Multiattribute Decision Making with Ordinal Preferences on Attribute Weights

Byeong Seok Ahn
Department of Business Administration, Hansung University
e-mail: bsahn@hansung.ac.kr

Abstract

In a situation that rank order information on attribute weights is captured, two solution approaches are presented. An exact solution approach via interaction with a decision-maker pursues progressive reduction of a set of non-dominated alternatives by narrowing down the feasible attribute weights set. In approximate solution approach, on the other hand, three categories of approximate methods such as surrogate weights method, the dominance value-based decision rules, and three classical decision rules are presented and their efficacies in terms of choice accuracy are evaluated via simulation analysis. The simulation results indicate that a method, which combines an exact solution approach through interactions with the decision-maker and the dominance value-based approach is recommendable in a case that a decision is not made at a single step under imprecisely assessed weights information.

1. Research background

In a multiattribute decision making (MADM) problem, one usually considers a finite discrete set of alternatives, $A=\{x,y,z,\cdots\}$, which is valued by a finite discrete set of attributes, $I=\{1,2,\cdots,n\}$. Let $v_i(x)$ be the value of alternative $x\in A$ on attribute $i\in I$ and w_i a scaling factor to represent the relative importance of the ith attribute. A classical evaluation of alternative leads to the aggregation of all criteria into a unique criterion called a value function under certainty and a utility function under uncertainty. In this paper, we assume that there exists additive value functions under preferential independence [4, 10] and thus the underlying model is a multiattribute value (MAV) model of the form:

$$V(x)=\sum_{i\in I} w_i v_i(x) \qquad (1)$$

where V is an overall multiattribute value, $0\leq V\leq 1$; alternative x is a vector of attribute levels $(x_1, x_2,\cdots, x_n)$; $v_i(x)$ is a single attribute value function, $0\leq v_i(x)\leq 1$; $w_i\geq 0$ are weights reflecting the relative importance of the range of the attribute values, and $\sum_{i\in I} w_i=1$.

During the past several decades, there have been research efforts to deal with imprecise information in an multicriteria decision making (MCDM) field. Under uncertainty case, if the set of feasible probability distributions is non-empty and contains more than one element, the dominance relations have to be checked. Initiated by Fishburn [2], authors, presuming different types of imprecise probabilities ranging from ordinal to the most general linear constraints, suggest verifying dominance with the use of linear programs [7]. Sage and White [8] admit possibly imprecise description of both attribute weights and utilities in the forms of linear set inclusion, and present a novel solution method. Kirkwood and Sarin [6] present a method for ranking multiattribute alternatives using a weighted additive evaluation function with partial weighting constants and further present an algorithm that partially rank-orders the complete set concerning alternatives based on the pairwise ranking information. The induced dominance relations under partial information about attribute weights are not sufficient to resolve a decision problem. Kirkwood and Corner [5] show in their simulation study that imprecise information about the ordinal preference information of weights is often not sufficient to determine the most preferred alternative for realistic decision problems.

2. Approximate solution approach

2. 1 Decision aid with surrogate weights method

In a case where a rank order about attribute weights is assessed from the decision maker as in SMARTS [10], several methods for determining approximate attribute weights have been presented. Stillwell et al. [9] present three surrogate weighting methods for determining the attribute weights that preserve the rank order of weights: rank sum weights, rank reciprocal weights, and rank exponent weights. Assuming that the significance of attribute weights is arranged in a descending order from the most important attribute to the least important attribute such as $w_1\geq w_2\geq\cdots\geq w_n$, they present approximate weights having the following forms:

(a) Rank sum (RS) weights

$$w_i=(n+1-i)/\sum_{j=1,\cdots,n} j=2(n+1-i)/n(n+1),\ i=1,\cdots,n$$

(b) Rank reciprocal (RR) weights

$$w_i=(1/i)/\sum_{j=1,\cdots,n} 1/j,\ i=1,\cdots,n.$$

A third method by Stillwell et al. [9], described as rank exponent weights, is not used in our comparative study since it requires one judged number from the respondent in order to specify a parameter used in the formula. Another surrogate weights that are computed by taking the mean of the extreme points of weights are rank order centroid (ROC) [1]:

(c) Rank Order Centroid (ROC) weights

$$w_i=(1/n)/\sum_{j=i,\cdots,n}1/j,\ i=1,\cdots,n.$$

Barron and Barrett [1] suggest that ROC weights among other approximate weights are more adequate to accurately rank alternatives. In their simulation study, the superiority of ROC weights is evaluated and verified in terms of the degree of identification of the best alternative and value losses with respect to the various combinations of the number of alternatives, the number of attributes, and four different distributions from which attribute values are generated.

2.2 Decision aid with aggregated dominance values

It is said that a pairwise weak dominance relation holds between alternative x and y if and only if it holds that $\xi_{min}(x, y)> \xi_{min}(y, x)$ or equally $\xi_{max}(x, y)> \xi_{max}(y, x)$. We now direct our attention to the pairwise dominance value, $\xi_{min}(x, y)$ obtained from solving (1) as helpful decision-relevant information for the use in establishing preference relations among the alternatives.

Step 1) Solve the problem (1) to obtain pairwise strict or weak dominance values.

Step 2) Using the paired dominance values in Step 1, compute aggregated dominating preference intensities for each of alternatives.

$$\varphi^+(x)= \sum_{y\square A-\{x\}}\xi_{min}(x, y),\quad \forall x\in A.$$

Step 3) Using the paired dominance values in Step 1, compute aggregated dominated preference intensities for each of alternatives.

$$\varphi^-(x)= \sum_{y\square A-\{x\}}\xi_{min}(y, x),\quad \forall x\in A.$$

Step 4) Compute the net preference intensity for each of alternatives by means of the difference between the dominating and the dominated values.

$$\varphi^N(x)= \varphi^+(x)- \varphi^-(x),\quad \forall x\in A.$$

Step 5) Establish the preference relations among the alternatives according to the following rules:

$$xPy \text{ if } \varphi^N(x)> \varphi^N(y)$$
$$xIy \text{ if } \varphi^N(x)= \varphi^N(y)$$
$$yPx \text{ if } \varphi^N(x)< \varphi^N(y)$$

where xPy means that alternative x is preferred to alternative y if the net preference strength of x is greater than that of y and xIy represents indifferent preference between x and y. For a brief reference, we denote a decision method by the magnitude of aggregated dominating values as the OUT I (i.e., Step 1-2) and a decision method by the magnitude of the net dominance values as the OUT II (i.e., Step 1-5).

2.3 Other decision rules

- maximax (OPTimistic): $\max_{x\in A}[\xi_{max}(x)]$
- maximin (PESSimistic): $\min_{x\in A}[\xi_{min}(x)]$
- minimax regret (REG):

$$\min_x[\max_{y\neq x}\max_W[V(y)-V(x)]]$$

In addition to the three classical decision rules, we consider a choice of an alternative which is the greatest in the midpoint of the value intervals, that is

- central values (CENT): $\max_{x\in A}[\xi_{min}(x)+\xi_{max}(x)]$.

2.4 Comparative analysis via simulation

In the simulation study, we demonstrate the performance of approximate weights methods (i.e., ROC, RR, RS, and ES) in terms of selection of the best alternative and overall rank ordering of alternatives, compared with aggregated dominance values (OUT I, OUT II, and CENT) and three classical decision rules (i.e., OPT, PESS, and REG) as a function of decision problem size. The quality of decisions is assessed by comparing decisions resulting from the use of each of approximate methods with those arising from knowledge of "true" weights, which are generated from random numbers which satisfy the rank order of attribute weights. The simulation study can be outlined by the following five steps:

Step 1) Create the simulated decision problems. Each sequentially generated random number from independent uniform distribution ranging in (0, 1) constitutes the $m\square n$ matrix of attribute values.

Step 2) Perform the simple dominance checks.

Step 3) Compute the attribute weights. The five different sets of attributes weights need to be generated; one is the randomly generated weights constrained so as to satisfy the rank order (hereafter we call them TRUE weights and the decision made by TRUE weights is called TRUE method), and ROC, RR, RS, and EW weights are generated according to the formulae. To generate the TRUE weights for the n attributes, we first select n-1 independent random numbers from a uniform distribution on (0, 1), then rank these numbers. Suppose the ranked numbers are $1>r_{n-1}\geq\cdots\geq r_2\geq r_1>0$. The differences of these consecutively ranked numbers can be obtained as the weights of the n-attributes, that is $w_n=1-r_{n-1}$, $w_{n-1}=r_{n-1}-r_{n-2},\cdots,w_1=r_1$. Then, the set of weights will sum to 1 and be uniformly distributed on the possible domain of weights [3].

<Table 1> Simulation Results in Terms of Average Hit Ratio

Alternative	Attribute	Rank-based weights ROC	RR	RS	EW	Dominance value-based rules OUT I	OUT II	CENT	Classical decision rules OPT	PESS	REG
3	3	0.891	0.881	0.873	0.721	0.884	0.883	0.841	0.737	0.812	0.832
	5	0.898	0.887	0.864	0.705	0.878	0.844	0.833	0.723	0.848	0.836
	7	0.889	0.866	0.857	0.686	0.848	0.830	0.815	0.692	0.836	0.826
	10	0.898	0.860	0.835	0.654	0.824	0.792	0.776	0.665	0.823	0.787
	15	0.875	0.835	0.829	0.648	0.762	0.720	0.719	0.620	0.759	0.723
5	3	0.846	0.830	0.816	0.654	0.816	0.833	0.782	0.590	0.680	0.759
	5	0.864	0.845	0.826	0.618	0.835	0.811	0.780	0.573	0.775	0.784
	7	0.855	0.830	0.805	0.593	0.809	0.786	0.753	0.558	0.773	0.756
	10	0.859	0.808	0.787	0.578	0.783	0.725	0.699	0.518	0.757	0.708
	15	0.876	0.799	0.773	0.565	0.730	0.680	0.652	0.486	0.719	0.658
7	3	0.815	0.793	0.792	0.622	0.785	0.815	0.695	0.576	0.605	0.701
	5	0.836	0.819	0.800	0.577	0.802	0.793	0.753	0.505	0.719	0.769
	7	0.825	0.799	0.773	0.551	0.782	0.767	0.717	0.483	0.730	0.714
	10	0.842	0.790	0.762	0.541	0.768	0.712	0.668	0.433	0.721	0.674
	15	0.831	0.781	0.757	0.525	0.713	0.652	0.620	0.411	0.704	0.618
10	3	0.760	0.735	0.734	0.578	0.729	0.737	0.719	0.477	0.718	0.711
	5	0.827	0.803	0.783	0.537	0.778	0.781	0.742	0.430	0.713	0.744
	7	0.806	0.779	0.748	0.507	0.727	0.723	0.678	0.402	0.696	0.692
	10	0.829	0.783	0.745	0.504	0.715	0.691	0.642	0.354	0.693	0.664
	15	0.849	0.774	0.740	0.489	0.702	0.645	0.611	0.350	0.681	0.608

Step 4) Determine the final ranking of a set of alternatives, applying the weights derived in Step 3, for each of generated decision problems.

Step 5) Compare the decision results by each of proposed methods with those by TRUE method in terms of efficacy measures.

Two measures for the performance evaluation include hit ratio and rank order correlation (Kendall's τ). The hit ratio evaluates how frequently the coincidence of best alternative occurs between methods under consideration and TRUE method throughout simulation runs. Thus, the best alternative resulted from each of proposed methods is compared with the best alternative chosen from TRUE method. As another indicator representing the accuracy of considered methods, we use Kendall's τ for calculating rank order correlations between the methods under consideration and TRUE method, and Kendall's τ is defined as follows:

$$\tau = 1 - \frac{2(\text{Number of Pairwise Preference Violations})}{\text{Total Number of Pairs of Preferences}}.$$

The value +1 in Kendall's τ, which ranges from -1 to +1, means perfect correspondence between the two rank orders. In addition to two efficacy measures, Kirkwood and Sarin [6] algorithm (shortly K-S algorithm) provides a criterion about whether the final ranking of a set of alternatives derived by each of methods preserves the (partial) ranking of a set of alternatives derived by the K-S algorithm, which can be stated as follows: if (1) holds and $w_1 \geq w_2 \geq \cdots \geq w_n$, then x is guaranteed to be preferred to y if and only if $\sum_{m=1,\cdots,i}[v_m(x)-v_m(y)] \geq 0$, $i=1,2,\cdots,n$. It is trivial to show that the final rankings by approximate weights methods except EW method preserve the (partial) ranking by the K-S algorithm. It is, however, meaningful to see the final ranking by the OUT (OUT I and OUT II) method preserves the ranking by the K-S algorithm since the final ranking by the OUT method is derived by using various extreme points of weights and then performing numerical manipulations.

We design the simulation with four different levels of alternatives (m = 3, 5, 7, 10) and with five different levels of attributes (n = 3, 5, 7, 10, 15). For each of 20 design elements (alternatives × attributes), the process of generating and analyzing decision problems was repeated until 10 replications of 10,000 trials had been obtained. The simulation results for each of 20 design elements are arranged with respect to two efficacy measures in <Table 1>. Throughout the different combinations of alternatives and attributes numbers, ROC method shows the highest degree of coincidences of best alternative with TRUE method in terms of hit ratio criterion whereas the OUT I method is consistently superior to the OUT II, CENT and three classical decision rules. With relatively less number of alternatives and attributes (m = 3, 5 and n = 3, 5, 7), the OUT I maintains correspondence of more than 80% with TRUE method and shows peculiar declines in hit ratio as the number of both alternatives and attributes increases (m =10 and n =10, 15) whereas ROC and RR show stable rates of prediction. Therefore, it can be said that ROC>RR>RS>EW on rank-based weights methods, OUT I>OUT II>CENT on dominance value-based rules, and no regular trends on classical decision rules. As was expected, EW method is the worst method to adopt in resolving a decision problem with rank-ordered attribute weights.

With regard to Kendall's τ, ROC method shows predominantly higher correlations with the ranking by TRUE method than other methods, irrespective of the various alterations of simulation parameters. From the smallest to the largest number of alternatives and attributes, ROC method, which ranges from 88% to 86%, shows only 2% decreases in rank order correlations with TRUE method whereas RR and RS method show rapid decreases of 7% - 8%. On the other hand, the OUT I method, which is in between 84% and 70%, consistently shows better performance than OUT II, CENT and three classical decision rules. The results in terms of rank order correlations are analogous to the results in terms of hit ratio criterion due to the high correlation between two efficacy measures. To summarize, the overall simulation results based on these two efficacy measures reveal ROC>RR>RS>EW on rank-based weights methods, OUT I>OUT II>CENT on dominance value-based rules, and no regular trends on classical decision rules.

3. Discussion

If we do not force the decision maker to specify parameters as input data to the extent that this becomes overly stressful or behaviorally and physically irrelevant in view of the inherent imprecision associated with domain knowledge of parameters characterizing the decision situation, the decision-maker provides his/her knowledge or preference information on the weights $\{w_i\}_{i \in I}$ of which the precise values are not known possibly on some of attributes in such way that information is to satisfy combinations of linear constraints: (a) $w_i \geq w_j$ or $w_i - w_j \geq \varepsilon$, where ε is a small positive number, (b) $w_i \geq \alpha_{ij} w_j$, (c) $l_i \leq w_i \leq u_i$, (d) $w_i - w_j \geq w_k - w_l$, for $i \neq j \neq k \neq l$.

In a case that a mixture of imprecise preferences is provided, it is not easy to obtain approximate weights by the use of formulae and hence their usage is somewhat limited. In the OUT method, however, we only have to solve small linear programs for checking paired dominance relations between alternatives, which are, in turn, utilized for determining the strength of preference of each of alternatives. In the simulation study, the OUT method (especially OUT I method), which is characterized by both exact solutions from paired dominance checks and their aggregation, shows outstanding performance in all cases studied except the rank-based attribute weights. It is believed that this has occurred due to the fact that the weights in the OUT method are disseminated into the alternatives the way in which they can take under imprecisely specified weights and that the dominance results (i.e., strict or weak) are then combined for identifying a preferred alternative based on a reasonable way of aggregation. In other words, the OUT method utilizes as pertinent weights as possible depending on the value scores which alternatives in paired comparison take rather than using point estimates which only satisfy the ordinal relation of weights.

References

[1] Barron, F.H. and B.E. Barret, "Decision Quality Using Ranked Attribute Weights," *Management Science*, Vol.42 (1996), pp.1515-1523.

[2] Fishburn, P.C., "Analysis of Decisions with Incomplete Knowledge of Probabilities," *Operations Research*, Vol.13 (1965), pp.217-237.

[3] Jia, J., G.W. Fischer, and J.S. Dyer, "Attribute Weighting Method and Decision Quality in the Presence of Response Error: a Simulation Study," *Journal of Behavioral Decision Making*, Vol.11 (1998), pp.85-105.

[4] Keeney, R.L. and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, Wiley, New York 1976.

[5] Kirkwood, C.W. and J.L. Corner, "The Effectiveness of Partial Information about Attribute Weights for Ranking Alternatives in Multiattribute Decision Making," *Organization Behavior and Human Decision Processes*, Vol.54 (1993), pp.456-476.

[6] Kirkwood, C.W. and R.K. Sarin, "Ranking with Partial Information: a Method and an Application," *Operations Research*, Vol.33 (1985), pp.38-48.

[7] Kmietowcz, Z.W. and A.D. Pearman, "Decision Theory, Linear Partial Information and Statistical Dominance," *Omega*, Vol.12 (1984), pp.391-399.

[8] Sage, A.P. and C.C. White, "ARIADNE: a Knowledge-based Interactive System for Planning and Decision Support," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol.14 (1984), pp.35-47.

[9] Stillwell, W.G., D.A. Seaver, and W. Edwards, "A Comparison of Weight Approximation Techniques in Multiattribute Utility Decision Making," *Organization Behavior and Human Decision Processes*, Vol.28 (1981), pp.62-77.

[10] Winterfeldt, D. von and W. Edwards, *Decision Analysis and Behavioral Research*, Cambridge University Press, England, 1986.