FRACTAL SURFACE ROUGHNESS OF CONCRETE

Y.S. Roh[1], Yunping Xi[2], and L. Chung[3]

ABSTRACT: In this study, the roughness of fracture surfaces in cementitious material has been characterized by roughness number (RN). A systematic experimental investigation was carried out to examine the dependency of fracture parameters on the aggregate sizes as well as the loading rates. Three aggregate sizes (0.1875 in, 0.5 in, and 0.75 in) and two loading rates (slow and fast loading rate) were used. A total of 52 compression tests and 53 tension tests were performed. All fracture parameters exhibited an increase, to varying degrees, when aggregates were added to the mortar matrix. The fracture surfaces of the specimens were digitized and analyzed. Fracture roughness was monotonically increased as maximum aggregate sizes increase.

***Key Words**: roughness; fracture surface; fractal; concrete; aggregate size*

1. INTRODUCTION

Fracture properties of concrete cannot be fully characterized by conventional linear elastic fracture mechanics. The nonlinear and inelastic behaviors have been attributed to irregularity of crack surfaces and nonhomogenity of internal structure of the material [1]. Therefore, quantitative descriptions of the correlation between the features of the internal structure and the fracture surface of concrete have been of great interest in recent years [1-4]. Observations on fracture surfaces of concrete indicated that different fracture processes could be characterized by different fracture surface topography. Hence, characterization of fractography becomes very important. Fracture surface provides us with invaluable information about energy absorption in the process of fracture, and it may bridges gap between micro-mechanical behaviors of fracture and the corresponding macroscopic fracture behavior of quasibrittle solids, such as concrete. In the present study, the effects of aggregate and loading rate on fracture properties of concrete are studied experimentally. The surface characteristics of fracture are evaluated by fractal theory [5]. The main purpose of this paper is to establish a quantitative correlation among fractality (fractal dimension, or roughness) of fracture surface of concrete, degree of heterogeneity (size of the aggregate), and loading rate effect. A carefully designed testing program was conducted and a theoretical model is developed based on the obtained test data.

2. EXPERIMENTAL PROGRAM

[1] *Researcher, Seismic Retrofitting and Remodeling Research Center, Seoul, roro3939@hanmail.net*
[2] *Assistant, Professor, C EAE, University of Colorado at Boulder, USA, xiy@colorado.edu*
[3] *Professor, Architectural Engineering, Dankook University, Korea, Lanchung@dku.edu*

Experimental investigation was carried out to examine the effects of aggregate size and loading rate on compressive and tensile strengths of concrete. Four different aggregate sizes were used. With different sizes of aggregates specimens were called hardened cement paste without any aggregate, mortar with fine sand, concrete with fine aggregate, and concrete with coarse aggregate (see Table 1). Two different loading rates were used: static loading rate (0.0001 to 0.00002 in/sec) and moderately fast loading rate (0.1 to 0.0625 in/sec.). Testing parameters are listed in Table 1. The water-cement ratio was kept as a constant $w/c = 0.5$, except for the cement paste specimens in which $w/c = 0.45$ was used. The ratio of total aggregate volume fraction to total volume of concrete, a/T, is kept constantly (a/T =0.7).

Table 1 Testing Parameters

	Cement Paste	Mortar	Fine agg.	Coarse agg.
d(avg)	-	#20	#8	#4
d(max)	-	0.1875in	0.5in	0.75in

3. SURFACE ROUGHNESS

Previous researches showed that the fracture surface of composite materials could be effectively described by fractal geometry [5-7]. There are numerous devices and methods for tracking and recording profile of fracture surface. A three-dimensional scanner was used to evaluate surface profiles. A mechanical probe is in contact with fracture surface, and elevations (distance from reference probe position to crack surface) are measured at a regular interval along crack profile in $25\mu m$ resolution. The topological information provided by 3D scanner is then used to determine surface roughness number (RN), which is defined as $RN = A_a(\delta)/A_o(\delta)$, where δ is the grid size or measurement interval; A_a and A_o are actual surface area and projected surface area between grids, respectively. The sum of all elemental areas provides an approximation for actual surface area. The surface roughness number can be calculated as $RN = \Sigma\Delta A_a(\delta)/\Sigma\Delta A_o$. From experimentally determined RN, fractal dimension, D, can be evaluated from $A_a(\delta) = A_o r^{-(D-d)}$, where d is the topological dimension, and r is a unit length scale. Fractal dimension, D, can also be evaluated by plotting roughness number versus measurement size in Richardson plot [8]. Slope of the linear region in log-log plot can be given by relationship: $D = d - slope$.

Figs. 1 to 4 show fracture surfaces obtained by 3-dimensional scanner from direct compression test specimens with different aggregate sizes at static loading rate. The interval of measurement resolution was from $25\mu m$ to $0.1m$. It is noticed that measurement interval larger than the maximum aggregate size should be included in the range of measurement scale.

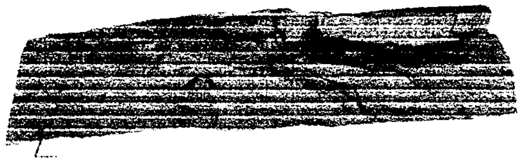


Fig. 1 Surface profile of cement paste

Fig. 2 Surface profile of mortar

Fig. 3 Profile of fine agg. surface

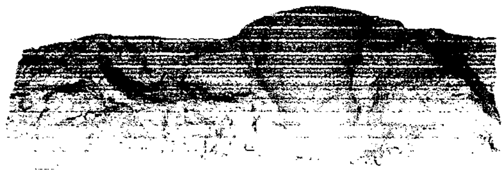


Fig. 4 Profile of coarse agg. surface

Fig. 5 shows maximum aggregate size versus strength. Both compressive strength and tensile strength increase with increasing aggregate size. Fractal dimension also increases with increasing aggregate size (Fig. 6). In both figures, the increases of strengths and fractal dimension are larger for compressive loading than for tensile loading (splitting tension specimen).
Fig. 7 shows the effect of loading rate on concrete strength. Compressive strength is increased at higher loading rate. Tensile strength does not change with increasing loading rate. This may due to limited range of loading rate used in experimental test. From Fig. 8, fractal dimensions for both compression and tension specimen decrease with increasing loading rates.

4. FRACTAL FRACTUE MODEL

From experimental observations, one can see clearly that the roughness of fracture surface depends on aggregate sizes, loading rates, and loading configurations (tension or compression). There are many possible scenarios of crack development in cementitious materials. Among many scenarios, four fracture types were selected to simulate different fracture surfaces. The four fracture types were modeled based on fractal theory originally developed in rock mechanics [3]. Three fracture types (fractal fracture type A, B, C, and D) simulated different crack paths which propagate around aggregate particle, and one type (fractal fracture type D) simulated in case of crack propagates inside of aggregate. The major advantage of this fractal model is that fractal dimension of fracture surface can be correlated to volume fraction and size distribution of aggregates used. The hexagon is selected as an example in this paper to demonstrate the formulation of model. Proposed fractal fracture model can simulate any shape of aggregates. A unit element of the size $L_o \times w_o$ can be used to represent a section of a crack path (Fig. 9). It needs to be mentioned that, the difference of fractal characteristics between crack propagation direction and transverse direction has not been considered in this study.

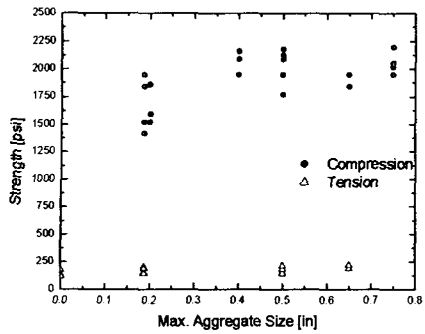


Fig. 5 Aggregate size vs. strength

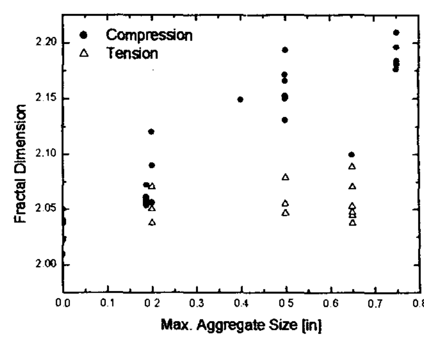


Fig. 6 Max agg. size vs. fractal dimensions

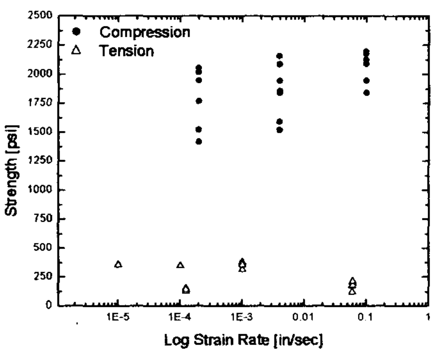


Fig. 7 Loading rates vs. strengths

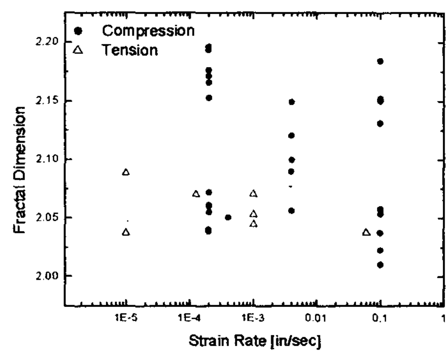


Fig. 8 Loading rates vs. fractal dimensions

The fractal fracture type A consists of a hexagon (aggregate shape) exposed two sides of hexagon circumference to the crack path as shown in Fig. 9. Fig. 9 illustrates fractal fracture model A, B, C, and D. Taking into account fractal fracture type A, let us consider N being number of crack segments, $N = L_t / \varepsilon_i$, where, L_t is total length of crack path. The number of crack segments is the summation of crack in matrix as well as in aggregate-matrix interface, $N = N_{matrix} + N_{agg}$. r is defined as a ratio of similarity and it is a function of relative aggregate size ε_i. From the basic geometry and fractal theory [3,5], r can be evaluated as; $r = \varepsilon_i / L_o = 1/1+\sqrt{3}$, where ε_i is the length of crack segment or one side of aggregate called relative aggregate size; and L_o is height of unit element and has been chosen so that ε_i is a constant for all three segments. w_o is width of the unit. The fractal dimension is then defined by, $D = \log N / \log(1/r) = 1.0931$. The volume fraction of aggregate can be calculated: $V = V_{aggregate} / V_{total} = (3\sqrt{3}/2) \bullet (\varepsilon_i^2 / w_o L_o) = 0.3481 L_o / w_o$. In case that the more aggregate surface is exposed to crack path, type B, as shown in Fig. 9 (B), similar procedure can be applied to derive fractal parameters which are $N = 4$, $r = 1/3$, $D = 1.262$, and $\bar{V}_{agg.} = 0.2887 L_o / w_o$. Comparing fractal dimensions of fractal fracture type A and type B, one can see that the larger portion of aggregate exposed to crack, the higher fractal dimension. The fracture type B contains less aggregate volume fraction compared to type A, because ε_{Ai} (ε_{Ai} for the fracture type A) has a larger length at the fixed L_o and w_o. As shown in Fig. 9 (C), fractal fracture type C has 50% of aggregate exposed to crack path like fracture type B, however, it is rotated 90^o with respect to longer aggregate axis. This particular crack path has a dramatic change of crack angle (90^o) when crack deflects from matrix to aggregate. In this case, the number of crack segments is 8 due to special geometry of crack path. The other fractal parameters are $r = 1/(2+2\sqrt{3})$, $D = 1.2245$ and $\overline{V}_{agg.} = 0.3481 L_o / w_o$.

Instead of crack going around aggregates, crack can also pass through aggregates as shown in Fig. 9 (D), in which crack path deflects from its direction with angle α.

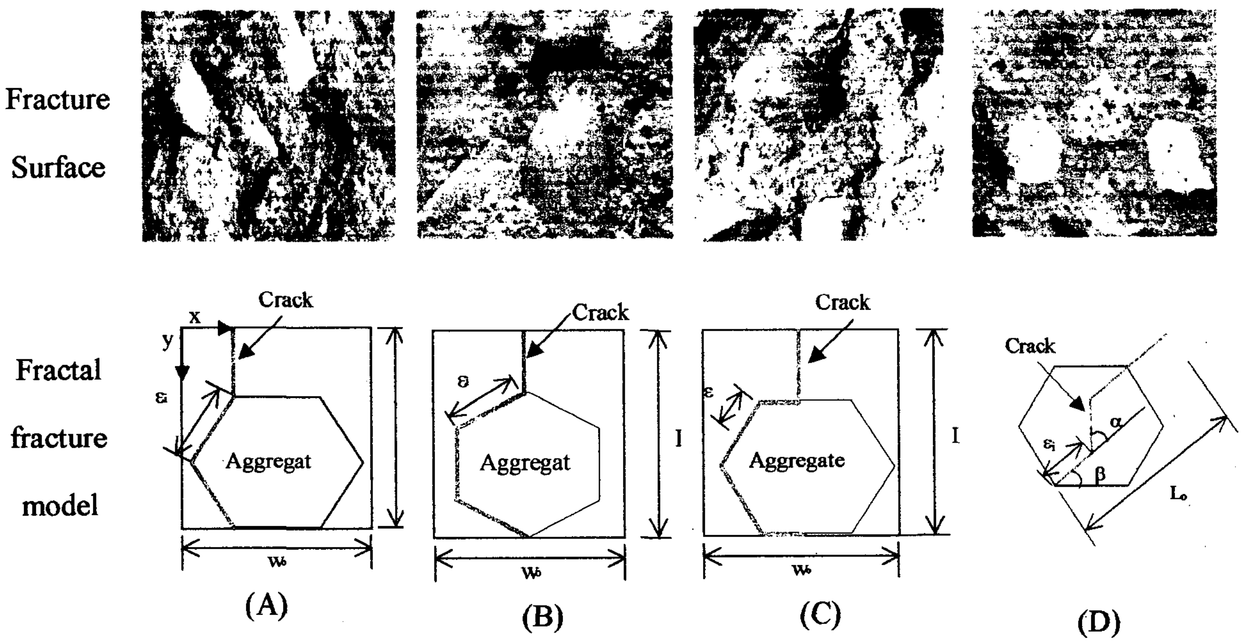


Fig. 9 Fractal fracture model for type A, B, C, and D

In type D, crack orientation may change randomly throughout crack surface, that is, the crack may deflect inside aggregate. To simulate the change of crack orientation, deflection angle α is introduced (see Fig. 9 (D)). The deflection angle affects fractal dimension, which means that fractal parameters of model depend on the deflection angle α.

Table 2 shows fractal dimension and volume fractions of the fractal fracture type D with different angle α. In case that α is zero, crack has no deflection inside the aggregate, going straightly through aggregate. Therefore, fractal dimension is simply 1.0, corresponding to flat surface. In actual fracture surface, many different aggregate sizes and fracture types are involved, which make the actual fracture surface very complicated. Different aggregate sizes can also be simulated using proposed model.

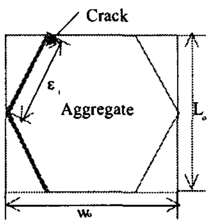
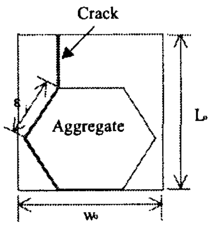
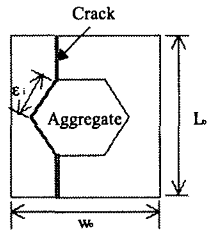
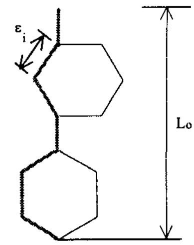
Table 2 Fractal parameters for the fractal type D

$N=4$	$\alpha=0^o$	30^o	45^o	60^o	90^o
$r=\frac{\varepsilon_i}{L_o}=\frac{1}{3+\cos\alpha}$	$\frac{1}{4}$	$\frac{1}{3.866}$	$\frac{1}{3.707}$	$\frac{1}{3.5}$	$\frac{1}{3}$
$D=\frac{\log N}{\log(1/r)}$	1.000	1.0252	1.0580	1.1066	1.2619
$\overline{V}=\frac{V_{aggregate}}{V_{total}}$	$0.3654\frac{L_o}{w_o}$	$0.3678\frac{L_o}{w_o}$	$0.3700\frac{L_o}{w_o}$	$0.3712\frac{L_o}{w_o}$	$0.3608\frac{L_o}{w_o}$

Table 3 shows crack paths with same fractal fracture type (A) but different aggregate sizes (A1, A2, and A3). Fractal fracture type A_1 represents the geometry maximum aggregate sizes in which the height of the maximum aggregate size is L_o. Then, type A_2 can be obtained by adding one segment of the crack in the matrix so that aggregate size is smaller than type A_1. In the same manner, the size of aggregate can be further reduced to A_3, A_4, ... etc. Different fractal dimension and volume fraction for these sub-types are listed in Table 3. Since real fracture surface contains all different kind of aggregate sizes and types, combination of each fractal fracture types need to be considered. Combinations of each different fracture types are also possible using proposed model. In Table 3, combination of fracture type A and B is explained. Using proper combinations of all four fracture types, one can simulate fractal dimensions of fracture surface

The fractal dimensions of four different fracture types obtained from above derivation can be compared with test data compression and tension tests. In Figs. 10 and 11, type B and type C yield similar fractal dimensions as obtained from the compression tests both slow and fast loading cases. Our calculation shows that crack patterns in compression tests seem to be similar with type B and type C rather than type A or type D. By visual inspection, more Type B and Type C were observed in compression fracture surfaces. On the other hand, Type A and Type D was observed in tension fracture surfaces. As it can be seen in Fig. 10 and 11 simulation of fractal dimension shows satisfactory results comparing with experimental test results.

Table 3 Fractal parameters for different aggregate sizes for type A

A_1	A_2	A_3	Combination of A and B
			
$N=2$	$N=3$	$N=4$	$N=7$
$\varepsilon_i = \frac{L_o}{\sqrt{3}}$	$\varepsilon_i = \frac{L_o}{1+\sqrt{3}}$	$\varepsilon_i = \frac{L_o}{2+\sqrt{3}}$	$L_o = (4+\sqrt{3})\varepsilon_i$
$D=1.2619$	$D=1.0931$	$D=1.0526$	$r = \frac{1}{(4+\sqrt{3})}$
$d_{max} = 1.1547L_o$	$d_{max} = 0.7320L_o$	$d_{max} = 0.5359L_o$	$D = \frac{\log 7}{\log(4+\sqrt{3})} = 1.1144$
$V_i = 0.866\frac{L_o}{w_o}$	$V_i = 0.3481\frac{L_o}{w_o}$	$V_i = 0.1865\frac{L_o}{w_o}$	

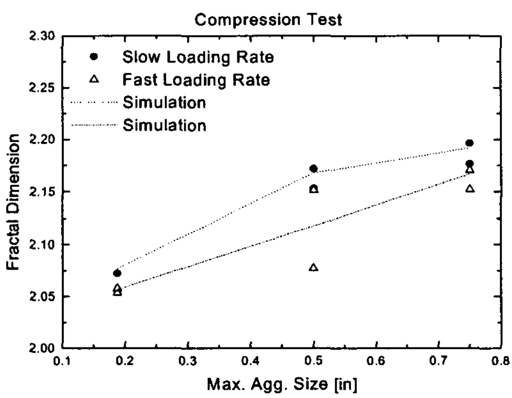


Fig. 10 Fractal dimension simulation

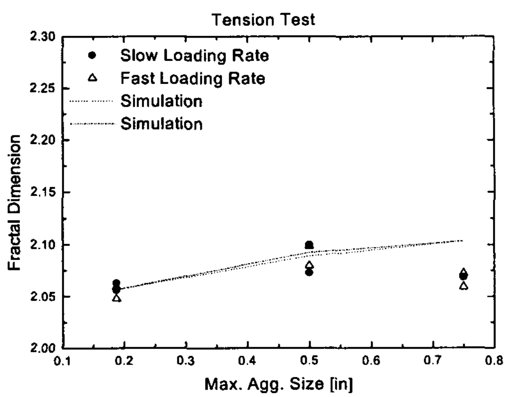


Fig. 11 Fractal dimension simulation

4. CONCLUSIONS

Fractal dimensions of concrete specimens with different aggregate sizes under different loading rates were measured. From experimental test, following conclusions are drawn.

1. The strength of concrete and the fractal dimension of fracture surface of concrete increased with increasing aggregate sizes.

2. The compressive strength of concrete increased with higher loading rate. The strength enhancement due to high loading rate did not appear in the tension test, within the range of the loading rates used in this study.
3. The fractal dimension decreased with increasing loading rate.

Fractal fracture model has been proposed, which is a combination of fractal theory and fracture mechanics. Fractal parameters can be correlated to the intrinsic parameters of microstructure, such as sizes and volume fraction of aggregates. The size and shape of aggregate can be incorporated in fractal fracture model. As numerical examples, fractal parameters of four different types of fracture paths were calculated and fracture surfaces for concretes with different types of aggregates can be simulated using the proposed fractal model.

REFERENCES

1. Carpinteri, A., "Experimental determination of Fracture Toughness K_{1c} and J_{1c} for Aggregative Material", Proceedings 5th International Concrete Fracture, Cannes, France, Vol. 4, 1981, pp. 1491-1498
2. Mecholsky, J.J., Passoja, D.E. and Feinberg-Ringel, K.S., "Quantitative Analysis of Brittle Fracture Surfaces Using Fractal Geometry", *Journal of American Ceramic Society*, Vol. 71, No. 1, 1989, pp. 60-65.
3. Xie, H., "Fractals in Rock Mechanics", A.A. Balkema, Brookfield, Vt. 1993.
4. Li, X.W., Tian, J., Kang, Y. and Wang, Z.G., "Quantitative Analysis of Fracture Surface by Roughness and Fractal Method", Scripta Metallurgica et Materialia, Vol.33, No.5, 1995, pp.803-809.
5. Mandelbrot, B.B., Passoja, D.E. and Paullay, A.,"Fractal Character of fracture surfaces of metals", Nature, Vol. 308, 1984, pp. 721-722.
6. Lung, C.W. and Mu, Z.Q., "Fractal dimension measured with perimeter-area relation and toughness of materials", Physics review Bulletin, Vol. 38, No. 16, 1987, pp.11781-11784.
7. Lange, D.A., Jennings, H.M. and Shah, S.P., "Relationship between Fracture Surface Roughness and Fracture Behavior of Cement Paste and Mortar", Journal of American Ceramic Society, Vol. 76, No. 3, 1993, pp 589-97.
8. Richardson, L.F.,"The problem of contiguity: An appendix of statistics of deadly quarrels", General system yearbook, Vol.6, 1961, pp. 139-187.
9. Roh, Y.S. and Xi, Y., "The Fracture Surface Roughness of Concrete with Different Aggregate Sizes and Loading Rates", ACI Special Publication, SP 201 - Fracture Mechanics for Concrete Materials: Testing and Applications, 2001, pp. 35-54.

ACKNOWLEDGEMENTS

The first author wishes to acknowledge the partial financial support of the Korea Science and Engineering Foundation (KOSEF) for the National Research Laboratory (NRL) project M1-0412-00-0068-04-J00-00-029-00. Partial financial support under NSF grant CMS-9872379 to University of Colorado at Boulder is gratefully acknowledged. Partial financial support under Colorado DOT grant 01 HAA 00197 to University of Colorado at Boulder is gratefully acknowledged.