

유한요소모델 개선을 위한 자동화된 매개변수 선정법 : 예제

An Automated Parameter Selection Procedure for Updating Finite Element Model : Example

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4. Examples

In this section, the proposed parameter selection procedure is applied to two example problems, one is the plate example given in section 2.2 and the other is a cover structure of hard disk drive (HDD).

4.1 Cracked clamped plate

The cracked plate given in section 2.2 is taken as an example problem. From Table 1, it can be noticed that the natural frequency error of the 2nd mode pair and the MAC values of 2nd and 3rd mode pairs show most undesirable correlations. Thus the updating parameter selection procedure is applied considering the following three criteria:

$$\{F_1, F_2, F_3\} = \left\{ \left(\frac{f_{a_2} - f_{x_2}}{f_{x_2}} \right)^2, 1 - MAC_{22}, 1 - MAC_{33} \right\} \quad (12)$$

As in section 2.2, it is assumed that only the stiffness matrix need to be updated. For each finite element in the region with the modeling error (see Figure 3), the sensitivities of the criteria (Eq. (12)) with respect to the chosen stiffness parameter are calculated and the resulting signs of the sensitivities are plotted in Figure 9. In this case, the sensitivities of F2 and F3 have the same sign. Using this information, the 1st phase of the parameter selection procedure is applied so that the

criteria given in Eq. (12) remain sensitive to the resulting parameters. Figure 10 shows the selected parameters after the 1 phase. And the stiffness correction matrix is written as

$$\Delta K = \sum_{i=1}^2 p_k K_i, \quad (13)$$

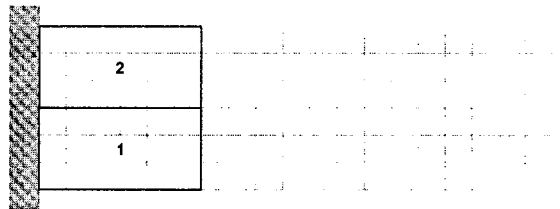


Figure 10: Two updating parameters after the 1st phase of the parameter selection procedure.

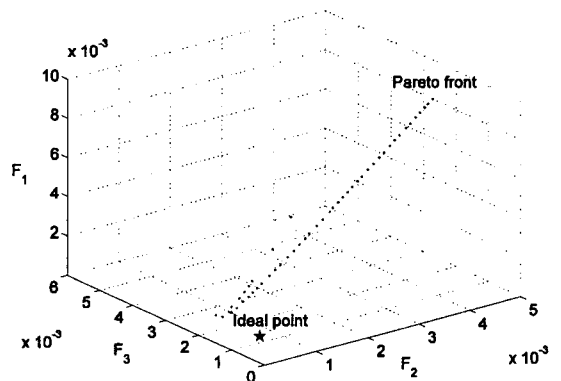


Figure 11: Cracked plate - Pareto front and ideal point.

where p_k and K_i are the updating parameter and stiffness matrix associated with the i^{th} substructure. Obviously, the total sensitivity of the criteria (Eq. (12)) remains unchanged and is evaluated as

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$$\left\{ \sum_{i=1}^2 \left| \frac{\partial F_1}{\partial p_k} \right|, \sum_{i=1}^2 \left| \frac{\partial F_2}{\partial p_k} \right|, \sum_{i=1}^2 \left| \frac{\partial F_3}{\partial p_k} \right| \right\} = \{0.0422, 1.5004, 2.2371\} \quad (14)$$

Since the number of the updating parameters is acceptable, the 2nd phase of the updating parameter selection procedure is not necessary. Thus the parameter selection procedure stops here.

The initial FE model is updated using the selected parameters. Figure 11 shows the Pareto front of the multiobjective function of Eq. (12) under the same constraints as in section 2.2. And the ideal point is obtained as

$$\{F_1, F_2, F_3\} = \{0.0000, 0.0007, 0.0013\} \quad (15)$$

Compared to the results in section 2.2, it can be noticed that the initial FE model is improved drastically. Table 2 summarizes modal properties of an updated FE model. The updated parameter have physical meaning because, due to the crack, p_{k2} is negative and p_{k1} is close to zero as shown in Table 2. From these observations, it can be said that the model updating was successful.

4.2 Hard disk drive (HDD) cover structure

The suggested parameter selection procedure is applied to an FE model of an hard disk drive (HDD) cover structure. The HDD cover is a

TABLE 2: Comparison of modal properties of cracked plate and updated FE model

Natural frequency (Hz)				
Mode	Simulated experiment	Updated model ^a	Error (%)	MAC
1	3.6011	3.4526	-4.1231	0.9999
2	22.7184	22.0003	-3.1606	0.9694
3	23.7103	23.7655	0.2327	0.9694
4	65.0973	62.4133	-4.1231	0.9912

$$^a p_{k1} = 0.1772, p_{k2} = -0.5495$$

rather complex three dimensional structure. In the FE model development, simplifications are made in thickness because the actual HDD cover shell has tapered and abrupt changes in

thickness. The resulting FE model is shown in Figure 12, which consists of solid, shell and beam elements (total 1115 elements, 6732 DOFs).

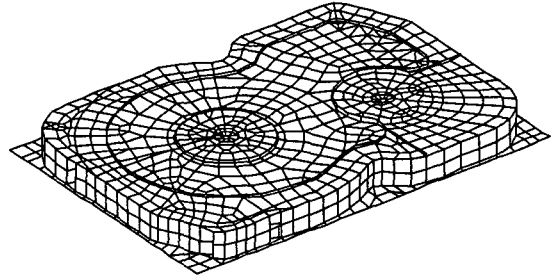


Figure 12: Finite element model of hard disk drive (HDD) cover structure.

TABLE 3: Comparison of the experimental and analytical modal properties before updating

Natural frequency (Hz)				
Mode	Experiment	Initial FE model	Error (%)	MAC
1	409.68	404.13	-1.3507	0.9847
2	908.15	931.94	2.6206	0.9831
3	1707.65	1669.00	-2.2633	0.8326
4	1748.86	1709.13	-2.2717	0.7754
5	1793.23	1757.94	-1.9681	0.8382
6	2474.99	2399.10	-3.0633	0.9496
7	2843.29	2723.27	-4.2213	0.9496
8	2976.06	2878.29	-3.2853	0.9360
9	3113.84	3016.39	-3.1298	0.9582
10	3268.98	3182.76	-2.6374	0.8905

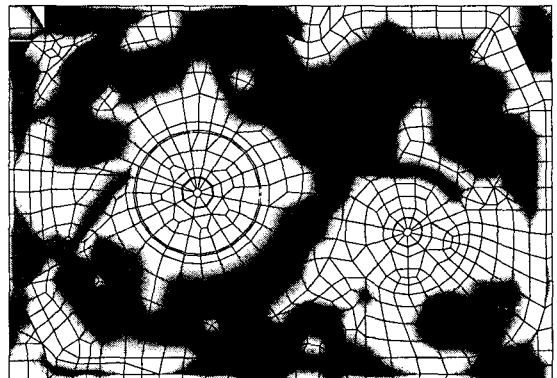


Figure 13: Error location of the initial FE model.

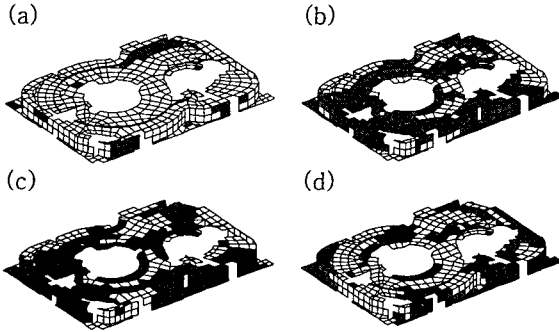


Figure 14: Signs of sensitivities:

$$(a) \frac{\partial F_1}{\partial t_i^e} \quad (b) \frac{\partial F_2}{\partial t_i^e} \quad (c) \frac{\partial F_3}{\partial t_i^e} \quad (d) \frac{\partial F_4}{\partial t_i^e}$$

; □ positive ; ■ negative

To validate the initial FE model, a modal test of the target structure is conducted. The frequency range of interest is from 0 to 3kHz where high vibration and noise levels were observed during an operational test of the HDD. To simulate free-free boundary condition, the cover is supported using soft rubbers. The structure is excited by an impact hammer and responses are measured at 66 points by a laser doppler vibrometer. A CADA-X system is used to measure frequency response functions and extract natural frequencies and mode shapes. The modal properties are compared in Table 3. It tells that the natural frequency errors of the 6th, 7th, 8th, and 9th mode pairs are larger than 3%. Also the MAC values of 3rd, 4th, 5th, and 10th mode pairs are below 0.9. To improve these unsatisfactory correlations, an FE model updating is performed.

First, using an error location technique[5], the region with dominant errors are located as in Figure 13. According to the error location results, 628 shell elements out of 1115 elements turn out to contain the dominant modeling errors. Now the suggested parameter selection procedure is applied to these shell elements. From Table 3, it can be noticed that the natural frequency error of the 7th mode pair is much larger than 3%. And the 3rd, 4th and 5th mode pairs show undesirable

correlations as their MAC values designate. Thus, the updating parameters are selected considering the following four criteria:

$$\{F_1, F_2, F_3, F_4\} = \left\{ \left(\frac{f_{a_2} - f_{s_2}}{f_{s_2}} \right)^2, 1 - MAC_{33}, 1 - MAC_{44}, 1 - MAC_{55} \right\} \quad (16)$$

For each of the shell elements having modeling errors, the sensitivities of the criteria (Eq. (16)) with respect to thickness parameter are calculated and the resulting signs of the sensitivities are shown in Figure 14. The total sensitivities of the criteria are calculated as

$$\left\{ \sum_{i=1}^{628} \left| \frac{\partial F_1}{\partial t_i} \right|, \sum_{i=1}^{628} \left| \frac{\partial F_2}{\partial t_i} \right|, \sum_{i=1}^{628} \left| \frac{\partial F_3}{\partial t_i} \right|, \sum_{i=1}^{628} \left| \frac{\partial F_4}{\partial t_i} \right| \right\} = \{0.0598, 8.1355, 16.4920, 5.2681\} \quad (17)$$

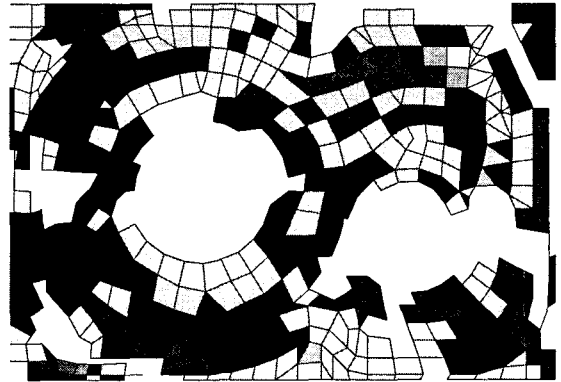


Figure 15: Updating parameters after the 1st phase of the parameter selection procedure.

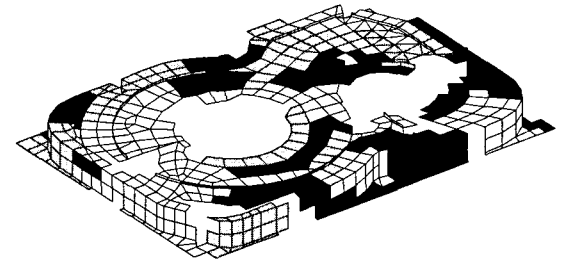


Figure 16: Updating parameters after the 2nd phase of the parameter selection procedure. From these information, the 1st phase of the parameter selection procedure is applied.

Figure 15 shows the resulting updating parameters after the 1st phase. The number of the parameters can be reduced from 628 to 150. Obviously the total sensitivities are not changed:

$$\left\{ \sum_{i=1}^{150} \left| \frac{\partial F_1}{\partial t_i} \right|, \sum_{i=1}^{150} \left| \frac{\partial F_2}{\partial t_i} \right|, \sum_{i=1}^{150} \left| \frac{\partial F_3}{\partial t_i} \right|, \sum_{i=1}^{150} \left| \frac{\partial F_4}{\partial t_i} \right| \right\} = \{0.0598, 8.1355, 16.4920, 5.2681\} \quad (18)$$

Although the number of the updating parameters are reduced to 150 without any sacrifice of the total sensitivities, it is still too many. Thus numerical difficulties are expected in the optimization process. Thus, the 2nd phase of the parameter selection procedure is applied to further reduce the number of updating parameters. The substructures are grouped until the number of the updating parameters become 20 with the minimal sacrifice of the potential sensitivity at each step. Although the total sensitivities are decreased slightly from Eq. (18) to

$$\left\{ \sum_{i=1}^{20} \left| \frac{\partial F_1}{\partial t_i} \right|, \sum_{i=1}^{20} \left| \frac{\partial F_2}{\partial t_i} \right|, \sum_{i=1}^{20} \left| \frac{\partial F_3}{\partial t_i} \right|, \sum_{i=1}^{20} \left| \frac{\partial F_4}{\partial t_i} \right| \right\} = \{0.0568, 7.7484, 15.6068, 4.9034\} \quad (19)$$

but the number of the updating parameter is drastically reduced from 150 to 20. The final updating parameter are shown in Figure 16. The initial FE model is updated by varying the selected 20 thickness parameters. The allowed maximum change of the parameters is set to 5% (about $50\mu\text{m}$) considering only measurement error. Modal properties of an updated model are compared with experimental results in Table 4. The updated results give quite acceptable correlations. For all the mode pairs, the natural frequency errors are less than 3%, and the MAC values are larger than 0:93.

TABLE 4: Comparison of the experimental

and analytical modal properties after updating

Natural frequency (Hz)				
Mode	Experiment	Initial FE model	Error(%)	MAC
1	409.68	403.77	-1.4389	0.9846
2	908.15	934.95	2.9520	0.9834
3	1707.65	1682.01	-1.5016	0.9560
4	1748.86	1724.98	-1.3658	0.9356
5	1793.23	1760.03	-1.8513	0.9356
6	2474.99	2427.18	-1.9318	0.9547
7	2843.29	2761.50	-2.8766	0.9547
8	2976.06	2890.45	-2.8766	0.9525
9	3113.84	3035.52	-2.5154	0.9672
10	3268.98	3195.55	-2.1849	0.9356

5. Conclusion

The problem of updating parameter selection was addressed in this work. The importance of updating parameter was demonstrated through case studies. By introducing the concept of total sensitivity, an updating parameter selection procedure was suggested. The suggested procedure is accomplished by a sequence of two different selection phases. The outstanding feature of the 1st phase of the parameter selection procedure is that the objective functions of concern are kept sensitive to the resulting parameters while the number of the parameters are reduced as small as possible. After the 1st phase, the parameter selection procedure can stop if the number of the resulting parameter is acceptable. Otherwise, the parameter selection procedure proceeds to the 2nd phase. In this phase, the updating parameters are grouped at the sacrifice of sensitivities. But a procedure was provided to minimize such sacrifice. Using the suggested parameter selection method, the objective function of interest remain most sensitive to the resulting parameters. Again it should be noticed that the parameter selection procedure must be followed by precise error localization. The effectiveness of the suggested method is proved by both a simulated case study and a real engineering

problem.

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REFERENCES

- [1] Berman, A., Multiple Acceptable Solutions in Structural Model Improvement, *AIAA Journal*, 33(5), 1995, pp.924-927.
- [2] Friswell, M. I. and Mottershead, J. E., *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic, Norwell, MA, 1995.
- [3] Levin, R. I. and Lieven, N. A. J., Dynamic Finite Element Model Updating using Simulated Annealing and Genetic Algorithms, *Mechanical Systems and Signal Processing*, 12(1), 1998, pp.91-120.
- [4] Avitabile, P., Model Updating: Endless Possibilities, *Proceedings of the 18th International Modal Analysis Conference*, 2000, pp.562-570.
- [5] Fissette, E. and Ibrahim, S., Error Location and Updating of Analytical Dynamic Models using a Force Balance Method, *Proceedings of the 6th International Modal Analysis Conference*, 1988, pp.1063-1070.
- [6] Zitzler, E., *Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications*. Ph.D. Thesis, Swiss Federal Institute of Technology Zurich, Germany, 1999.
- [7] Miettinen, K., *Nonlinear Multiobjective Optimization*, Kluwer Academic, Norwell, MA, 1999.
- [8] Kim, G. H., *Finite element model updating using multiobjective optimization technique*. Ph.D. Thesis, Dept. of Mechanical Engineering, Korea Advanced Institute of Science and Technology, 2003