CASE 3

WARRANTY COST ANALYSIS [(i) AIRCRAFT WINSHIELD and (ii) MICROWAVE ANTENNAS]

CASE STUDIES IN RELIABILITY

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WARRANTY COST ANALYSIS
[(i) AIRCRAFT WINSHIELD and
(ii) MICROWAVE ANTENNAS]

WARRANTY CONCEPT

- Contractual agreement
- Established on sale of product
- Establishes
  - Buyer responsibility
  - Limitations
  - Seller liability

ROLE AND USES

1. Protection for buyer
   - Remedy on failure of item
   - Assurance of performance

   This is important as customers cannot assess a new product.

   Many countries have warranty legislation to ensure this.

ROLE AND USES

2. Protection for Seller
   - Specifies Buyer responsibilities
   - Limits liability
   - Limits cost
3. Marketing tool
   - Signal quality
   - Differentiate from others

PRODUCT WARRANTY

- Nearly every kind of product is sold with warranty (Consumer Non-durables, Consumer Durables, Software, Commercial and Industrial Products, and Defense Acquisition)
- Many different kinds of warranty policies are offered

FREE REPLACEMENT POLICY

The manufacturer agrees to repair or provide replacements for failed items free of charge up to a time W from the time of the initial purchase. The warranty expires at time W after purchase.
(Denoted as FRW Policy.)

PRO-RATA POLICY

The manufacturer agrees to refund a fraction of the purchase price should the item fail before time W from the time of the initial purchase. The buyer is not constrained to buy a replacement item.
(Denoted as PRW Policy.)

FURTHER READING

Blischke, W. R., and D. N. P. Murthy (1994). *Warranty Cost Analysis*, New York: Marcel Dekker, Inc.

_____, Eds. (1996). *Product Warranty Handbook*, New York: Marcel Dekker, Inc.

Murthy, D.N.P. And Blischke, W.R. (2004). *Warranty Management and Manufacturing*, Under preparation

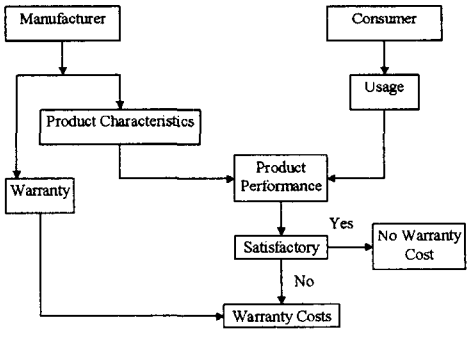
WARRANTY COST ANALYSIS

- Offering warranty results in additional costs to the manufacturer due to servicing claims over the warranty period
- The cost depends on
  - Product reliability
  - Warranty duration and terms
  - Actions taken (repair, replace, refund etc)

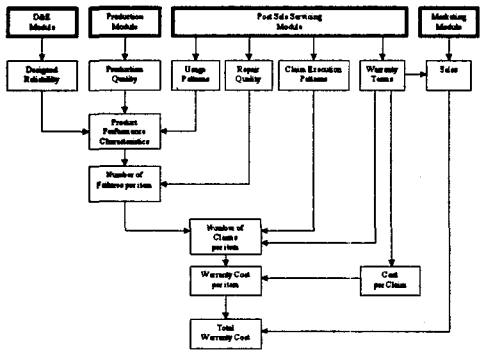
MODELLING

- Simple versus Detail models: See the next two slides
- The simple model is adequate for the cost analysis of FRW and PRW policy for an existing product
- The detail model is needed for new product yet to be developed

SIMPLE MODEL



DETAILED MODEL



SOME NOTATION

C_s: seller's average cost per item

C_r: average repair cost

C_b: buyer's cost of item (selling price)

F(t): Failure distribution for the item

M(t): Renewal function associated with F(t)

(Obtained from tables – See *Warranty Cost Analysis* by Blischke and Murthy)

FRW POLICY

<u>NONREPAIRABLE ITEMS</u>

Average cost per unit sold = c_s M(W)

<u>REPAIRABLE ITEMS</u>

Repaired items are "good-as-new" (true for Case-1)

Average cost per unit sold = c_rM(W)

PRW POLICY

Average cost per unit sold = $c_b[F(W) - \mu_W/W]$

(μ_T = Average time to failure of all items that fail with lifetimes less than T)

Formulas for μ_W are given in *Warranty Cost Analysis* book

CASE: AIRCRAFT WINDSHIELD

From Chapter 13 of
Warranty Cost Analysis

PROBLEM

- Item supplied without warranty
- Customers requests two-year warranty
- Select warranty terms (duration – time or flying hours)
- Asess warranty costs

WINDSHIELD FAILURES

Windshield has several layers of material, including a very strong outer skin with a heated layer just beneath it, all laminated under high temperature and pressure.

Failures result from damage or delamination of the non-structural outer ply, or failure of the heating system.

These failures do not result in damage to the aircraft, but do result in replacement of the windshield.

DATA FOR MODELLING

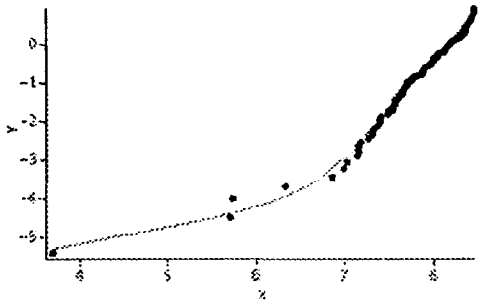
- Failure data: Time to failure
- Censored data: Items have survived for this length
- This makes the estimation problem a bit more difficult

Failure Times				Service Times		
0.040	1.866	2.385	3.443	0.046	1.436	2.592
0.301	1.876	2.481	3.467	0.140	1.492	2.600
0.309	1.899	2.610	3.478	0.150	1.580	2.670
0.557	1.911	2.625	3.578	0.248	1.719	2.717
0.943	1.912	2.632	3.595	0.280	1.794	2.819
1.070	1.914	2.646	3.699	0.313	1.915	2.820
1.124	1.981	2.661	3.779	0.389	1.920	2.878
1.248	2.010	2.688	3.924	0.487	1.963	2.950
1.281	2.038	2.823	4.035	0.622	1.978	3.003
1.281	2.085	2.890	4.121	0.900	2.053	3.102
1.303	2.089	2.902	4.167	0.952	2.065	3.304
1.432	2.097	2.934	4.240	0.996	2.117	3.483
1.480	2.135	2.962	4.255	1.003	2.137	3.500
1.505	2.154	2.964	4.278	1.010	2.141	3.622
1.506	2.190	3.000	4.305	1.085	2.163	3.665
1.568	2.194	3.103	4.376	1.092	2.183	3.695
1.615	2.223	3.114	4.449	1.152	2.240	4.015
1.619	2.224	3.117	4.485	1.183	2.341	4.628
1.652	2.229	3.166	4.570	1.244	2.435	4.806
1.652	2.300	3.344	4.602	1.249	2.464	4.881
1.757	2.324	3.376	4.663	1.262	2.543	5.140
1.795	2.349	3.385	4.694	1.360	2.560	

MODELLING

- **Data: 88 failure times & 65 service times**
- **Repairable item: Repaired back to new**
- **WPP Plot used to select the model**
- **Mixture Weibull distribution fits the data best (See next slide)**
- **Ignoring the few early failure (by treating them as outliers) the two-parameter Weibull is a good fit (Shape parameter > 1)**

WPP PLOT



POLICIES CONSIDERED

1. **FRW, W = 5000 fl. hrs.**
2. **FRW, W = 2 years, calendar time**
3. **PRW, W = 5000 fl. hrs.**
4. **PRW, W = 2 years**

(Average usage rate: 3061 flight hours per year)

WARRANTY COSTS

Costs: $C_s = \$9000$ $C_b = \$17500$ $C_r = \$5400$

Policy	Estimated Cost
1	$15,669
2	18,978
3	13,300
4	16,098

CASE: MICROWAVE ANTENNAS

MICROWAVE ANTENNAS

- Used for sending and receiving microwave signals
- Major Components
  - Crystal Receiver
  - Crystal Transmitter
  - 2Mb card
  - 2Mb PCM card

SOME CHALLENGES

- Sold in lots (size varying from 1 - 100)
- Sold with 3 year FRW policy
- Failed items returned in batches
- No information about
  - the time at which the item was put in use
  - the time at which the item failed

Manufacturing Cost Data

Customer Order Number	Number of Systems in Batch	Labour Cost ($)	Material Cost ($)	Overhead Recovered ($)	Direct Expenses ($)	Total Cost ($)	Average Cost per System ($)
034-1605	2	1329	11474	929	0	13732	6866.00
034-1616	24	18155	131811	12355	1035	163356	6806.50
034-1758	2	2665	11940	1865	0	16470	8235.00
034-1809	38	30600	178243	21415	0	230258	6059.42
034-1899	2	1800	10250	1260	0	13310	6655.00
056-1976	4	2812	29448	1966	0	34226	8556.50
068-1838	4	3966	27528	2776	0	34270	8567.50
072-1955	2	1238	18782	867	0	20887	10443.50
099-1429	100	72900	529900	51100	7100	661000	6610.00
106-1682	16	14180	132379	9928	1386	157873	9867.06

Repair Cost Data

Repair Job Number	Number of Failed Parts	Labour Cost ($)	Material Cost ($)	Overhead Recovered ($)	Direct Expenses ($)	Total Cost ($)	Average Cost per System ($)
034-W345	4	230	435	161	0	826	206.50
034-W355	1	83	0	62	0	145	145.00
034-W364	2	108	2	75	0	185	92.50
034-W378	1	54	0	37	0	91	91.00
040-R215	1	63	0	44	0	107	107.00
040-W241	1	63	0	44	0	107	107.00
040-W252	3	111	2	78	0	191	63.67

Failure Records

Repair Job #	Failure Date	Customer Job #	Despatch Date	Age (Days)	Serial No
040-W285	9/16/94	040-1145	7/14/93	429	87
196-R344	5/3/95	196-1306	7/30/93	642	1376
040-W376	3/8/95	040-1168	9/20/93	534	1396
040-W376	3/8/95	040-1168	9/20/93	534	1397
040-W376	3/8/95	040-1168	9/20/93	534	1398
040-W376	3/8/95	040-1168	9/20/93	534	1300
040-W252	7/11/94	040-1359	10/29/93	255	1559
040-W252	7/11/94	040-1360	10/29/93	255	1498

Survival Records

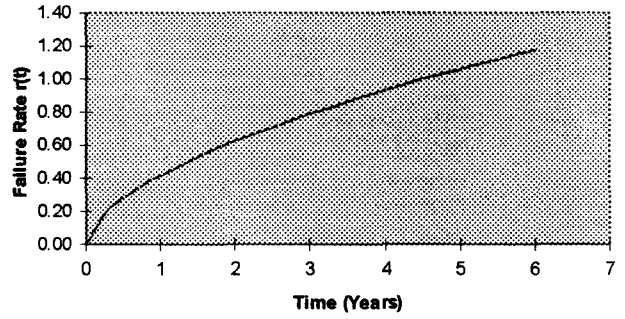
Customer Job Number	Despatch Date	Number of Systems	Date of Data Collection	Age (Days) at Date of Data Collection
196-1306	7/30/93	2	6/9/95	679
196-1322	9/11/93	2	6/9/95	636
355-1336	9/24/93	2	6/9/95	623
040-1358	10/29/93	2	6/9/95	588
040-1359	10/29/93	2	6/9/95	588
040-1360	10/29/93	2	6/9/95	588
040-1361	10/29/93	2	6/9/95	588
034-1393	11/12/93	10	6/9/95	574

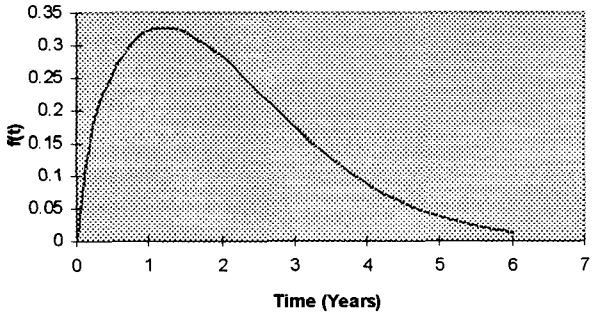
Data used in MATLAB program for analysis

Batch Number j	Batch Size N_j	Age of Batch T_j	Number of Failures in Batch n_j	Failure Times x_{ij}
1	2	679	1	642
2	2	636	0	
3	2	623	0	
4	2	588	0	
5	2	588	2	255
				350
6	2	588	3	255
				354

PARAMETER ESTIMATES

- **Manufacturing Cost per Item, C_s = \$7316.18**
- **Repair Cost per Item, C_r = \$143.94**
- **Weibull scale parameter, $\lambda(= 1/\eta)$ = 0.43233**
- **Weibull shape parameter, β = 1.57479**

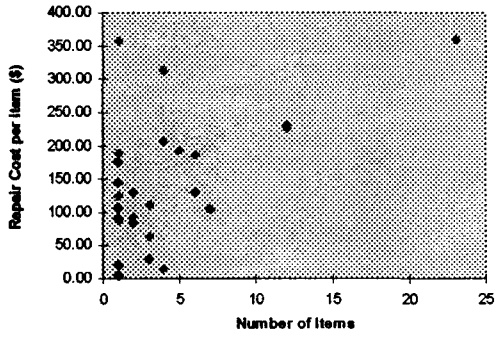




Warranty Servicing Cost for different Warranty Periods

Warranty Period (Years) W	Expected Number of Failures in Warranty Period M(W)	Warranty Servicing Cost ($) c*M(W)	Warranty Servicing Cost as a percentage of the manufacture cost
1	0.27	38.43	0.53
2	0.80	114.48	1.56
3	1.51	216.78	2.96
4	2.37	341.02	4.66
5	3.37	484.61	6.62
6	4.49	645.78	8.83

REPAIR COST PER ITEM vs NUMBER OF ITEMS RETURNED



WARRANTY SERVICING COSTS FOR DIFFERING REPAIR COSTS

Warranty Period (Years)	Repair Cost $143.94	Repair Cost plus 1 Standard Deviation $238.87	Repair Cost plus 2 Standard Deviations $333.81	Repair Cost plus 3 Standard Deviations $428.75
1	38.43	63.78	89.12	114.47
2	114.48	189.99	265.49	341.00
3	216.78	359.77	502.76	645.75
4	341.02	565.95	790.89	1015.82
5	484.61	804.25	1123.90	1443.55
6	645.78	1071.73	1497.69	1923.64

WARRANTY SERVICING COSTS FOR VARYING SCALE PARAMETER λ

Warranty Period (Years)	99.9% Confidence Interval for λ Lower Limit 0.42295	99% Confidence Interval for λ Lower Limit 0.42499	95% Confidence Interval for λ Lower Limit 0.42674	Point Estimate for λ 0.43233	95% Confidence Interval for λ Upper Limit 0.43793	99% Confidence Interval for λ Upper Limit 0.43968	99.9% Confidence Interval for λ Upper Limit 0.44172
1	37.12	37.41	37.65	38.43	39.21	39.46	39.75
2	110.59	111.43	112.15	114.48	116.82	117.56	118.42
3	209.42	211.01	212.38	216.78	221.21	222.61	224.24
4	329.43	331.93	334.10	341.02	347.99	350.19	352.75
5	468.14	471.70	474.77	484.61	494.52	497.64	501.28
6	623.84	628.58	632.68	645.78	658.98	663.15	668.00

WARRANTY SERVICING COSTS FOR VARYING SCALE PARAMETER β

Warranty Period (Years)	99.9% Confidence Interval for β Lower Limit 1.56540	99% Confidence Interval for β Lower Limit 1.56744	95% Confidence Interval for β Lower Limit 1.56920	Point Estimate for β 1.57479	95% Confidence Interval for β Upper Limit 1.58038	99% Confidence Interval for β Upper Limit 1.58214	99.9% Confidence Interval for β Upper Limit 1.58418
1	38.73	38.67	38.61	38.43	38.25	38.19	38.13
2	114.63	114.60	114.57	114.48	114.38	114.35	114.32
3	216.25	216.37	216.47	216.78	217.10	217.20	217.31
4	339.27	339.65	339.98	341.02	342.06	342.39	342.78
5	481.11	481.87	482.52	484.61	486.70	487.36	488.13
6	640.03	641.27	642.35	645.78	649.23	650.32	651.58