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파라미터 불확실성을 갖는 어핀 T-S 퍼지 시스템의 제어기 설계

Controller Design for Affine T-S Fuzzy System with Parametric Uncertainties

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Abstract

This paper proposes a stability condition in affine Takagi-Sugeno (T-S) fuzzy systems with parametric uncertainties and then, introduces the design method of a fuzzy-model-based controller which guarantees the stability. The analysis is based on Lyapunov functions that are continuous and piecewise quadratic. The search for a piecewise quadratic Lyapunov function can be represented in terms of linear matrix inequalities (LMIs).

Keywords: Affine T-S fuzzy model, Parametric uncertainty, Piecewise quadratic Lyapunov function, Linear matrix inequality.

1. Introduction

Most plants in the industry have uncertainties and it make hard to control the general nonlinear, uncertain plants. In order to surmount these difficulties, fuzzy control is developed recently. It has been shown that fuzzy logic control (FLC) is a successful control approach for complex nonlinear systems [1]. There are a number of systematic analysis and controller design methodology in the literature, where the Takagi-Sugeno (T-S) fuzzy model is used. Original T-S fuzzy system has not only linear but also affine terms in the consequent part [1]. But affine terms in the consequent part are ignored in almost all paper and called homogeneous T-S fuzzy system [6]. Based on homogeneous T-S fuzzy systems Lee *et al.* proposed the stabilization methodology of nonlinear systems with parametric uncertainties in [2].

The T-S fuzzy systems considered in this paper are allowed to have an affine term. This can be an advantage, because affine T-S fuzzy systems may be able to approximate nonlinear functions to high accuracy with fewer rules than the homogeneous T-S fuzzy systems with linear consequents only [3-6]. Motivated by the stabilization methodology in [2], this paper aims at studying the control problem for the affine T-S fuzzy systems.

This paper is organized as follows: In the Section 2 we review the basic notation of affine T-S fuzzy systems and assumption of uncertainty model. We propose a stability analysis methodology of affine T-S fuzzy systems with parametric uncertainties in the Section 3. Section 4 presents a numerical example. Finally conclusion and some discussions are given in Section 5.

2. Preliminaries

Consider continuous-time affine T-S fuzzy system in which the ith rule is formulated in the following form:

Plant rules:

R^i: If $x_1(t)$ is Γ_1^i and $\cdots$ $x_n(t)$ is Γ_n^i,

Then $\dot{x} = (A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t) + (a_i+\Delta a_i)$, (1)

where Γ_j^i ($i=1,\cdots,q$, $j=1,\cdots,n$.) is the fuzzy set, $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input. The defuzzified output of the affine T-S fuzzy system (1) is represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{q}\mu_i(x(t))((A_i+\Delta A_i)x(t) + (B_i+\Delta B_i)u(t)+(a_i+\Delta a_i)) \quad (2)$$

where

$$\omega_i(x(t))=\prod_{j=1}^{n}\Gamma_j^i(x_j(t)), \quad \mu_i(x(t))=\frac{\omega_i(x(t))}{\sum_{i=1}^{q}\omega_i(x(t))},$$

and $\Gamma_j^i(x_j(t))$ is the membership value of $x_j(t)$ in Γ_j^i.

Throughout this paper, a state feedback affine T-S fuzzy-model-based control law is utilized for the stabilization of the T-S fuzzy system (2).

Controller rules:

R^i: If $x_1(t)$ is Γ_1^i and $\cdots$ $x_n(t)$ is Γ_n^i,

Then $u(t)=K_i x(t)$. (3)

The defuzzified output of the controller rules is given by

$$u(t)=\sum_{i=1}^{q}\mu_i(x(t))K_i x(t). \quad (4)$$

The closed-loop system with (2) and (4) is represented as

$$\dot{x}(t)=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))((A_i+\Delta A_i + (B_i+\Delta B_i)K_j)x(t)+a_i+\Delta a_i) \quad (5)$$

For convenient notation, we introduce followings:

$$\overline{x}=\begin{bmatrix} x \\ 1\end{bmatrix}, \quad \overline{A}_i=\begin{bmatrix} A_i & a_i \\ 0 & 0\end{bmatrix}, \quad \Delta\overline{A}_i=\begin{bmatrix} \Delta A_i & \Delta a_i \\ 0 & 0\end{bmatrix}$$

$$\overline{B}_i=\begin{bmatrix} B_i \\ 0\end{bmatrix}, \quad \Delta\overline{B}_i=\begin{bmatrix} \Delta B_i \\ 0\end{bmatrix}, \quad \overline{C}_i=[\,C_i \;\; 0]$$

Using this notation, the system (5) can be expressed as

$$\begin{aligned}\dot{\overline{x}}(t) &= \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(\overline{x}(t))\mu_j(\overline{x}(t))(\overline{A}_i+\Delta\overline{A}_i + (\overline{B}_i+\Delta\overline{B}_i)\overline{K}_j)\overline{x}(t) \\ &= \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(\overline{x}(t))\mu_j(\overline{x}(t))\overline{G}_{ij}\overline{x}(t) \end{aligned} \quad (6)$$

where $\overline{G}_{ij}=\overline{A}_i+\Delta\overline{A}_i+(\overline{B}_i+\Delta\overline{B}_i)\overline{K}_j$. For $l \in L_0$ (6) becomes

$$\dot{x}=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))G_{ij}x(t) \quad (7)$$

Since the affine T-S fuzzy systems (6) have time-varying uncertain matrices, it is difficult to decide the stability of the system. In this reason the parametric uncertainties considered here are removed under some reasonable assumptions.

Assumption 1 The parametric uncertainties considered here are norm-bounded, in term

$$[\Delta A_i \;\; \Delta a_i \;\; \Delta B_i] = D_i F_i(t)[E_{1i} \;\; E_{2i} \;\; E_{3i}] \quad (8)$$

$$[\Delta\overline{A}_i \;\; \Delta\overline{B}_i] = \overline{D}_i \overline{F}_i(t)[\overline{E}_{1i} \;\; \overline{E}_{3i}] \quad (9)$$

where D_i, E_{1i}, E_{2i}, and E_{3i} are known real constant matrices, and $F_i(t)$ is unknown matrix function satisfies $F_i(t)^T F_i(t) \le I$. And extended variables have following notation.

$$\overline{D}_i=\begin{bmatrix} D_i & 0 \\ 0 & 0\end{bmatrix}, \quad \overline{F}_i=\begin{bmatrix} F_i & 0 \\ 0 & 0\end{bmatrix}, \quad \overline{E}_{1i}=\begin{bmatrix} E_{1i} & E_{2i} \\ 0 & 0\end{bmatrix}$$

$$\overline{E}_{2i}=\begin{bmatrix} E_{2i} \\ 0\end{bmatrix} \quad \overline{E}_{3i}=\begin{bmatrix} E_{3i} \\ 0\end{bmatrix}$$

3. Robust Stability of Affine T-S Fuzzy Systems

Before proceeding main theorem of paper, we call the following which will be need throughout the proof.

Lemma 1 Given constant matrices D and E and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:

$$S+DFE+E^TF^TD^T<0 \tag{10}$$

where F satisfies $F^TF \le R$, if and only if for some $\varepsilon>0$

$$S+[\varepsilon^{-1}E^T \ \ \varepsilon D]\begin{bmatrix} R & 0 \\ o & I \end{bmatrix}\begin{bmatrix} \varepsilon^{-1}E \\ \varepsilon D^T \end{bmatrix}<0 \tag{11}$$

Since we are interested in analyzing exponential stability of the origin, we let $I_0 \subseteq I$ be the set of indices for cells that contain origin and $I_1 \subseteq I$ be the set of indexes for cells that do not contain origin. It is assumed that $a_k=0, \Delta a_k=0$ for all $k \in K(i)$ with $i \in I_0$.

The main result on robust stability of affine T-S fuzzy system with parametric uncertainties is summarized in the following theorem.

Theorem 1 For each $i \in I$, If there exists a symmetric matrix T and some scalars ε_{ij}, $(i,j=1,\cdots,q)$ such that the following LMIs are satisfied, then the continuous-time affine T-S fuzzy system is asymptotically stable:

$$P_{ij}=F_{ij}^T T F_{ij}, \quad (i,j)\in I_0 \tag{12}$$

$$\overline{P}_{ij}=\overline{F}_{ij}^T T \overline{F}_{ji}, \quad (i,j)\in I_1 \tag{13}$$

satisfy

$$0<P_{ij} \tag{14}$$

$$0<\overline{P}_i \tag{15}$$

$$\begin{bmatrix} \Phi_{ii} & * & * \\ E_{1i}+E_{3i}K_i & -\varepsilon_{ii}I & * \\ D_i^TP_{ii} & 0 & -\varepsilon_{ii}^{-1}I \end{bmatrix}<0 \tag{16}$$

$$\begin{bmatrix} \overline{\Phi}_{ii} & * & * \\ \overline{E}_{1i}+\overline{E}_{3i}\overline{K}_i & -\varepsilon_{ii}I & * \\ \overline{D}_i^T\overline{P}_{ii} & 0 & -\varepsilon_{ii}^{-1}I \end{bmatrix}<0 \tag{17}$$

$$\begin{bmatrix} \Theta_{ij} & * & * & * & * \\ E_{1i}+E_{3i}K_j & -\varepsilon_{ij}I & * & * & * \\ E_{1j}+E_{3j}K_i & 0 & -\varepsilon_{ij}I & * & * \\ D_i^TP_{ij} & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ D_j^TP_{ij} & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix}<0 \tag{18}$$

$$\begin{bmatrix} \overline{\Theta}_{ij} & * & * & * & * \\ \overline{E}_{1i}+\overline{E}_{3i}\overline{K}_j & -\varepsilon_{ij}I & * & * & * \\ \overline{E}_{1j}+\overline{E}_{3j}\overline{K}_i & 0 & -\varepsilon_{ij}I & * & * \\ \overline{D}_i^T\overline{P}_{ij} & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ \overline{D}_j^T\overline{P}_{ij} & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix}<0 \tag{19}$$

where,

$$\Phi_i=A_i^TP_{ii}+P_{ii}A_i+K_i^TB_i^TP_{ii}+P_{ii}B_iK_i$$

$$\overline{\Phi}_i=\overline{A}_i^T\overline{P}_{ii}+\overline{P}_{ii}\overline{A}_i+\overline{K}_i^T\overline{B}_i^T + \overline{P}_{ii}\overline{B}_i\overline{K}_i$$

$$\Theta_{ij}=A_i^TP_{ij}+P_{ij}A_i+A_j^TP_{ij}+P_{ij}A_j+K_j^TB_i^TP_{ij} +P_{ij}B_iK_j+K_i^TB_j^TP_{ij}+P_{ij}B_jK_i$$

$$\overline{\Theta}_{ij}=\overline{A}_i^T\overline{P}_{ij}+\overline{P}_{ij}\overline{A}_i+\overline{A}_j^T\overline{P}_{ij} + \overline{P}_{ij}\overline{A}_j+\overline{K}_j^T\overline{B}_i^T\overline{P}_{ij}+\overline{P}_{ij}\overline{B}_i\overline{K}_j + \overline{K}_i^T\overline{B}_j^T\overline{P}_{ij}+\overline{P}_{ij}\overline{B}_j\overline{K}_i$$

Proof: It is omitted by lack of space.

To reduce the conservatism of Theorem 1, we introduce S-procedure. After applying the S-procedure, Stability condition of Affine T-S fuzzy system will be represent as follows.

Corollary 1 In solving the inequalities in Theorem 1, if we replace (12) and (13) to (20) and (21) respectively, and P_{ij} $\overline{P}_{ij}$ in (16)-(19) to (22) and (23) respectively, then the continuous-time affine T-S fuzzy system is asymptotically stable in the relaxed condition.

$$0<P_{ij}-E_{ij}^TU_{ij}E_{ij} \tag{20}$$

$$0<\overline{P}_i-\overline{E}_{ij}^T\overline{U}_{ij}\overline{E}_{ij} \tag{21}$$

$$0<P_{ij}+E_{ij}^TW_{ij}E_{ij} \tag{22}$$

$$0<\overline{P}_i+\overline{E}_{ij}^T\overline{W}_{ij}\overline{E}_{ij} \tag{23}$$

where U_{ij}, W_{ij} and E_{ij} are defined in [3].

Proof: It is omitted by lack of space.

4. Simulation

Consider the following nonlinear differential system:

$$\dot{x}(t)=\sum_{i=1}^{4}\mu_i(x(t))((A_i+\Delta A_i)x(t) + (B_i+\Delta B_i)u(t)+a_i+\Delta a_i) \tag{24}$$

where $A_1=\begin{bmatrix} -10 & 11 \\ 10 & 9 \end{bmatrix}$, $A_2=\begin{bmatrix} -1 & 5 \\ 2 & 8 \end{bmatrix}$, $A_3=\begin{bmatrix} -1 & 5 \\ 2 & 8 \end{bmatrix}$, $A_4=\begin{bmatrix} -10 & 10 \\ 10 & 5 \end{bmatrix}$, $a_1=\begin{bmatrix} 0 \\ -2 \end{bmatrix}$, $a_2=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $a_3=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $a_4=\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $B_i=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for

$i=1,\cdots,4$ and normalized membership value is $\mu_1(x_1(t))=1\,(x_1<-2)$, $\mu_2(x_1(t))=1\,(-2\le x_1<0)$, $\mu_3(x_1(t))=1\,(0\le x_1<2)$, $\mu_4(x_1(t))=1\,(2\le x_1)$.

From Assumption 1 and Corollary 1, we find the P, K, W, U matrices. we can find the following P matrices and K matrices:

$$P_{11}=\begin{bmatrix}230.4587 & 0.0000 & 12.4572\\ 0.0000 & 255.3732 & 0.0000\\ 12.4572 & 0.0000 & 249.1446\end{bmatrix}$$
$$P_{22}=\begin{bmatrix}280.2876 & 0.0000\\ 0.0000 & 255.3732\end{bmatrix}$$
$$P_{33}=\begin{bmatrix}280.2876 & 0.0000\\ 0.0000 & 255.3732\end{bmatrix}$$
$$P_{44}=\begin{bmatrix}230.4587 & -0.0000 & -12.4572\\ -0.0000 & 255.3732 & 0.0000\\ -12.4572 & 0.0000 & 249.1446\end{bmatrix}$$
$$K_1=[-0.8711\ \ -4.2935\ \ -2.8035]$$
$$K_2=[0.6486\ \ -2.5686]$$
$$K_3=[-1.3973\ \ -2.5686]$$
$$K_4=[-1.0760\ \ -4.0934\ \ -2.9523]$$

Thus one can verify that the system is asymptotically stable in the large. By using gain matrices, we simulate with 4 initial condition $[x_1\ x_2]^T=[2\ 0]^T$, $[4\ 0]^T$, $[6\ 0]^T$, $[8\ 0]^T$ and result are shown in Figure. 1.

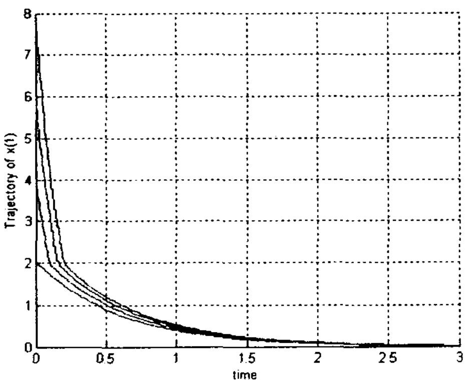

Figure. 1 State Trajectory of the system with 4 initial conditions

5. Closing Remarks

In this paper, we have developed and analyzed a new robust fuzzy controller design method for affine T-S fuzzy systems. The search for piecewise quadratic Lyapunov functions for affine T-S fuzzy system is convex optimization problem in terms of linear matrix inequalities.

6. Reference

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