A Study on Mesh Sensitivity of 3-D Homoginized Crack Model for Concrete Fracture Analysis

Nam, Jin Won*　Song, Ha Won**　Byun, Keun Joo**　Bang, Choon Suk***

ABSTRACT

Since quasi-brittle materials like concrete show strain localization behavior accompanied by strain softening, a numerical drawback such as mesh sensitivity is appeared in the finite element analysis. In this paper, the so-called homogenized crack model which was introduced for three dimensional finite element analysis of fracture in concrete is studied for the mesh size dependence problem in fracture analysis. A homogenized crack element having a velocity discontinuity is averaged to remove the mesh sensitivity in finite element analysis of concrete fracture. Numerical examples show that softening behavior of concrete fracture is successfully predicted without mesh sensitivity using the homogenized crack model.

1. Introduction

The failure process in concrete can be associated to the formation of single cracks, crack bands and shear bands. Many models have been proposed to simulate the behavior of concrete fracture. In the eighties the effort was on the development of the discrete crack model and the smeared crack model. In the nineties, it was developed a type of finite element model capable of modeling localized damage without remeshing, resulted in the embedded discontinuities approximations[1).

Numerical application of the strain softening makes loss of ellipticity of governing equation and boundary value problem becomes ill-posed. Consequently, a numerical drawback such as mesh sensitivity including the orientation of the mesh is appeared in the finite element analysis. Recently, various models have been proposed to solve these problems. For example, as an effort to develop the models, Oliver[2)] proposed discontinuity model that presumes fracture of concrete as phenomenon of discontinuity in strain localization problem.

In this paper, the so-called Homogenized Crack Model (HCM), which was proposed to simulate the fracture behavior of concrete[3)], is studied as a solution for the mesh sensitivity problem in the finite element analysis of concrete fracture using the model. The solution of mesh objectivity of crack models are reviewed and their advantages or disadvantages as the solution are discussed.

* Graduate Student, School of Civil and Environmental Eng., Yonsei University

** Professor, School of Civil and Environmental Eng., Yonsei University

*** Researcher, Korea Railroad Research Institute

2. Regularization of Continuum Model

For the sake of attaining mesh objectivity of continuum model, several regularization techniques have been developed during the past decades. These remedies can be classified into two main groups. The first group including crack band model and embedded crack model is well known to solve the mesh sensitivity without material stability but can be mesh dependent in the infinitely small softening region. The second group including non-local continuum damage model and gradient model can solve the mesh dependancy with preserving material stability. All these models have a similarity with regard to having a certain limit, i.e. an additional internal length scale, l, which controls the size of crack or localization zone must be needed.

Among those regularization techniques, embedded displacement discontinuity elements have been shown to be an effective tool to describe the propagation of cracks independent of element boundaries and most of the work has been devoted to embedded discontinuities into displacement-based finite elements[5]. There are two main approaches in the embedded discontinuity model: continuum and discrete (weak discontinuity and strong discontinuity, Fig. 1). See Table 1 for the comparison of the characteristic length for the models. Those, non-linear continuum constitutive models (stress-strain laws), commonly used in continuum approached to failure, and the discrete laws (tension-softening laws), considered in non-linear fracture mechanics, are not independent from each other. When successful in modeling failure of concrete, both types of constitutive models must be closely related[6].

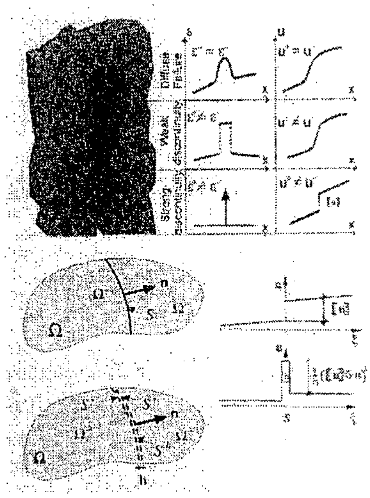

Fig. 1 Kinematics of a discontinuity[6]

From these points of view, the HCM can be an effective technique for attaining mesh objectivity without additional internal length, which will be discussed at next chapter, for concrete fracture analysis by interlinking continuum and discrete approaches. HCM also has a strong point by considering velocity discontinuities using stiffness of crack. Consequently, HCM can be considered as a modified embedded discontinuity continuum approach.

Table 1. Comparison of crack models as a mesh sensitivity solutions

Models	Basic equations	Internal variables	Notes
crack band model	$\varepsilon_c = \frac{w_c}{l_r}$	l_r, the reference length of the finite element	-simply applicable -useless in the infinitely small softening region
non-local continuum model	$\bar{f}(x) = \int_V \alpha(x,\xi) f(\xi) d\xi$, $\alpha_0(r) = \exp(-\frac{n_{\dim} r^2}{2l^2})$	l, internal length of the nonlocal continuum	-the loading-unloading conditions have a nonlocal character, -the stress return algorithm cannot be performed at each Gauss integration point separately.
gradient model	$f(\sigma, \bar{x}) = F(\sigma) - h(\bar{x}),\ \bar{x} = x + c\nabla^2 x$	c, material parameter proportional to the square of the internal length	
embedded crack model	$\varepsilon(x,t) = \nabla^s \bar{u} + H_k \nabla^s [[u]] + \mu_k \frac{1}{k}([[u]] \otimes n)^s$	κ, localization zone width	-stress locking is observed as the crack growth
homogenized crack model	$[\delta]\dot{\varepsilon} \approx [\delta]\dot{\varepsilon}^i + \mu \dot{g}$	μ, volume fraction	-no additional internal length scale is necessary

3. Homogenized Crack Model

In this paper, the basic concept of 3-D HCM is introduced. A detailed procedure for constitutive equation of 3-D HCM is shown elsewhere (Song et al[3]).

Stress rate and strain rate of concrete and of crack within a representative elementary volume (REV, Fig. 2) are denoted by $\dot{\sigma}^i$, $\dot{\varepsilon}^i$ and $\dot{\sigma}^j$, $\dot{\varepsilon}^j$, respectively, and following mixture rule can be established:

$$\dot{\sigma} = \mu_i \dot{\sigma}^i + \mu_j \dot{\sigma}^j , \quad \dot{\varepsilon} = \mu_i \dot{\varepsilon}^i + \mu_j \dot{\varepsilon}^j \tag{1}$$

where, μ_i and μ_j are the volume fraction of concrete and crack. If the thickness of a crack inside concrete is negligibly small compared to the size of the finite element, i. e. t≪H, each volume fraction approximates to 1 and t/H , i. e. $\mu_i = \frac{BW(H-t)}{HBW} \cong 1$, $\mu_j = \frac{BWt}{HBW} \cong \frac{t}{H}$.

The equilibrium equations as well as compatibility conditions in the interface between concrete and crack surface can be established by assuming that the crack has finite material properties such as normal and shear stiffness. In local coordinate system, these equations are as follows:

$$\dot{\sigma}_{yy} = \dot{\sigma}^i_{yy} = \dot{\sigma}^j_{yy} , \quad \dot{\tau}_{xy} = \dot{\tau}^i_{xy} = \dot{\tau}^j_{xy} , \quad \dot{\tau}_{yz} = \dot{\tau}^i_{yz} = \dot{\tau}^j_{yz}$$
$$\dot{\varepsilon}_{xx} = \dot{\varepsilon}^i_{xx} = \dot{\varepsilon}^j_{xx} , \quad \dot{\varepsilon}_{zz} = \dot{\varepsilon}^i_{zz} = \dot{\varepsilon}^j_{zz} , \quad \dot{\gamma}_{zx} = \dot{\gamma}^i_{zx} = \dot{\gamma}^j_{zx} \tag{2}$$

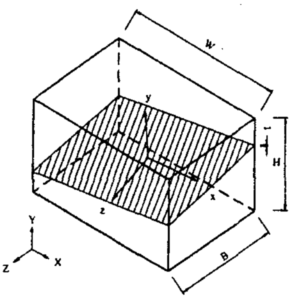

Fig. 2 Representative elementary volume[6] (REV).

Then the following velocity discontinuities in the normal and shear direction of the crack can be introduced:

$$\dot{g} = \{ \dot{g}_y, \ \dot{g}_x, \ \dot{g}_z \}^T, \quad [\delta] \dot{\sigma}^j = [K] \dot{g} \tag{3}$$

and average crack strain can be expressed as:

$$\frac{1}{t} \dot{g} = [\delta] \dot{\varepsilon}^j \tag{4}$$

Let μ be the ratio of the crack area and REV, i.e. μ=1/H, then, eq. (1) becomes:

$$[\delta] \dot{\varepsilon} \approx [\delta] \dot{\varepsilon}^i + \mu \dot{g} \tag{5}$$

Meanwhile, constitutive equation of the concrete can be modeled with a constitutive matrix [*D*] as:

$$\dot{\sigma}^i = [D] \dot{\varepsilon}^i \tag{6}$$

Eqs. (3)~(6) can be rearranged as follows for an equation for the strain rate of concrete by using appropriate matrixes, $[A]$, $[B]$ and $[S_1]$:

$$[\delta] \dot{\varepsilon}^i = [A] \dot{\varepsilon} + [B] \dot{g}, \quad \dot{\varepsilon}^i = [S_1] \dot{\varepsilon} \tag{7}$$

Finally, an averaged constitutive equation of the concrete having a crack inside can be derived as:

$$\dot{\sigma} = [D^{eq}] \dot{\varepsilon} , \quad [D^{eq}] = [D][S_1] \tag{8}$$

It is worthwhile to note that the averaged constitutive equation does not include the thickness of crack but thickness of crack finite element is included instead as a solution of the mesh sensitivity.

4. Numerical Analysis

Tensile failure analysis using HCM is carried out for a plain concrete specimen with double notches (Fig. 3) and the result is compared with an experimental data[7].

As shown in Fig. 4, the axial stress-axial deformation curves show softening behavior with a small discrepancy between peak strengths. Fig. 5 shows that the mesh sensitivity can be solved in the numerical analysis which has different number of elements by using HCM.

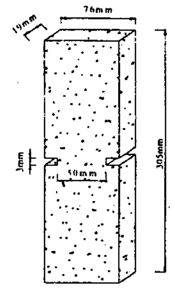

Fig. 3 Model specimen with double notches.

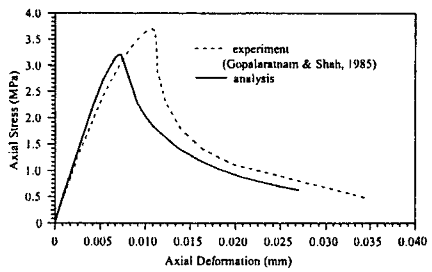

Fig.4 Axial stress-displ. relationship.

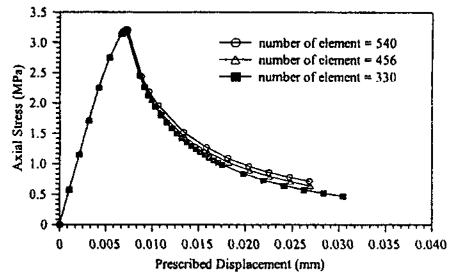

Fig. 5 Stress-displ. relationship for different mesh designs.

5. Conclusions

In this paper, the homogenized crack model (HCM) is studied for problem of mesh size dependence in the FEM analysis of concrete fracture. The HCM is manifested to a remedy for mesh sensitivity problem in compared with conventional regularization models in the point of no need for additional internal length. Results from the numerical analysis agree well with the experimental data and show mesh objectivity. The 3-D HCM for finite element analysis of concrete fracture shows that the mesh sensitivity is successfully overcome and failure behaviors of concrete are well simulated.

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