TIGHTENED CRITICAL VALUE DEGRADATION TEST

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Abstract

Determination of sample sizes and the inspection intervals for degradation tests is considered. The cases of degradation rate model and degradation path model are analyzed with some examples. Tightened critical value tests are also considered that are shown to be advantageous over non-tightened ones.

1 Introduction

A degradation test records changes over time of some performance characteristics and a failure is said to occur if the amount of degradation of the item exceeds the pre-specified threshold level. Many reliability test standards require that degradation of some characteristics should be measured to check if a failure occurs. In manufacturing industries, such degradation data are taken at regular time basis or at predetermined time epochs. Those measurements may provide some useful information to assess reliability and have some important practical advantages.

Degradation tests are performed usually under elevated stresses which is called an accelerated degradation test (ADT). Many authors have studied ADT's (see Nelson[4], chapter 11). Most of them are concerned with the investigation of physical properties of degradation. However, to assess the reliability of the item under test or to perform such experiments efficiently, statistical analysis such as inference for failure time distribution or test design should be considered.

This study considers the problem of design a degradation test. To design a degradation test, a lot of factors should be determined physically or statistically; objective of test, degrading performance characteristics, method of measurement, failure criteria, stresses and their levels, sample size, inspection interval, stress and life relationship, *etc.* Among them, sample size and inspection interval need statistical decision.

There are four different approaches in designing a life test statistically; confidence interval approach, minimizing variances of estimators approach, economic design approach and acceptance sampling (i.e. hypothesis testing) approach. The first approach is used in bogey testing in automobile industry and MIL-STD 690, while the second one is mostly used in design accelerated life tests. Boulanger and Escobar [1] studied the

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problem of designing an ADT where the sample size and observation times determined to minimize the asymptotic variances of estimators. They also proposed a method of determining the required sample size based on the relative percentage of estimation error, which may considered as a variant of the confidence interval approach.

Economic design is to minimize the average cost of testing or to optimize some objective functions such as the variance of estimators under cost constraints. Wu and Chang[6], Yu and Tseng[9], Yu and Chiao[8],Yu[7] have considered the problem of economic design of a degradation test. The acceptance sampling approach is adopted in reliability qualification test standards such as MIL-STD 781, MIL-HBDK 108, *etc.* Sohn and Jang[5] proposed an acceptance sampling plan by attribute based on ADT.

This study proposes a method of designing a degradation test based on acceptance sampling approach. To analyze degradation test data, models that can describe the degradation behaviors of the performance characteristics are required. Several different models are proposed (see Nelson [4]). Among them, degradation rate model and degradation path model are the most frequently used ones in the literatures. In this study, we proposed methods of determination of sample size and the inspection interval for both models. Degradation rate model is considered in Section 2 and degradation path model is considered in Section 3.

2 Degradation Rate Model

Notation

Y_t : performance level at time t
t_o : pre-specified test time
C_o : critical value
N : number of total samples
k: number of inspections
τ: length of inspection interval ($\equiv t_o/k$)
n : number of samples inspected at inspection times $\tau, 2\tau, \cdots, k\tau$ ($\equiv N/k$)
Y_{ij} : measured value of Y_t of the i^{th} item at time $j\tau$, $i=1,\cdots,n$, $j=1,\cdots,k$
$F(t: \beta, \theta_t)$: distribution function of Y_t
β: constant parameter
θ_t: parameter varying over time
h(t) : known increasing function of t
$L(\cdot)$: probability of accepting a lot
$C_T(t)$: tightened critical value at time t
$\Phi(\cdot)$: distribution function of a standard normal random variable

Assume N items are available for the degradation test. The sampling plan is as follows:

Acceptance sampling plan: N items are put to test at time 0. At $\tau, 2\tau, \cdots, k\tau$, n items are taken out without replacement and the values of the performance characteristic are measured. If there are no failed items at each time τ, $2\tau, \cdots, k\tau$, the lot is accepted. Otherwise, the lot is rejected.

In the sampling plan, an item is defined to be failed if the performance characteristic exceeds the critical value C_o. A degradation rate model is to describe the degradation behavior in terms of the time varying parameters of the distribution function of the performance characteristic. We assume that the time varying parameter θ_t is of the type

$$\theta_t = \theta h(t), \tag{1}$$

where θ is a constant.

We assume that the test time t_o and the critical value C_o are predetermined. Hence, to design the test, the values of n and k (or τ) should be determined. Assume that we can find an acceptable value θ_a of θ, which means that the lots with θ smaller than θ_a are considered to be good ones. We are to determine the values of n and k so that a lot with θ_a is accepted with probability not less than $1-\alpha$, $0<\alpha<1$.

Since Y_{ij}'s are independent, the probability of accepting a lot with θ is

$$L(\theta_a) = \prod_{j=1}^{k} [F(C_o : \beta, \theta h(j\tau))]^n. \tag{2}$$

Hence the values of n and k can be obtained as the least solutions satisfying

$$L(\theta_a) \geq 1-\alpha . \tag{3}$$

Numerical search analysis is needed to obtain the solutions of (3). Note that there may be many combinations of (n,k). Among them, suitable one may be selected in consideration of practical limitation such as testing facilities and costs of samples.

In degradation tests, there are many cases where the amounts of degradation of some performance characteristics in earlier times of the test are so large that it is not necessary to continue the test until the prescribed test time t_o. In this case, a tightened critical value test may be used to shorten the test time. The appropriate critical value at time t is

$$C_r(t) = C_o \frac{h(t)}{h(t_o)}, \tag{4}$$

An item is judged to be failed if $Y_t > C_r(t)$. In this case, we have

$$F(C_r(t); \beta, \theta_t) = F(C_o; \beta, \theta_{t_o}). \tag{5}$$

Hence, the values of n and k can be obtained as the least solutions satisfying

$$nk \geq \frac{\ln(1-\alpha)}{\ln(F(C_o;\beta,\theta_a h(t_o))}. \tag{6}$$

Example 1. Assume that Y_t follows an exponential distribution with expected value $\theta_t = \theta h(t)$. Also assume that $t_o = 2$ and $C_o = 30$. The values of n and k satisfying (3) are listed in Table 1 for some h(t)'s, where θ_a is selected as 2.35 and α=0.05.

Table 1. Values of (k,n) of Example 1.

k \ h(t)	t	$\sqrt{2t}$	$\sqrt[4]{8t}$	$3.16(1-e^{-t/2})$
1	30	30	30	31
2	30	28	23	31
3	29	24	19	29
4	27	21	15	27
5	25	18	13	25
6	23	16	11	23
7	21	15	10	21
8	20	13	9	20
9	18	12	8	18
10	17	11	7	17

In this example, h(t)'s are accommodated to have the same values at t_o. Note that the values of n do not decrease greatly as k increases. This is because that the values of θ_t's at earlier times of the test are small so that the results of the earlier inspections do not contribute much to the determination of n.

However, if the critical values are tightened as in (4), n is obtained as

$$n = \frac{N*}{k}, \tag{7}$$

where N* is the value of n in the first row of Table 1. Note that in the tightened critical value test considered a lot is rejected if any item turns out to be failed at any inspection times. Hence we may reject the lot with bad performance characteristics in the earlier times of the test. This is an advantage of the tightened critical value degradation test.

Example 2. Jang et al. [2] analyzed the degradation behavior of an optical pickup, where the photo diode balance is taken as the degrading characteristic. The test is performed until t_o = 192 hours with C_o = 30. The degradation distribution is assumed to be a Weibull with constant shape parameter β and time varying scale parameter η_t. From the test results, β is estimated as 2.2 and the scale parameter satisfies

$$\eta_t = \eta(1 - e^{-t/71}), \tag{8}$$

where η was estimated to be 21. In this case, the failure probability of an item at t_o is estimated as 8.12%, which seems to be too large. Hence, the acceptable level of η_a is chosen to be 15, which makes the failure probability at t_o 0.5%. Table 2 shows the values of n and k satisfying (3) with α=0.1.

Table 2. Values of (k,n) of Example 2.

k	1	2	3	4	5	6	7	8
n	20	19	16	14	12	10	9	8

For the tightened critical value test, the values of n and k can also be determined as in (7) with N* = 20. If we let k=4, then we have n=5. Hence, we put 20 items to the test and take out 5 items for inspection at 48, 96, 144 and 192 hours. If there comes any failed item against the tightened critical value at those inspection times, the lot is rejected.

3 Degradation Path Model

A degradation path model is to explain the trajectory of degrading characteristic. The model seems to be appropriate for the cases where n items are put to the test and their performance values are measured at some predetermined times repeatedly while the items are put back to the test after measurement.

This study assumes that

$$Y_t = \theta h(t) + \varepsilon_t, \tag{9}$$

where ε_t denotes the independent measurement error. We assume that the distribution of ε_t is $N(0, \sigma_\varepsilon^2)$.

In this case, it is assumed that the lot is accepted if

$$\overline{Y}_{\cdot j} \le C_o, \quad j=1, 2, \cdots, k, \tag{10}$$

where $\overline{Y}_{\cdot j} = \sum_i Y_{ij} / n$.

Here, θ may be considered as a constant or a random variable. Let's first assume that θ is a positive constant, which means that there is no item-to-item variability. Then we have

$$L(\theta_a) = P\{\overline{Y}_{\cdot 1} \le C_o, \cdots, \overline{Y}_{\cdot k} \le C_o\} = \prod_{j=1}^{k} P\{\overline{Y}_{\cdot j} \le C_o\} = 1 - \alpha, \tag{11}$$

since the measurements errors are assumed to be independent. The values of n and k may be obtained numerically.

Example 3. Table 3 shows the relative brightness level changes over time under room temperature of 5 inorganic electroluminescence backlights. In this case, C_o = 0.4.

Table 3. Brightness level of electroluminescence back lights

t \ item	1	2	3	4	5
135	0.81507	0.8166	0.81435	0.81435	0.81581
357	0.7254	0.72431	0.72331	0.72331	0.72472
500	0.67283	0.67227	0.67032	0.67032	0.67429
1000	0.53514	0.53708	0.53541	0.53541	0.542
1537	0.46122	0.46081	0.4593	0.4593	0.46763
2008	0.41163	0.40665	0.40579	0.40579	0.41667

To analyze the results, the following model is assumed;

$$Y_t = \theta \ln(t/\eta) + \varepsilon_t, \tag{12}$$

where Y_t = lnln(1/brightness level at t). The least squares estimates of θ and η are 0.56 and 2449, respectively. We use these results in the design of a new test where the acceptable value θ_a is selected to be as 0.522. Then the value of n satisfying (11) with α=0.1 and t_o = 1000 is 17 for $1 \le k \le 10$. In this case, the values of n do not vary because the error variance was too small.

Assume now that θ is a normal random variable with mean μ_θ and variance σ_θ^2. Then, we have

$$Y_{ij} = \theta_i h(j\tau) + \varepsilon_{ij}, \tag{13}$$

where ε_{ij}'s are the copies of $\varepsilon_t, i = 1, \cdots, n, j = 1, \cdots, k$.

If the effects of measurement errors are negligible, we have

$$L(\theta_a) = P\{\overline{Y}_{.1} \le C_o, \cdots, \overline{Y}_{.k} \le C_o\} = P\{\overline{\theta} \le \frac{C_o}{t_o}\} = 1 - \alpha, \tag{14}$$

where $\bar{\theta} = \sum_i \theta_i / n$. (14) means that intermediate measurements have no effect on accepting a lot. It is because that $Y_{ij} = \theta_i h(j\tau)$ is increasing in j for the ith item since θ_i is a fixed constant with respect to an item. Hence, we can only determine the value of n as follows;

$$\Phi\left(\frac{C_o/t_o - \mu_\theta}{\sigma_\theta/\sqrt{n}}\right) = 1 - \alpha, \tag{15}$$

or equivalently

$$n = \left(\frac{Z_\alpha \sigma_\theta}{C_o/t_o - \mu_\theta}\right)^2, \tag{16}$$

where $\Phi(Z_\alpha) = 1 - \alpha$.

Example 4. Percent increases in operating current for GaAs lasers tested at 80°C with t_o=40 (hundred hours) are listed in the textbook by Meeker and Escobar[3], p642. A non-intercept linear model is adopted to model the degradation paths. For this device, C_o=10(%) increase is assumed to be a failure level. From the Figure 13.14 in the textbook, p338, we see that the slopes of the degradation lines seem to be randomly varying. By plotting the estimated slopes on a normal probability paper, we see that the normality assumption is not violated. The parameters $\mu_\theta, \sigma_\theta$, σ_ε are estimated as 0.2, 0.046 and 0.19, respectively. Thus from (16) we can obtain n = 3 when α=0.05.

When the measurement errors are not negligible, we can obtain the values of k and n from

$$\prod_{i=1}^{n} \Phi((\boldsymbol{\Sigma}^{-1/2}(C_r\mathbf{1} - \mu_\theta \mathbf{h}))_i) = 1 - \alpha, \tag{17}$$

where $\mathbf{h} = [h(\frac{t_o}{k}), h(\frac{2t_o}{k}), \cdots, h(\frac{kt_o}{k})]^T$, $\boldsymbol{\Sigma} = \sigma_\theta^2 \mathbf{h}\mathbf{h}^T + \sigma_\varepsilon^2 \mathbf{I}$, and $\mathbf{1} = [1,1,\cdots,1]^T$. This follows from the fact that the distribution of $\overline{\mathbf{Y}} = [\overline{Y}_{\cdot 1}, \overline{Y}_{\cdot 2}, \cdots, \overline{Y}_{\cdot k}]^T$ is a k-dimensional multivariate normal with mean vector $\mu_\theta \mathbf{h}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. In this case, we may reduce the critical value test as in (4). The values of k and n of the tightened critical value test can be obtained from

$$\prod_{i=1}^{n} \Phi((\frac{C_r}{h(t_o)} - \mu_\theta)(\boldsymbol{\Sigma}^{-1/2}\mathbf{h})_i) = 1 - \alpha. \tag{18}$$

4 Conclusion

This study presents an approach of determining the sample size and the inspection interval for a degradation test. The cases of degradation rate model and degradation path model are considered. This study also proposes tightened critical value tests which are advantageous over non-tightened critical value tests. Some actual examples are analyzed with the proposed approaches.

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