Optimal search plan for multiple moving targets with search priorities incorporated

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Abstract

This paper deals with a one-searcher multi-target search problem where targets with different detection priorities move in Markov processes in each discrete time over a given space search area, and the total number of search time intervals is fixed. A limited search resource is available in each search time interval and an exponential detection function is assumed. The searcher can obtain a target detection award, if detected, which represents the detection priority of target and is non-increasing with time. The objective is to establish the optimal search plan which allocates the search resource effort over the search areas in each time interval in order to maximize the total detection award. In the analysis, the given problem is decomposed into intervalwise individual search problems each being treated as a single stationary target problem for each time interval. An associated iterative procedure is derived to solve a sequence of stationary target problems. The computational results show that the proposed algorithm guarantees optimality.

1. Introduction

Victory or defeat in modern warfare has been dominated by the ability to perform information operations. Accordingly, many weapon systems to implement intelligence, surveillance, reconnaissance (ISR) mission in information operation have been developed and operated in military field so far. One of those is the target-acquisition system. In the system, the most important issue is how to find and detect enemy's targets accurately.

Search theory has given reasonable answers for these problems. It began in World War II in response to Navy's antisubmarine operation and became an important one of operations research areas.

Search problem deals with search plan or strategy which allocates search resource to increase the detection probability, including three major elements; probability distribution for target's location and motion, detection function, constraint on search resource.

The information about the target's position at some initial time and it's subsequent motion can be quantified in terms of probability distribution. The detection function relates the amount of resource placed in an area to the probability of detecting target given that it is located in that area. Generally, the searcher has a limited amount of resource available to conduct the search. The limitation of resources may restrict to distribute search resource infinitely over search areas.

As an object of interest, there are two types of target's movement. A stationary target is assumed to be located in one of the discrete cells which partition the search area and do not change it's location during the search. A moving target moves from one cell to another cell as time goes by. Earlier studies on search problems have been focused on detecting a single stationary target or a moving target. However, in the real circumstance of modern warfare there are so many targets which are acting in operational area. In most cases, the searcher is interested in detecting some of them, not only a particular one.

Generally, these targets are not fixed at one position during the military operation. They displace their location frequently during the operation in order to secure their survivability. Furthermore, they all have different degrees of threat. Therefore, it is important for the searcher to establish the detection priorities of targets when he is planning for search operation.

Motivated by these notice, the issue of search plan for detecting multiple moving targets with different detection awards will be addressed in this thesis. A single searcher, say UAV (Unmanned Aerial Vehicle), carrying limited search resource is going to perform a reconnaissance mission. Searches are conducted in discrete time intervals and mutually independent of each other distinct time interval. Multiple targets are moving among discrete spaces called cells in time intervals according to Markov motion. Detection priorities of targets are represented by a variety of different detection awards and the award varies with time. The probability distribution of the targets' initial location, the transition probability of each target and the exponential detection function are known to the searcher. Then, the remaining problem for the searcher is to establish the optimal search plan which allocates the given limited resources to the search cells at each time interval in order to maximize the total detection award.

2. Problem description

The proposed problem considers the situation where friendly ground forces are confronting some enemy troops for offensive or defensive ground operation and preparing for reconnaissance plan. The operational field is partitioned into several smaller areas, called cells, for effective land operation. It is assumed that the terrain analysis of each cell is immediately completed.

The goal of the friendly search operation is to detect some threatening moving targets (enemy troops),, which keeps displacing their positions from one cell to another cell to secure their survivability after the beginning of the search operation.

Some awards are obtained as the targets are detected. The detection awards depend on targets and each target award decreases with time, which implies that it is motivated to detect the targets as soon as possible.

The friendly reconnaissance troop with an UAV(Unmanned Aerial Vehicle) is put on the surveillance mission carrying a limited search resource, say flight time, to detect the hostile (enemy) targets. An UAV can search for targets over the operational areas consecutively during the scout flight.

The objective of the proposed problem is to find an optimal search plan which allocates the flight time of the UAV in each search area during each given time interval to maximize the detection awards subject to the limited total flight time.

Some assumptions are considered as follows;

① The searcher to detect the enemy' s targets is a single UAV, and the targets do not respond to the searcher' s action such that the targets are passive and so not either evading or hiding.

② The number of targets, the probability distribution of each target' s initial location and its transition probability are assumed to be known in advance.

③ The search is conducted during each time interval and the targets move among the cells. The numbers of cells and time intervals are finite.

④ Each target occupies exactly one cell during each time interval, so that their moving processes follow Markov processes. It is assumed that target transitions are independent of each other.

⑤ It is not considered that targets are destroyed or demolished to vanish, any new target are added during the whole search operation, and targets are divided into several targets or merged together during the search operation.

⑥ The detection function is assumed as an exponential function, which represents detection probability of the search resource.

⑦ The search cost is also changed for the situation where the flight is ignored.

⑧ The searcher is provided with a positive award for any target in any time interval, where the award amount depends on targets and is non-increasing with time.

Consequently, the given problem can be formulated as follows;

$$\text{Min } f(X) = \sum_{i=1}^{n}\sum_{j=1}^{m} \Delta V_{ij}\, g_j(x,i)$$

$$\text{subject to } \sum_{k\in C} x_{ik} \le R_i \text{ for all } i,k$$

$$x_{ik} \ge 0 \text{ for all } i,k$$

where $$g_j(x,i) = \sum_{\omega_j\in\Omega} \pi(\omega_j)\prod_{h=1}^{i}\exp(-\alpha_{h,k_h} x_{h,k_h}) \quad ,$$

$\Delta V_{ij} = V_{ij} - V_{i+1,j}$ and $V_{n+1,j} = 0$

3. Analysis

The objective of the given problem is to determine the optimal distribution of the search resource among all the cells during each time interval to maximize the total expected detection award.

For the problem analysis, consider the situation where a certain target with no priority moves among the partitioned search areas during finite time intervals. Let $g(x,i)$ be defined as the probability that a certain target will be undetected until the i^{th} time interval. The problem to minimize the non-detection probability of the target, that is, to minimize $g(x,i)$ subject to Eq. (2-5), can be viewed as a single moving target problem. There is an efficient method to find optimal search plan for a single moving target problem, which has been provided by Brown[2].

However, as shown in Eq. (2-4), the objective function *f(X)* of the given problem is a linear combination of the functions, $g_j(x,i)$ for $i = 1,\Lambda\,,n$ and $j = 1,\Lambda\,,m$. To minimize each $g_j(x,i)$ can be viewed as the single moving target problem for target j. Nevertheless, any one of the optimal solutions to each single moving target problem may not be the optimal one to the proposed problem. It is because each target has its own probability of the initial location and transition, and priority, different from any others. Thus, it may not be appropriate to decompose the given problem into several single moving target problems, each being solved separately.

Therefore, several solution properties will be characterized to derive the efficient algorithm to guarantee the optimality for the given problem in this chapter.

Proposition 1

The function *f(X)* is convex in x_{ik}.

Because the objective function $f(x)$ is convex and all constraints are linear, the given problem becomes a convex programming problem.

Consider the special case where n=1 and m=1, called the case of a single stationary target problem. This is then a the problem of minimizing non-detection probability $\sum_{k \in C} p(k)\exp(-\alpha_k x_k)$ during a single time interval, where $p(k)$ is the probability that a target is located in cell k.

In the general case where $n \neq 1$ and $m \neq 1$, consider movement of each target during a particular time interval. Each target occupies exactly one cell during each of n time intervals. That is, there are no other targets to move to another cell during each time interval. Thereupon, the targets during a particular time interval can be considered as stationary targets. Thus, the stationary target problem is investigated for the given problem.

Review the single moving target case. It is about to minimize $g(x,i)$. For a particular time interval t for $1 \le t \le i$, the overall non-detection probability is equivalent to the probability that no target was detected before time t, is detected at time t, and will be detected till time i after time t. Therefore, the function $g(x,i)$ can be derived as follows;

(3-1) $g(x,i) = \sum_{k \in C} p(x,t)\exp(-\alpha_{tk} x_{tk}) q(x,t,i)$

where $p(x,t) \equiv$ Probability that a target arrives at cell k during the t^{th} time interval without being detected before the t^{th} time interval

and $q(x,t,i) \equiv$ Probability that a target is in cell k during the t^{th} time interval and will not be detected till the i^{th} time interval after the t^{th} time interval, given no detection during the time interval [1, t]

The function of Eq. (3-1) is required for a stationary target search problem. If $p(x,t)$ and $q(x,t,i)$ are known, the stationary target problem can be solved.

Similarly, the given problem can be reduced to a stationary target problem an shown in Proposition 2.

Proposition 2

Given any search plan X fixed except for a particular time interval, the given problem can be reduced to a stationary target problem.

Proposition 3

Given any feasible solution X which satisfies (2-5), let another feasible solution which is obtained after the *single stage search problem* be X'. Then $f(X) \ge f(X')$ for every t.

By Propositions 1, 2, and 3, given any feasible search plan, the problem can be reduced to a stationary target problem, called the *single stage search problem* for any time interval $1 \le t \le n$. Such new search plan generated by the *single stage search problem* updates current solutions so as to improve the objective function value.

Therefore, to find the optimal search plan, these solution properties will be used to establish an effective algorithm in the next chapter.

4. Algorithm

The optimization algorithm proceeds as follows;

Step 1: Let X_n^0 satisfying Eq. (2-4) be the initial solution for the search.

Step 2: Let ε be a small positive number.

Step 3: Set r = 1.

Step 4: Perform Steps 5-6 for t=1,...,n.

Step 5: Set $X_t^r = Y_t(X_{t-1}^r)$.

Step 6: Go to the next value t.

Step 7: If $\left| f(X_n^r) - f(X_n^{r-1}) \right| < \varepsilon$, stop.

Step 8: Increase r by 1.

Step 9: Go back to Step 4.

5. Conclusions

This thesis deals with a search problem for multiple moving targets with search priorities incorporated. Most of the search problems in the literature have considered a single moving target to maximize the detection probability, while this thesis considers multiple targets that move in Markov processes in discrete time over a given space. The worth of each target is devaluated as decreasing with time. The consideration of multiple targets appears to the more-like realistic for actual ground warfare because it s often to confront many hostile targets moving dynamically in most battle areas and their detection priorities are dependent on both time and friendly forces. Thus, the objective is to determine the optimal search plan which allocates the limited search resource(effort) over the operational search areas in all the allowed discrete time intervals not only to maximize the total detection probability but also to maximize the overall total detection award.

It is shown in the analysis that the given problem can be decomposed into intervalwise individual search problems each being treated as a single stage target problem for each given time interval. Therefore, the problem can be solved by working on a sequence of single stage target problems iteratively. At each iteration, search resource reallocation may be needed to improve the objective function value in view of the total detection award in the term. By doing so, the proposed iterative procedure shows that the objective function value converges to the limit point, implying that the optimality is guaranteed. The computational results show that the optimal search plan obtained by the proposed algorithm provides more improved objective function value and detection probability of each target than those of the myopic search plan.

As a further study, the following issues can be considered to extend the problem;

① Any other search models considering different detection functions

② Additional considerations including dynamic search cost

③ Any constraint on the searcher's path

References

[1] Benkoski, S.J., and Weisinger J. R., *A survey of the search theory literature*, Naval Research Logistics, 38, 469-494, 1991.

[2] Brown, S.S., *Optimal search for a moving target in discrete time and space*, Operations Research, 28, 1275-1289, 1980.

[3] Charnes, A. and Cooper, W. W., *The theory of search: Optimum distribution of search effort*, Management Science, 5, 44-50, 1958.

[4] Dambreville, F. and Cadre, J. P., *Detection of a Markovian target with optimization of the search efforts under generalized linear constraints*, Naval Research Logistics, 49, 117-142, 2002.

[5] Discenza, J. H., *Optimal survivor search with multiple states*, Operations Research, 28, 309-323, 1980.

[6] Eagle, J.N. and Dell, R. F., *Using multiple searchers in constrained path moving target search roblem*, Naval Research Logistics, 43, 463-480, 1996.

[7] Guenin, J., *Optimum distribution on effort: An extension of the Koopman basic theory*, Operations Research, 9, 1-7, 1961.

[8] Haley K.B., and Stone, L.D., *Search Theory and Application*, Plenum Press, 1980.

[9] Hozhaki R. and Iida, K., *Optimal strategy of route and look for the path constrained search prblem with reward criterion*, European Journal of Operational Research, 100, 236-249, 1997.

[10] Kadane, J. B., *Surveillance search for a moving target*, Operations Research, 31, 720-738, 1983.

[11] Koopman, B.O., *Search and Screening*, 2nd ed, Pergamon Press, Newyork, 1989.

[12] Smith, F. H. and Kimeldorf, *Discrete sequential search for one of many objects*, Annals of statistics, 3, 906-915, 1975.

[13] Stone, L. D., *What's happened in search theory since the 1975 Lanchester Prize?*, Operations Research, 37, 501-506, 1989.

[14] Stone, L.D., *Theory of optimal search*, 2nd ed. Operation Research society of America, Arlington, VA, 1989.

[15] Thomas, L.C. and Eagle, J.N., *Criteria and approximate method for path-constrained moving target search problems*, Naval Research Logistics, 42, 27-38, 1995.

[16] Washburn, A.R., *Branch and bound method for a search problem*, Naval Research Logistics, 45, 98-113, 1998.

[17] Washburn, A.R., *Search for a moving target: The FAB algorithm*, Operations Research, 31, 739-751, 1983.

[18] Washburn, A.R., *On a search for a moving target*, Naval Research Logistics, 27, 315-322, 1980.