한국소음진동공학회 2004년도 춘계학술대회논문집, pp. 597~600.

접촉 강성을 고려한 디스크브레이크의 면외진동 해석
Analysis of Out-of-plane Motion of a Disc Brake System Considering Contact Stiffness

신기홍*·조용구**·오재응***
Kihong Shin, Yong-Goo Joe and Jae-Eung Oh

Key Words : Disc brake(디스크 브레이크), Contact Stiffness(접촉강성), Stability Analysis(안정성해석), Complex Eigenvalue Analysis(복소고유치해석) Nonlinear Analysis(비선형해석).

ABSTRACT

A two-degree-of-freedom out-of-plane model with contact stiffness is presented to describe dynamical interaction between the pad and disc of a disc brake system. It is assumed that the out-of-plane motion of the system depends on the friction force acting along the in-plane direction. Dynamic friction coefficient is modelled as a function of both in-plane relative velocity and out-of-plane normal force. When the friction coefficient depends only on the relative velocity, the contact stiffness has the role of negative stiffness. The results of stability analysis show that the stiffness of both pad and disc are equally important. Complex eigenvalue analysis is conducted for the case that the friction coefficient is also dependent on the normal force. The results further verify the importance of the stiffness. It has also been found that increasing the gradient of friction coefficient with respect to the normal force makes the system more unstable. Nonlinear analysis is also performed to demonstrate various responses. Comparing the responses with experimental data has shown that the proposed model may qualitatively well represent a certain type of brake noise.

1. Introduction

Dynamic instability of a disc brake is strongly related to a non-linear oscillation induced by friction interaction at the vibrating interface. Its most usual phenomenon may be the squeal noise and stability analysis for the disc brake noise has been extensively studied over the last few decades. Many of these researches are based on the finite element method [1~3]. However, these methods generally consider many degrees of freedom and many coefficients to constitute complicated expressions. As a result, it may not be easy to find basic roles of the frictional interface from such methods. Experimental results show that the disc brake squeal noise may be resulting from the coupling of out-of-plane motions of the pad and the disc through frictional instability [4, 5]. Thus, a simple model needs to be developed that describes the fundamental dynamics of the friction mechanism of a brake system.

Recently, Shin et al. [6] introduced a two-degree-of-freedom model which describes a 'one mode' interaction between the pad and disc. However, the model only accounts for the in-plane dynamics, and so, in this paper, an out-of-plane model is introduced to find the role of the contacting surface. It is assumed that the friction coefficient depends on both in-plane relative velocity and out-of-plane normal force to describe the relationship between the in-plane and out-of plane vibrations.

The friction mechanism has been found to act as negative stiffness in the out-of-plane vibration if the friction coefficient is only a function of the relative velocity (it is named as 'μ-system' in this paper). Stability analysis is carried out to find the conditions for the noisy state of the system. The results show that the damping is not an important parameter in this case, whereas the stiffness is the essential parameter to overcome the effect of negative stiffness. When the friction coefficient is also dependent on the normal force, a parameter β is introduced which is a partial derivative of the friction coefficient with respect to normal force. This is named as 'β-system' in this paper. The results of complex eigenvalue analysis further verify that the stiffness is the key parameter to control the out-of-plane vibration, and both stiffness parameters of the pad and the disc are equally important.

Nonlinear analysis is also conducted to examine the responses of the system. The results are then compared with an experimental result. In a certain condition, it has been found that both the limit cycle and auto-spectrum from the nonlinear simulation are qualitatively similar to those constructed from the experimental signal.

2. Out-of-plane Model and Its Stability Analysis

Consider the out-of-plane model as shown in Fig. 1. This represents the pad and disc as single-degree-of-freedom systems which are connected through a sliding friction interface and contact stiffness. It is assumed that

* 책임저자; 정회원, 국립안동대학교 기계공학부
E-mail : kshin@andong.ac.kr
Tel : (054) 820-5885, Fax : (054) 820-5885

** 정회원, 한양대학교 대학원

*** 한양대학교 기계공학부

the dynamic friction coefficient is dependent on the in-plane relative velocity and the out-of-plane normal force, i.e., $\mu(v_r, N)$.

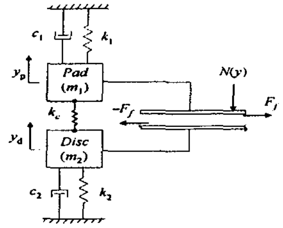

Fig. 1 Two-degree-of-freedom out-of-plane model

The normal force acting on the interface varies with the vertical relative displacement, i.e., $N(y)$ which is influenced by the contact stiffness k_c. Thus, the model describes how the in-plane motion affects the out-of-plane motion through the change of friction force. The equations of motion of the system can be written as

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K+K_c(y)]\{y\}=0 \tag{1}$$

where, the normal force can be written as

$$N(y)=k_c(y_p-y_d) \tag{2}$$

Thus, the friction force on the pad side becomes

$$F_f=-\mu(v_r,N)\cdot k_c\cdot(y_p-y_d) \tag{3}$$

where v_r is the relative velocity between the pad and disc along the in-plane direction. Although the friction force acts along the horizontal axis, it depends on the vertical relative displacement. As a result, the effect of friction force is imbedded in the contact stiffness. The linearized contact stiffness matrix can be found by [3].

$$[K_c]=-\mu(v_r,N)\begin{bmatrix} k_c & -k_c \\ -k_c & k_c \end{bmatrix} -k_c\begin{bmatrix} \frac{\partial\mu}{\partial N}k_c(y_p-y_d) & -\frac{\partial\mu}{\partial N}k_c(y_p-y_d) \\ \frac{\partial\mu}{\partial N}k_c(y_p-y_d) & -\frac{\partial\mu}{\partial N}k_c(y_p-y_d) \end{bmatrix} \tag{4}$$

If the friction coefficient is not greatly affected by the change of the normal force, the slope $\frac{\partial\mu}{\partial N}$ approaches zero, then the contact stiffness matrix becomes

$$[K_c]=-\mu(v_r)\begin{bmatrix} k_c & -k_c \\ -k_c & k_c \end{bmatrix} \tag{5}$$

Considering the friction coefficient is always positive, $\mu(v_r)>0$, the stiffness elements in the *symmetric* contact stiffness matrix $[K_c]$ all act as a *negative stiffness*. The system with this contact stiffness is named as 'μ-system' in this paper for convenience. In this case, the equations of motion can be written as

$$\begin{aligned} m_1\ddot{y}_p+c_1\dot{y}_p+(k_1-\mu(v_r)k_c)y_p+\mu(v_r)k_cy_d=0 \\ m_2\ddot{y}_d+c_2\dot{y}_d+(k_2-\mu(v_r)k_c)y_d+\mu(v_r)k_cy_p=0 \end{aligned} \tag{6}$$

It is important to note that the term $\mu(v_r)k_c$ is the most important parameter that acts as a negative stiffness. Note also that the friction coefficient depends on the in-plane relative velocity, but it constitutes the negative stiffness in the out-of-plane direction. Therefore, it can be considered that the in-plane vibration affects on the out-of-plane motion through the change of the friction coefficient.

If $\mu(v_r)$ is assumed to be a positive constant, linear stability analysis may be performed. For stability analysis, the characteristic equation becomes

$$\det\begin{bmatrix} \lambda^2+c_{11}\lambda+k_{11} & k_{12} \\ k_{21} & \lambda^2+c_{22}\lambda+k_{22} \end{bmatrix}=0 \tag{7}$$

where $c_{11}=\frac{c_1}{m_1}$, $c_{22}=\frac{c_2}{m_2}$, $k_{11}=\frac{k_1-\mu k_c}{m_1}$, $k_{22}=\frac{k_2-\mu k_c}{m_2}$, $k_{12}=\frac{\mu k_c}{m_1}$, $k_{21}=\frac{\mu k_c}{m_2}$. After some mathematical manipulations, a simple stability criterion can be obtained as

$$\bar{k}_1+\bar{k}_2\le\bar{k}_1\bar{k}_2 \tag{8}$$

where $\bar{k}_1=\frac{k_1}{\mu k_c}$ and $\bar{k}_2=\frac{k_2}{\mu k_c}$. Several straight lines ($\bar{k}_1$ versus $\bar{k}_2$) can be drawn for various values of m_1 and m_2. These are shown in Fig. 2(a). The curve $\frac{1}{\bar{k}_1}+\frac{1}{\bar{k}_2}=1$ that satisfies the stability criterion is also shown in this figure where the curve forms the boundary between stable and unstable regions. It is also shown that the stability of the system does not depend on mass parameters. The stability criterion is further verified by some numerical simulations. The Routh-Hurwitz criterion is directly applied to equation (7) by using the same method in [6]. The stability of the system for various parametric conditions is examined, and the results are shown in Fig. 2(b). Note that the curve $\frac{1}{\bar{k}_1}+\frac{1}{\bar{k}_2}=1$ in Fig. 2(a) coincides with the stability boundary in Fig. 2(b). It has also been found that the results are always the same regardless of mass and damping values. In other words, unlike the case of the in-plane vibration [6], the stability of the system does not depend on the damping parameters or the masses, but only the magnitude of stiffness parameters affects the stability of the system. It can be seen that the stability criterion given in equation (8) implies that the stiffness of *both* pad and disc must be sufficiently large enough to suppress the effect of negative stiffness.

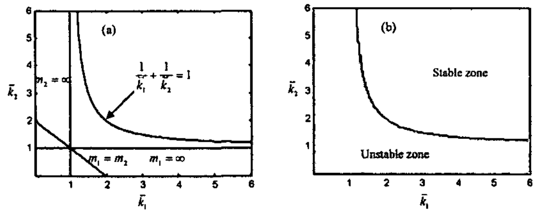

Fig. 2 (a) Plots of $\bar{k}_1$ versus $\bar{k}_2$ for various values of m_1 and m_2, (b) Stability area using the Routh-Hurwitz criterion for any values of mass and damping.

3. Complex Eigenvalue Analysis

If the friction coefficient is a function of both relative velocity and normal force, the non-symmetric part of the contact stiffness matrix must be considered. In this case, the slope $\frac{\partial\mu}{\partial N}$ has a great influence on the stability of the system and is an important parameter. Now let such system to be 'β-system' and $\beta = \frac{\partial\mu}{\partial N}$ for convenience. In this section, considering this non-symmetric stiffness matrix, a qualitative change in the dynamics such as the bifurcation is examined as the parameter β varies. The real parts of the eigenvalues of the model (equation (1)) are monitored as a system parameter changes, and the corresponding bifurcation diagrams are examined.

It is assumed that the out-of-plane normal force and the in-plane relatively velocity are independent of each other. And let $\mu(v_r)$=1 for convenience. Then, the contact stiffness matrix becomes

$$[K_c] = -\mu(N)\begin{bmatrix} k_c & -k_c \\ -k_c & k_c \end{bmatrix} - k_c\begin{bmatrix} \beta k_c(y_p - y_d) & -\beta k_c(y_p - y_d) \\ \beta k_c(y_p - y_d) & -\beta k_c(y_p - y_d) \end{bmatrix} \quad (9)$$

where the contact stiffness matrix is composed of a symmetric negative stiffness part and a non-symmetric part. In this case, the complex eigenvalues are obtained numerically to find the main parameters that make the system unstable. After the complex eigenvalues are obtained, bifurcation diagrams are investigated.

Since the most important factor is the stiffness, its effects are examined in detail by varying the parameter β from 0 to 3, where a constant value of $\mu(N)$=0.6 is used..

The results are shown in Fig. 3 and Fig. 4. When the stiffness of both pad and disc is increased, the bifurcation point moves away to the right and hence the system becomes more stable (Fig. 3). However, if the stiffness is increased in the system only on one side, for example in the pad, initially the bifurcation point moves to the left side and then it moves to the right side as the stiffness is further increased as shown in Fig. 4. Note that moving the bifurcation point to the left side means a detrimental effect on the system stability. Despite this negative effect, Fig. 4 shows the possibility of making the β-system more stable if the stiffness of either the pad or the disc is increased greatly. This result is slightly different from the stability analysis in the previous section where the stability criterion states that it is impossible to make the μ-system stable by increasing the stiffness only on one side if the other stiffness is too small. Nevertheless, from the results shown in Fig. 3 and 4, it can be seen that stiffness reinforcement must be considered to avoid the instability in the out-of-plane motion. It also suggests that it is most effective when the stiffness of both pad and disc are increased simultaneously.

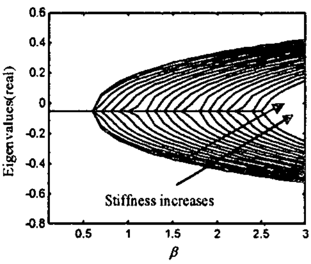

Fig. 3 Bifurcation diagrams when both stiffness parameters (k_1, k_2) are increased. (k_c=0.5, k_2=2k_1, m_1=m_2=1, c_1=c_2=0.1, k_1=0.5~5)

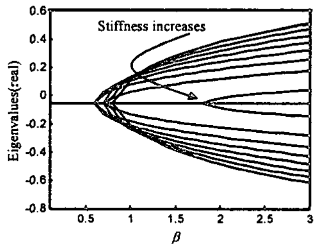

Fig. 4 Bifurcation diagrams when only k_1 is increased. (k_c=0.5, k_2=2, m_1=m_2=1, c_1=c_2=0.1, k_1=0.5~5)

4. Nonlinear Simulation and Experiments

Recently, many experimental results have been published on friction-induced brake noise, especially for disc brake squeal noise [7, 8]. A significant fact found from these experiments is that the peak frequencies of the squeal occur with multiples of the fundamental frequency, which is a typical nonlinear characteristic. It is also found that the contacting surfaces are not always in contact with each other but experience intermittent separations. This phenomenon may be the main source of the nonlinearity. In this section, numerical simulations are performed by taking account of such nonlinearity.

And the aims of nonlinear analysis are to verify the results of linear stability analysis and to find whether the nonlinear simulation results are correlative to the experimental results. A switching nonlinearity is assumed to satisfy the intermittent contact, i.e., when two masses are separated, the system loses its contact stiffness value until they meet again. Then, the stiffness matrix of the system in equation (1) may be given as

$$[K_s]=\begin{cases}[K]+[K_c] & \text{for } \eta\le\varepsilon\\ [K] & \text{for } \eta>\varepsilon\end{cases} \tag{10}$$

where η is the relative displacement between the pad and the disc, and a small region ε is introduced to define the contact region. Numerical simulations are performed to determine the attractors for various system parameter values. For this numerical simulation, the β-system is considered and the system parameters are arbitrarily set to $m_1=m_2=1$, $k_1=1$, $k_2=2$, $c_1=c_2=0.01$, and $\beta=0.7$. The contact stiffness (k_c) is then varied to examine the responses of the system.

Many qualitatively different responses are obtained as k_c varies. When $k_c=4$, as in Fig. 5, the frequency spectrum shows that the fundamental frequency and the second harmonic component are dominant, and the corresponding limit cycle motion is also presented.

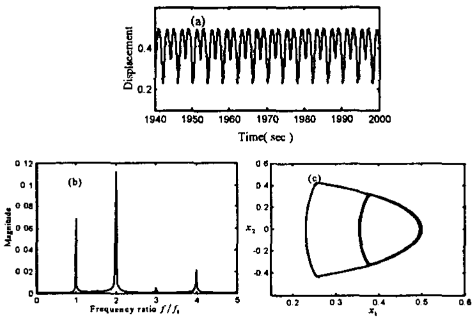

Fig. 5 Motions of the pad for $k_1=1$, $k_2=2$ and $k_c=4$.

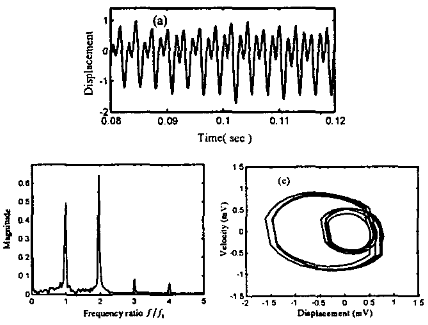

Fig. 6 Experimental results.

This numerical result is compared with an experiment. The displacement signal of the pad was measured with a laser displacement sensor. A typical displacement signal at squeal state is shown in Fig. 6(a), and the corresponding frequency spectrum and phase portrait are shown in Fig. 6(b) and Fig. 6(c). Comparing these figures with the results in Fig. 11, it can be seen that the nonlinear simulation results are qualitatively similar to the experimental results. Thus, it can be considered that the proposed two-degree-of-freedom out-of-plane model may qualitatively well represent this type of brake noise.

5. Conclusions

From stability analysis and complex eigenvalue analysis of the proposed two-degree-of-freedom model, the following results have been found: First, for the 'μ-system', a stability criterion has been derived, and it suggests that both stiffness parameters of the pad and the disc must be sufficiently large to suppress the effect of the negative stiffness. Second, for the 'β-system', the instability can be avoided if the stiffness of either the pad or the disc is increased greatly while the most effective method is increasing both stiffness of the pad and the disc simultaneously. Nonlinear numerical analysis has been conducted, and the results were found to agree well with the experimental results. Thus, it can be concluded that the proposed two-degree-of-freedom model may well describe the fundamental mechanism of the friction induced brake noise.

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