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Design of Intelligent Digital Controller with Dual-Rate Sampling

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Abstract

In this paper, a new dual-rate digital control technique for the Takagi-Sugeno (T-S) fuzzy system is suggested. The proposed method takes account of the stabilizablity of the discrete-time T-S fuzzy system at the fast-rate sampling points. Our main idea is to utilize the lifted control input. The proposed approach is to obtain the dual-rate discrete-time T-S fuzzy system by discretizing the overall dynamics of the T-S fuzzy system with the lifted control, and then to derive the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for this system.

Key Words : Digital control, Takagi-Sugeno (T-S) fuzzy system, stability, dual-rate sampling, linear matrix inequalities (LMIs).

1. Introduction

Dual-rate control for the linear system was shown to be very effetiveness [8]. However, the dual-rate digital control for the Takagi-Sugeno (T-S) system is still open.

This paper aims at developing a new dual-rate digital controller for the Takagi-Sugeno (T-S) system with careful reflection of stability of intersampling points. The main contribution of this paper is to derive some sufficient conditions, in terms of the linear matrix inequalities (LMIs), such that the digitally controlled system is asymptotically stable at every intersampling points. Specifically, our main idea is to utilize the lifted control input. The proposed approach is to obtain the dual-rate discrete-time T-S fuzzy system by discretizing the overall dynamics of the T-S fuzzy system with the lifted control, and then to derive the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for this system. An example is provided for showing the feasibility of the proposed discretization method.

2. Preliminaries and Problem Description

Consider the fuzzy rule of a sampled-data T-S fuzzy systems with the sampling time T governed by

$$R_i: \text{IF } z_1(t) \text{ is about } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{ip}$$
$$\text{THEN } \frac{d}{dt}x(t) = A_i x(t) + B_i u(t) \tag{1}$$

where $R_i, i \in I_q = \{1, 2, \cdots, q\}$, is the ith fuzzy rule, $z_h(t), h \in I_p = \{1, 2, \cdots, p\}$, is the hth premise variable, $\Gamma_{ih}, (i, h) \in I_q \times I_p$, is the fuzzy set, and $u(t) = u(kT)$ is the piecewise-constant control input vector to be determined time interval $[kT, kT+T)$. Using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of this samped-data T-S fuzzy model is described by

$$\dot{x}(t)=\sum_{i=1}^{q}\theta_i(z(t))(A_ix(t)+B_iu(t)) \quad (2)$$

where $w_i(z(t))=\prod_{h=1}^{p}\Gamma_{ih}(z_h(t))$, $\theta_i(z(t))=\frac{w_i(z(t))}{\sum_{i=1}^{q}w_i(z(t))}$,

and $\Gamma_{ih}(z_h(t))$ is the grade of membership of $z_h(t)$ in Γ_{ih}. Based on the PDC [6,7], we consider the following fuzzy digital control law for the fuzzy model (2)

R_i: IF $z_1(kT)$ is Γ_{i1} and $\cdots$ and $z_p(kT)$ is Γ_{ip}

THEN $u(t)=K_{di}x(kT)$ (3)

for $t\in[kT, kT+T)$. The overall state feedback fuzzy-model-based digital control law is represented by

$$u(t)=\sum_{i=1}^{q}\theta_i(z(kT))K_{di}x(kT) \quad (4)$$

Because the fuzzy model (2) is subject to the sampling time T, in general, stabilizing controller at the intersampling points do not exist. The aim of this paper is to design the digital control law (4) such that the closed-loop system is asymptotically stable at every intersampling points.

3. Dual-Rate Fuzzy-Model-Based Digital Control

3.1 Fast Discretization for Continuous-Time T-S Fuzzy System

Our proposed method of descritizing the continuous-time T-S fuzzy system is to apply the fast discretization technique [8] to the T-S fuzzy system. The fast discretization leads to a dual-rate discrete-time system which can be lifted to a single-rate discrete-time system. Specifically, the continuous T-S fuzzy system is dicretized with fast-rate sampling $T_f\left(=\frac{T_s}{n}\right)$. For this discretized version, lifting the control input so that the lifted signals correspond to the slow-rate sampling T_s results in a lifted system. To maintain the polytopic structure of the lifted system for designing the digital fuzzy model-based controller, we utilize the following assumption.

Assumption 1 [4,5] *Suppose that the firing strength $\theta_i(t)$ for $t\in[kT_s,(k+1)T_s)$ is $\theta_i(kT_s)$. That is*

$$\theta_i(t)\approx\theta_i(kT_s) \quad (5)$$

Then, the nonlinear matrices $\sum_{i=1}^{q}\theta_i(z(t))A_i$ and $\sum_{i=1}^{q}\theta_i(z(t))B_i$ of (2) can be approximated as the piecewise constant matrices $\sum_{i=1}^{q}\theta_i(z(kT_s))A_i$ and $\sum_{i=1}^{q}\theta_i(z(t))B_i$, repectively.

Theorem 1 *The sampled-data T-S fuzzy system (2) can be converted to the following pointwise dynamical behavior with a slow sampled system and a lifted sampled input.*

$$x[k+1]=G(\theta[k])x[k]+\overline{H}(\theta[k])\overline{u}[k] \quad (6)$$

where $x[k]=x(kT_s)$, $u[k]=u(kT_s)$, $\theta[k]=\theta(z(kT_s))$,

$G(\theta[k])=G_f^n(\theta[k])=\left(\sum_{i=1}^{q}\theta_i[k]G_{fi}\right)^n$,

$H(\theta[k])=(G_f^{n-1}(\theta[k])+G_f^{n-2}(\theta[k])+\cdots+I)H_f(\theta[k])$,

$H_f(\theta[k])=\sum_{i=1}^{q}\theta_i(z(kT_s))H_{fi}$, $G_{fi}=\exp(A_iT_f)$,

$H_{fi}=(G_{fi}-I)A_i^{-1}B_i$, *a lifted sampled input* $\overline{u}[k]$ *is defined as*

$$\overline{u}[k]=\begin{bmatrix}\overline{u}_1[k]\\ \overline{u}_2[k]\\ \vdots\\ \overline{u}_n[k]\end{bmatrix}=\begin{bmatrix}u(lT_f)\\ u((l+1)T_f)\\ \vdots\\ u((l+n-1)T_f)\end{bmatrix} \quad (7)$$

and the matrices $G(\theta[k])$ *and* $\overline{H}(\theta[k])$ *are given by*

$G(\theta[k])=G_f^n(\theta[k])$

$\overline{H}(\theta[k])=[\,G_f^{n-1}(\theta[k])H_f(\theta[k])\;\; G_f^{n-2}(\theta[k])H_f(\theta[k])\;\cdots\; H_f(\theta[k])\,]$

Proof: The proof is omitted due to lack of space.■

In the proposed discretization method, two approximations are performed as follows:

$$\exp(A(\theta[k])T_f) \approx G_{fi}(\theta[k]) \tag{7}$$

$$(G_f(\theta[k])-I)A^{-1}(\theta[k])B(\theta[k]) \approx H_f(\theta[k]) \tag{8}$$

To analyze, introduce approximation error defined by

$$e_1 = \|\exp(A(\theta[k])T_f) - G_{fi}(\theta[k])\|_2 \tag{9}$$

Applying Taylor series expansion from the right-hand side gives

$$\begin{aligned} e_1 &= T_f^2\left|\left(\frac{1}{2!}\left(\sum_{i=1}^{q}\theta_i[k]A_i\right)^2 - \frac{1}{2!}\sum_{i=1}^{q}\theta_i[k]A_i^2\right)+\cdots\right|_2 \\ &= O(T_f^2) \\ &= O\left(\left(\frac{T_s}{n}\right)^2\right) \end{aligned} \tag{10}$$

In the same manner, approximation error in (8) is

$$\begin{aligned} e_2 &= |(G_f(\theta[k])-I)A^{-1}(\theta[k])B(\theta[k]) - H_f(\theta[k])|_2 \\ &= O(T_f) \\ &= O\left(\frac{T_s}{n}\right) \end{aligned} \tag{11}$$

From (10) and (11), approximation error clearly goes to zero as n approaches the infinity.

Remark 1 *In the conventional discretization method with single-rate sampling [4,5], the approximations are also performed at above two cases, and the approximation results are exactly equal to (7) and (8) when $n=1$. Therefore, our discretized version (6) yields smaller approximation error than that of [4,5].*

Remark 2 *The proposed method needs no the fast-rate sampling device because our discretized version yields relatively small error for the long sampling time by virtue of the dual-rate sampling schemes.*

Corollary 1 *The fast-sampled discrete-time system of (2) is obtained as follows:*

$$x[l+1] = G_f(\theta[l])x[l] + H_f(\theta[l])u[l] \tag{12}$$

where $x[l]=x(lT_f)$ and $\theta[l]=\theta(lT_f)$.

Proof: When $n=1$ and $T=T_f$, it can be straightforwardly proved by Theorem 1. ■

3.2 Stability Conditions

In this subsection, we derive the stability conditions for the dual-rate T-S fuzzy system (6). Consider the open-loop system for (6).

$$x[k+1] = G(\theta[k])x[k] \tag{13}$$

The following theorem gives a set of conditions for ensuring the stability of (13).

Theorem 2 *The equilibrium of (13) is globally asymptotically stable in the sense of Lyapunov stability criterion if there exists a common positive definite matrix P such that*

$$G_{fi}^T P G_{fi}^T - P < 0 \quad I \in [1, q] \tag{14}$$

Proof: The proof is omitted due to lack of space.■

Remark 3 *From Theorem 2, we know that if $G_f(\theta[l])$ is globally asymptotically stable, so is $G(\theta[k])$. This is very useful property for the design of digital controller.*

3.3 Design of Stabilizable Digital Controller

Our main objective is to construct the stabilizable controller for (2) at fast-rate sampling points. We first design a stabilizable controller for the fast-sampled discrete-time system, and then convert the controlled system into (6).

We consider the following state feedback fuzzy control law for (12):

$$u[l] = \sum_{i=1}^{q}\theta[l]K_{di}x[l] \tag{15}$$

Then, the closed-loop system can be rewritten as

$$x[l+1] = \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i[l]\theta_j[l](G_{fi}+H_{fi}K_{dj})x[l] \tag{16}$$

The following theorem provides the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for (6).

Theorem 3 *For the dual-rate T-S fuzzy system (6), the closed-loop system under the state feedback controller law*

$$\tilde{u}[k] = K_d(\theta[k])\begin{bmatrix} I \\ (G_f(\theta[k])+H_f(\theta[k])K_d(\theta[k])) \\ \vdots \\ (G_f(\theta[k])+H_f(\theta[k])K_d(\theta[k]))^{n-1} \end{bmatrix} \times x[k] \tag{17}$$

is globally asymptotically stabilizable in the sense of Lyapunov stability criterion if there exist symmetric positive definite matrix Q and constant matrix F such that

$$\begin{bmatrix} -Q & * \\ G_{fi}Q+H_{fi}F_i & -Q \end{bmatrix} < 0 \quad i \in [1, q]$$

$$\begin{bmatrix} -Q & * \\ G_{fi}Q+H_{fi}F_j+G_{fj}Q+H_{fj}F_i 2 & -Q \end{bmatrix} < 0 \quad i < j \in [1, q] \tag{18}$$

*where * denotes the transposed element in symmetric position.*

Proof: The proof is omitted due to lack of space.■

4. Closing Remarks

In this paper, a new dual-rate digital control method has been proposed for the T-S fuzzy system. We have formulated and solved the intersampling stability problem for the T-S fuzzy system. The proposed fast discretization approach leads to the dual-rate T-S fuzzy system which can be lifted to a single-rate discrete-time system. For this system, the stability conditions at the fast-rate sampling points have been derived. Finally, for the digitally controlled T-S fuzzy system, the sufficient stabilization conditions in the sense of the Lyapunov asymptotic stability have been derived.

5. References

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