

# Zwicker 라우드니스에 대한 설계 민감도 해석 및 최적화 DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION OF ZWICKER'S LOUDNESS

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## ABSTRACT

The design sensitivity analysis of Zwicker's loudness with respect to structural sizing design variables is developed. The loudness sensitivity in the critical band is composed of two equations, the derivative of main specific loudness with respect to 1/3-oct band level and global acoustic design sensitivities. The main specific loudness is calculated by using FEM, BEM tools, i.e. MSC/NASTRAN and SYSNOISE. And global acoustic sensitivity is calculated by combining acoustic and structural sensitivity using the chain rule. Structural sensitivity is obtained by using semi-analytical method and acoustic sensitivity is implemented numerically using the boundary element method. For sensitivity calculation, sensitivity analyzer of loudness (SOLO), in-house program is developed. A 1/4 scale car cavity model is optimized to show the effectiveness of the proposed method.

## 1. INTRODUCTION

For the purpose of just noise reduction, the A-weighted sound pressure level is enough to design products. But considering a human's auditory system, loudness is the most important parameter for noise problems. Hellman and Zwicker [1] proved that a decrease in dB(A) can produce an increase in loudness. With the development of sound quality engineering, loudness has been more important as the basis of some objective sound quality metrics defined by Zwicker and Fastl [2]. To make more suitable product to human being, loudness can be selected instead of SPL or A-weighted SPL. Using this numerical method, engineers can reduce loudness directly. But there has been little study on reducing loudness using design sensitivity analysis (DSA) and optimization. In this paper the focus is on DSA of vibro-acoustic system.

In a gradient-based optimization, it is important to have the sensitivities of the object function and the constraints with respect to the design variables [3]. Formulation of sensitivities is the first step in an optimization process.

Wang [4] developed continuum DSA of a coupled vibro-acoustic system using FEM. Wang and Lee [5] and Kim and Choi. [6] calculate global acoustic design sensitivity using structural sensitivity in FEM and acoustic sensitivity in BEM for semi-coupled problem. But no research results have been reported in DSA of

loudness. The sensitivity of the main specific loudness was calculated by the chain rule to combine global acoustic sensitivity and the derivative of the main specific loudness. Also Coyette et al. [7] investigated for global sensitivity, structural sensitivities with respect to sizing design variables, i.e. thickness were calculated using semi-analytical method and acoustic sensitivity with respect to surface velocity was calculated. The derivative of the main specific loudness was formulated using loudness proposed by Zwicker et al. [8].

## 2. EVALUATION OF ZWICKER'S LOUDNESS

The evaluation model of Zwicker's loudness makes a start with the concept of specific loudness. Specific loudness comes from Stevens' law that a sensation belonging to the category of intensity sensation grows with physical intensity according to a power law [2].

In ISO 532 B, 1/3-oct band filters are used instead of critical-band filters for the practical reason. Some compensation is needed since there is the difference between critical and 1/3-oct bandwidth. And additional specific loudness produced by the cut off slope in the abutting filter towards lower frequencies is taken into account by changing the exponent of specific loudness slightly from 0.23 to 0.25. The formula of the main specific loudness is in ISO 532 B [8]:

$$NM = \left( 0.0635 \cdot 10^{0.025L_{TQ}} \right) \times \left[ \left[ 1 + 0.25 \cdot 10^{0.1(L_E - L_{TQ})} \right]^{0.25} - 1 \right] \text{ sone}_G / \text{Bark}$$

(1)

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$NM$  is the main specific loudness defined in 1/3-oct band,  $L_{TQ}$  is excitation level at threshold in quite and  $L_E$  is excitation level. To calculate excitation level, some corrections are added. For 1/3-oct band filters, low frequency range is added and correction factor is used at all bands. And the logarithmic transmission factor to represent the transmission between free field and our hearing system is incorporated.

$$L_E = P_{band} - a_0 - c_1 \quad (2)$$

where

$$P_{band} = 20 \log \left[ \left( \int_{\omega} p^2 d\omega / \Delta\omega \right)^{0.5} / p_{ref} \right] \text{ dB} \quad (3)$$

where  $\Delta\omega$  is frequency bandwidth,  $p_{ref}$  is reference sound pressure, 20e-6 pa.

### 3. GLOBAL ACOUSTIC DESIGN SENSIVITY ANALYSIS

#### 3.1 Structural Sizing Design Sensitivity Analysis

The variational equation of structural dynamic system can be written in the form [4]

$$j\omega d_u(v, \bar{z}) + c_u(v, \bar{z}) + \frac{1}{j\omega} a_u(v, \bar{z}) = l_u(\bar{z}) \quad (4)$$

where

$$\begin{aligned} d_u(z, \bar{z}) &= \iint_{\Omega^S} m z^T \bar{z}^* d\Omega \\ c_u(z, \bar{z}) &= \iint_{\Omega^S} C_u z^T \bar{z}^* d\Omega, \\ a_u(z, \bar{z}) &= \iint_{\Omega^S} A_u z^T \bar{z}^* d\Omega, \\ l_u(\bar{z}) &= \iint_{\Omega^S} f^T \bar{z}^* d\Omega. \end{aligned} \quad (5)$$

$\Omega^S$  is the structural domain;  $u$  the design variable;  $z(x, u)$  the displacement;  $m(x, u)$  the structural mass;  $C_u$  the damping effect;  $A_u$  the linear partial differential operator; and  $f(x, u)$  is the applied dynamic load. Since structure system is coupled to

acoustic system with the surface velocity, Eq. (4) is expressed by velocity instead of displacement

Eq. (6) is expressed below using the finite element method using shape function.

$$\left[ j\omega \mathbf{M} + \mathbf{C} + \frac{1}{j\omega} \mathbf{K} \right] \mathbf{v}(\omega) = \mathbf{f}(\omega), \quad (6)$$

$\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$  are mass, damping, stiffness matrix, respectively and  $\mathbf{v}$ ,  $\mathbf{f}$  are velocity, load vector, respectively.

By using the direct differentiation method, the derivative of Eq. (6) with respect to sizing design variable  $u$  is obtained as

$$\begin{aligned} \left[ j\omega \mathbf{M} + \mathbf{C} + \frac{1}{j\omega} \mathbf{K} \right] \mathbf{v}' \\ = \mathbf{f}' - \left[ j\omega \mathbf{M}' + \mathbf{C}' + \frac{1}{j\omega} \mathbf{K}' \right] \mathbf{v} \end{aligned} \quad (7)$$

In semi-analytical method, the derivative of mass matrix is calculated as

$$\mathbf{M}' \cong \frac{\mathbf{M}(u^0 + \Delta u) - \mathbf{M}(u^0)}{\Delta u} \quad (8)$$

In the same manner  $\mathbf{C}'$ ,  $\mathbf{K}'$  and  $\mathbf{f}'$  can be calculated. And the structural velocity sensitivity  $\mathbf{v}'$  can be computed.

#### 3.2 Global Acoustic Design Sensitivity Analysis

With the Neumann boundary condition  $\partial p / \partial \mathbf{n} = -j\omega \rho v_n$  where  $\rho$  is the fluid density and  $v_n$  is normal velocity, the boundary integral equation expressed as [5]

$$c(x_0) p(x_0) = \iint_{\Omega^S} \left[ p(x) \cdot \frac{\partial G}{\partial \mathbf{n}} + j\omega \rho v_n \cdot G \right] d\Omega \quad (9)$$

In Eq. (9)  $c(x_0)$  is the coefficient with respect to  $x_0$ ,  $x_0$  is the position of a field point,  $G(x, x_0)$  is Green's function and  $S$  is the acoustic boundary which equals to structural boundary.

If field points locate on the boundary, Eq. (10) leads to the boundary element equation:

$$\mathbf{A}(\omega)\mathbf{p}_s = \mathbf{B}(\omega)\mathbf{v}_n. \quad (10)$$

For vibro-acoustic system, nodal pressure can be computed:

$$\mathbf{p}_s = \mathbf{A}(\omega)^{-1}\mathbf{B}(\omega) = \mathbf{C}(\omega)\mathbf{v}_n, \quad (11)$$

Once  $\mathbf{p}_s$  has been computed, Eq. (10) can be used to compute the sound pressure at any field point as

$$p_e(x_0) = \mathbf{A}_e\mathbf{p}_s + \mathbf{B}_e\mathbf{v}_n. \quad (12)$$

$\mathbf{A}_e$  and  $\mathbf{B}_e$  are row vectors of influence coefficients. Sensitivity of field point pressure to the change of the design variable  $u$  can be evaluated through the differentiation of Eq. (12):

$$\frac{\partial p_e}{\partial u} = \frac{\partial \mathbf{A}_e}{\partial u}\mathbf{p}_s + \mathbf{A}_e \frac{\partial \mathbf{p}_s}{\partial u} - \frac{\partial \mathbf{B}_e}{\partial u}\mathbf{v}_n - \mathbf{B}_e \frac{\partial \mathbf{v}_n}{\partial u} \quad (13)$$

If only sizing sensitivity is considered, the design variable  $u$  does not influence row vectors. And  $\partial \mathbf{p}_s / \partial u$  is changed in terms of  $\partial \mathbf{v}_n / \partial u$  using Eq. (11).

$$\frac{\partial p_e}{\partial u} = (\mathbf{A}_e\mathbf{C}(\omega) - \mathbf{B}_e) \cdot \frac{\partial \mathbf{v}_n}{\partial u} \quad (14)$$

This is global acoustic design sensitivity formula. The first term, in parentheses, represents acoustic sensitivity and second term points to structural sensitivity calculated from Eq. (7).

#### 4. DESIGN SENSIVITY ANALYSIS OF ZWICKER'S LOUDNESS

In many cases loudness pattern diagram clearly shows which partial area is dominant. It is efficient to reduce the dominant part of the noise that produces the largest area in the loudness pattern. So reducing the main specific loudness contributes largely reducing total loudness. This procedure is efficient especially because of making effect.

In Eq. (1), the main specific loudness has not structural design variables. To calculate sensitivity with respect to structural design variable, chain rule is used as below.

$$\frac{\partial NM}{\partial u} = \frac{\partial NM}{\partial L_E} \cdot \frac{\partial L_E}{\partial u} \quad (15)$$

The first derivative can be derived directly from Eq. (1). Because  $L_{TQ}$  is constant in the specific octave band,  $\partial NM / \partial L_E$  is constituted by the excitation level.

And the second term is equal to  $\partial P_{band} / \partial u$  because of  $a_0$  and  $c_1$  which are constant in Eq. (2). By chain rule,  $\partial P_{band} / \partial u$  can be obtained as

$$\frac{\partial P_{band}}{\partial u} = \frac{10}{\ln 10} \cdot \frac{\left( \int_{\Delta\omega} p^2 d\omega / \Delta\omega \right)^{-1}}{\Delta\omega} \cdot \int_{\Delta\omega} p \cdot p' d\omega \quad (16)$$

where  $p'$  is global acoustic design sensitivity in Eq. (14).

If the critical band is selected, constants depend on the critical band are obtained, i.e.  $a_0$ ,  $c_1$  and  $L_{TQ}$ . And

$L_E$ , pressures and global acoustic design sensitivity of each frequency in critical band can be calculated. With these values the sensitivity of the main specific loudness is calculated.

#### 5. NUMERICAL EXAMPLE

A 1/4 scale car cavity model is studied and this model has 138 nodes and 136 elements. In this model, 4 nodes are simply supported and dynamic load is applied to one node as shown in Figure 1. The material property of this model is given in Table 1. After the structural finite element analysis, the sound pressure and main specific loudness are calculated within the frequency range of interest, 20 ~ 560 Hz, by the boundary element method and ISO 532 B. Figure 2 shows the analysis results for the frequency response function at field point, 1/3 octave band, and loudness pattern.

Table 1 Material properties for the vehicle cavity

Structural property	Young's modulus	Density	Poisson ratio
(Aluminium)	70 GPa	2600 kg/m <sup>3</sup>	0.33
Fluid property	Sound velocity	Density	
(Air)	340 m/s	1.21 kg/m <sup>3</sup>	

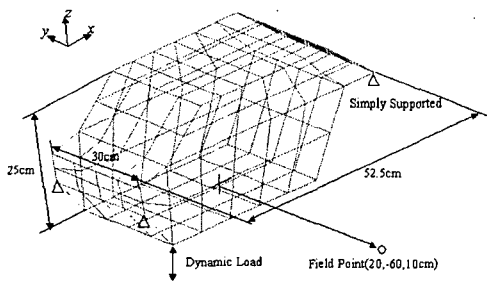
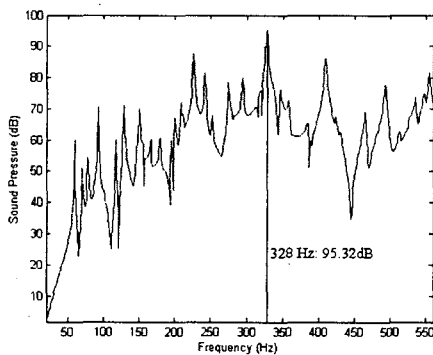
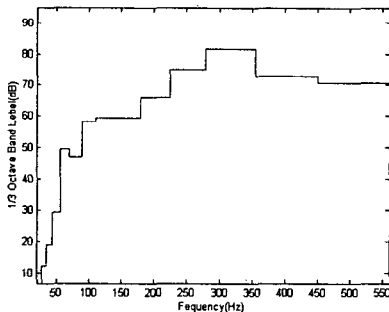


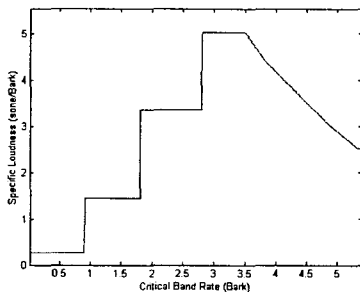
Figure 1. 1/4 scale vehicle cavity model and boundary condition



(a) Frequency response function of sound pressure



(b) 1/3 octave band



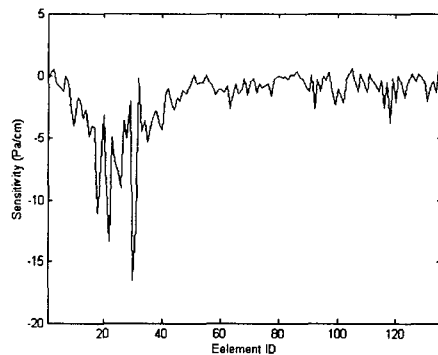
(c) Loudness pattern at field point

Figure 2. Analysis results for vehicle cavity model

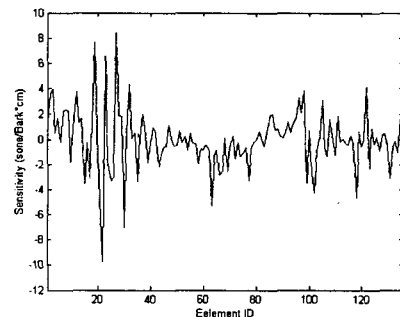
In Figure 2, the maximum value of FRF is 95.3 dB at 328 Hz and the largest value of loudness pattern is 5.035 sone/Bark at 4<sup>th</sup> band corresponding to the range, 280 ~ 355 Hz. If two figures are compared, the low frequency range of FRF is lower estimated in loudness pattern. This is caused by our hearing system shown in equal loudness contour. And the slope specific loudness of 4<sup>th</sup> band is covered 5<sup>th</sup> and 6<sup>th</sup> band level. So 4<sup>th</sup> band is considered dominant and selected as the object for the design sensitivity analysis and optimization in this system.

The design sensitivity analyses with respect to each element thickness are performed for 4<sup>th</sup> band and 328 Hz, respectively by *SOLO*. Plus sensitivity means that sound pressure or the main specific loudness will be increased when element thickness is increased. Minus sensitivity means in opposite. Figure 3 shows the sensitivity results.

In Figure 3, two sensitivity plots are not quite different because dominant peak is 328 Hz in 4<sup>th</sup> band. Figure 4 shows these sensitivity results as contour plots. In Figure 4, the largest values of sensitivity are distributed on the upper plate of cavity model.

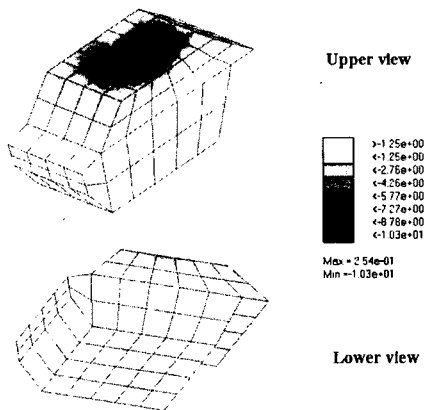


(a) Global acoustic sensitivity at 328 Hz

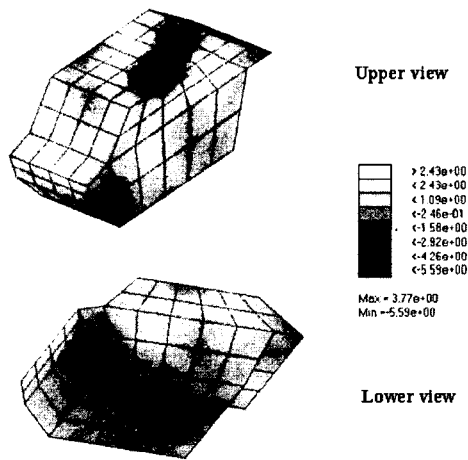


(b) Loudness sensitivity at 4<sup>th</sup> band

Figure 3. Sensitivity results with respect to element number



(a) Global acoustic sensitivity at 328 Hz



(b) Loudness sensitivity at 4<sup>th</sup> band

Figure 4 Contour plot of the loudness sensitivity

To verify the accuracy of loudness sensitivity coefficient, two elements are selected. Element 27 has the maximum plus sensitivity and element 22 has the maximum minus sensitivity. In Table 2, the convergence test is shown to verify sensitivity analysis by using the central finite difference method. Table 2 shows that *SOLO* computes sensitivity accurately.

In this paper, Sequential Linear Programming (SLP) is used for the sizing optimization problem. It is not practical to optimize each element thickness as design variable for real manufacturing. So each panel is selected for optimization as shown in Figure 5.

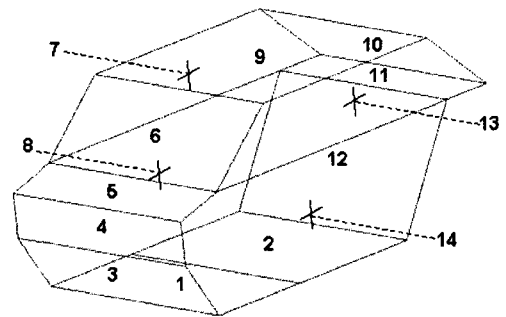


Figure 5 Design variables for optimization of vehicle cavity model

There are three design optimization problems for comparison between the case of SPL, the case of 1/3 octave band, and the case of loudness. A standard form of the design optimization is written as

**Minimize** (a) Sound pressure level at 328 Hz

(b) 1/3 octave critical band

(c) Main specific loudness at 4<sup>th</sup> band

**Subject to** No increase of total mass

**Side constraints:**  $0.05 \leq t_i \leq 0.15$  [cm]

where,  $i = 1$  to 14, number of design variables

Figure 6 shows the optimization results for 3 cases. The results shows that optimization of loudness gives much better improvement.

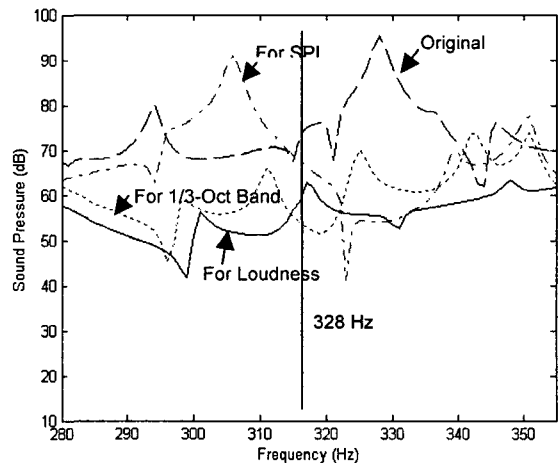


Figure 6. Comparison plot between original and optimum model

Table.2 Sensitivity verification for vehicle cavity model

Element Number	Perturbation $\delta d$ [%]	$\psi(d + \delta d)$ sone/Bark	$\psi(d - \delta d)$ sone/Bark	$\Delta\psi$ sone/(Bark*m)	$\psi'$ sone/(Bark*m)	Accuracy $\psi' / \Delta\psi$
27	10	5.112518	4.913717	9.940050		84.9
	1	5.042923	5.025886	8.518500	8.443110	99.1
	0.1	5.036214	5.034524	8.450000		99.9
22	10	4.932189	5.085041	-7.642600		127.1
	1	5.024581	5.043804	-9.611500	-9.711340	101.0
	0.1	5.034406	5.036343	-9.685000		100.3

## 6. CONCLUSION

The design sensitivity analysis of Zwicker's loudness is developed. The main specific loudness in the critical band of interest is selected as object instead of total loudness. To calculate the main specific loudness structural FEM, acoustic BEM and ISO 532 B method are used. Similarly there are three stages for loudness sensitivity. Structural and acoustic sensitivities and the derivative of loudness formula are needed.

Structural and acoustic sensitivities are combined into global sensitivity. Semi-analytical method is applied to calculate structural sensitivity with respect to element thickness. Acoustic sensitivity with respect to normal velocity is obtained by using BEM. With these two sensitivities global sensitivity can be calculated. The derivative of loudness with respect to excitation level is a straightforward process. By chain rule, the derivative of loudness with respect to excitation level and global sensitivities constitute the sensitivity of Zwicker's loudness.

The 1/4 scale vehicle cavity model is used for numerical example to verify loudness sensitivity formulation and optimization. *SOLO* calculates the loudness sensitivity of the vehicle cavity model and this sensitivity is verified by the central finite difference method. And design optimization procedure is achieved to reduce the main specific loudness in selected critical band. The optimization results of loudness shows much reduction compared to that of SPL and that of 1/3 octave band.

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