tilde{\xi}_{eq}\right)^m$ and $\Phi\left(\varepsilon_{eq}\right) = 1 + (\beta - 1)\left(1 - e^{-n\varepsilon_{eq}}\right)$. The result of the minimization is the optimal value of L. The perfectly plastic material is obtained at $\Phi\left(\varepsilon_{eq}\right) \equiv 1$ and $\Lambda\left(\xi_{eq}\right) \equiv 1$ (in this case $L \equiv L_{pp}$), the rigid-plastic, hardening material at $\Lambda\left(\xi_{eq}\right) \equiv 1$ ($L \equiv L_{ph}$), the viscoplastic material at $\Phi\left(\varepsilon_{eq}\right) \equiv 1$ ($L \equiv L_{vp}$). In the case of viscoplastic, hardening material $L \equiv L_{vph}$. A measure of the deviation from the design based on the perfectly/plastic material can be defined by

$$\delta_{ph} = \frac{\left|L_{ph} - L_{pp}\right|}{L_{pp}}, \ \delta_{vp} = \frac{\left|L_{vp} - L_{pp}\right|}{L_{pp}}, \ \delta_{vph} = \frac{\left|L_{vph} - L_{pp}\right|}{L_{pp}} . \quad (11)$$

As typical values of an aluminum alloy, the following values are considered: $n = 7.63$ and $\tilde{\xi}_{eq} = 6500 s^{-1}$ (3). In all cases, it is assumed that $\theta_f = \pi/2$, $r_f = \sqrt{5}/2$, $\Omega = \pi/2$, and $u = 1.0 s^{-1}$. The variation of the deviation parameters and the dimensionless plastic work, $E = W/(\sigma_0 \Omega)$, calculated at the optimum values of L with β and m are shown in Figs.3-6.

6. Conclusion

The main result of this work is that the effect of various constitutive laws on the ideal flow design is small and maybe even negligible in many cases. It is confirmed from Figs.3-6 in which the dependence of the measures of deviation from the perfectly/plastic design introduced in Eq.(11) on different material parameters is virtually minimal. The maximum deviation is less than 1.5%. It suggests that designs based on the perfectly/plastic

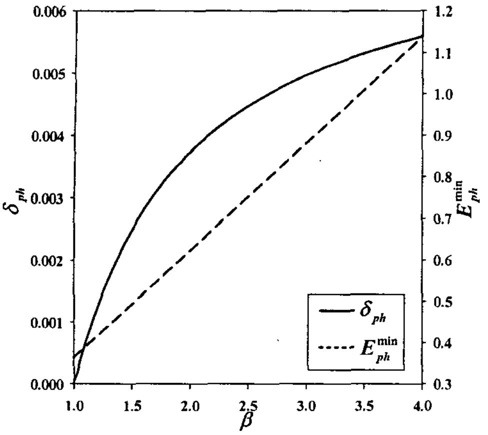

Fig. 3 Variation of δ_{ph} and E_{ph}^{min} with β for the rigid/plastic, hardening material

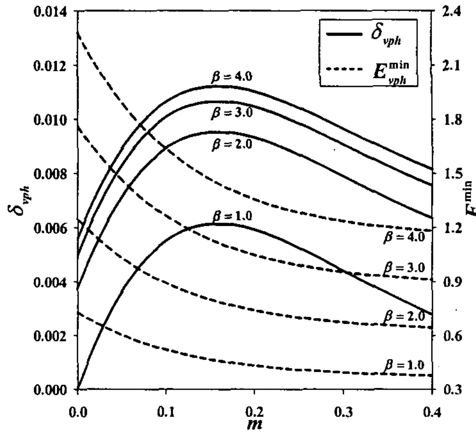

Fig. 5 Variation of δ_{vph} and E_{vph}^{min} with m at different β for the viscoplastic, hardening material

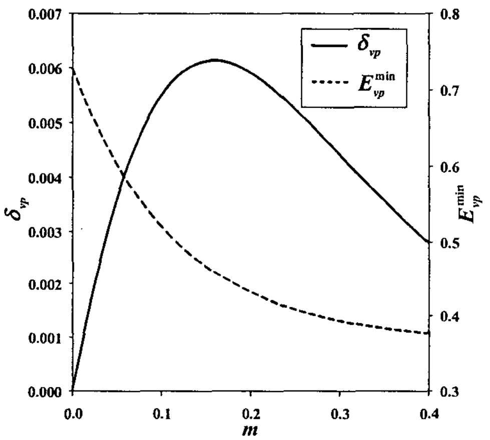

Fig. 4 Variation of δ_{vp} and E_{vp}^{min} with m for the viscoplastic material

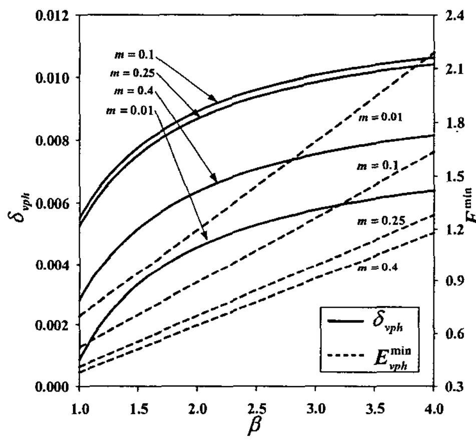

Fig. 6 Variation of δ_{vph} and E_{vph}^{min} with β at different m for the viscoplastic, hardening material

material provide reasonable design guidelines for quite an arbitrary constitutive law.

Acknowledgement

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