Economics of Supply Chain Contracting for Quality

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Abstracts

This paper models and solves contracting schemes for both when quality is perfectly observable and when quality is not perfectly observable in supply chain. When quality is perfectly observable, the first–best optimal solution which is that the marginal utility of procurer obtained from the quantity and quality supplied by suppliers (the price) is equal to the marginal cost to produce the quantity and quality is obtained. However, when quality is not perfectly observable to procurers the optimal solution cannot be the first–best but the second–best where the price is greater than the marginal cost to produce the quantity and quality and social welfare is less than that of the first–best solution.

Keywords: 1) Quality observability, 2) Supply chain contracting, and 3) Incentive compatibility, 4) Second–best social surplus

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I. Introduction

A key challenge facing today's procurement activity in supply chain is the management of quality, Kim and Weaver (1995) and Weaver (1993). Because quality of products supplied from suppliers in supply chain is one of the most important determinants of quality of products produced by procurers, procurers should get an appropriate quantity of products with appropriate quality from suppliers in order to guarantee the appropriate quality to final products. Recent experience in supply chain research provides an important case for analysis of opportunity for supply chain contracting to manage the quality in supply chain. Weaver and Kim (1999) demonstrated that grades and standards leave in place substantial incentives for supply chain management of quality. These incentives pose an important turning point for the industry since they may motivate either substantial vertical integration or achieve supply chain management through the structurally neutral approach of contracting. In fact, regulative alternatives such as grades and standard to manage quality are dominated by contracting alternatives. This paper deals with the microeconomics of supply chain contracting as a solution to this general problem and examines the impacts of quality management in supply chain on utility of procurers and suppliers and the social welfare.

From the property of quality, quality can be classified into two cases: quality is perfectly observable to all constituents in supply chain and quality is not perfectly observable to at least some constituents in supply chain. The contracting models in this paper cover both cases. In the first case, quality also becomes a decision variable as well as quantity and price and the solution shows that the marginal utility of procurer obtained from quality and quantity supplied by suppliers which is the price of quantity and quality is equal to the marginal cost of supplier to produce quality and quantity in the optimum. However, in second case, quality cannot be a decision variable but the type variable of suppliers and the price is determined to be greater than marginal cost of supplier to produce quality and quantity because of the adverse selection[1] committed by supplier. The difference between the marginal utility of procurer and the marginal cost of supplier is the loss in social welfare by imperfect observability of quality.

This paper is made up as follows. Next section provides an overview of a case of quality management in supply chain. The third section discusses the basic notations for modeling and

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[1]. Supplier will lie her/his type to obtain more utility. In this case, supplier pretends that quality produced by her/him is different from true quality.

the fourth section models and solves the contracting model. Finally, section V writes up conclusion.

II. Overview of the dairy supply chain as a case of study

Dairy industry presents a good context for the study of supply chain management of quality (Halloway, 1998). Raw milk varies in quality dimensions defined by attributes that have both positive and negative values. Milk processors at various stages as procurer value protein, fat, solids such as whey, and negatively value sematic cell, bacterial, and other contaminations/residues. Clearly, because the quality of raw milk is one of the most important determinants of quality of final dairy products such as yogurt, butter, and cheese, it should be managed by milk processors.

Some contaminations/residues in raw milk can be observed by milk processor while some cannot be observed by milk processors though suppliers can know the presence of the contaminations/residues. An important example of contamination that is of current concern is antibiotic residues. The presence of residues or contaminants in any supply impacts processors costs in the short run and producer revenues and all supply chain welfare in the long run if they are not observed by procurers and suppliers reveal the presence truly. Options include regulatory approaches such as grades and standards, or private sector strategies such as supply chain contracting.

Importantly, supply chain management actively feeds back to the supply chain's performance depending on whether contracts are used and depending on the transparency of those contracts. Recent experience in the dairy product supply chain provides an important case for analysis of the current state of opportunity for supply chain contracting as well as the current state of implementation supply chain contracts. Over the past several years, this supply chain has been increasing exposed to product price, quantity, and quality risks that suggest opportunity for supply chain management through contracting.

II. Fitting quality into microeconomics

The notion that quality can be explicitly managed through supply chain is the focus of extensive literature in management science under the banner of Total Quality Management (TQM). Though it has received little attention within the context of neoclassical microeconomics, see Chong (1996), De Vany and Saving (1983), Dotchin and Oakland (1992), or Lederer and Rhee (1995). Following and extending the models of them, this paper starts from the following notations.

Suppose a supplier (e.g. auto-part manufacturer, milk farmer) produces a quantity y that has continuously variable quality q to supply it to procurer. Thus production involves two processes, one that results in quantity (y) and one that results in quality (q) and the neoclassical cost function of the supplier becomes $C(y,q)$. The properties of cost function are assumed to be $\frac{\partial C}{\partial y} > 0$, $\frac{\partial C}{\partial q} > 0$, $\frac{\partial^2 C}{\partial y^2} \le 0$, $\frac{\partial^2 C}{\partial q^2} \le 0$, and $\frac{\partial^2 C}{\partial y \partial q} > 0$. As noted, this quality may be perfectly observable and imperfectly observable to procurer or to both procurer and supplier while quantity is always perfectly observable. For example, both buyer (procurer) and supplier (dealer) of used cars cannot observe the quality of used cars perfectly even though supplier can have more information on the cars than procurer (Akerlof, 1970). Dairy food manufacturer (procurer of raw milk) observes the quality of raw milk imperfectly. Before closing the layout of notation, it is possible to generalize the objective of quality management to allow for incentives (λ) for quality attainment q (Lederer and Rhee, 1995).

Based on the above notations, the utility function of suppliers is defined as $U^s(\pi^s) = U^s(py + \lambda yq - C(y,q))$ where U^s is a utility function for suppliers, π^s is the profit of suppliers, p is the market unit price of products supplied by suppliers, and λ is the quality incentive (price) for a unit of product (Lederer and Rhee, 1995). The utility function of procurers is defined as $U^p(B(y,q) - py - \lambda yq)$ where U^p is a utility function for procurers and $B()$ is the benefits from quantity y and quality q supplied by supplier.

IV. Contracting between procurer and supplier

1. Models

Based on the utility functions defined in the previous section, contracting models are developed to decide the contracting parameters, y, q, and λ. In this paper, procurer is principal who decides the contracting parameters and cannot observe the quality of products supplied by supplier in the case of information asymmetry where quality is not observable to procurer while it is perfectly observable to supplier and supplier is agent who takes procurer's decision on contracting parameters and can conceal the true quality of her/his products. There are two models available: contract model for information symmetry case where quality is perfectly observable to both procurers and suppliers and contract model for information asymmetry case. As noted, though quality may be unobservable to both procurer and supplier this paper consider the case where quality is not observable only to procurer in order to examine what happens to utility of procurer and supplier and social welfare when supplier can lie on her/his quality.

Model for information symmetry case

Model for information symmetry case where both procurer and supplier can observe the quality of products supplied by suppliers perfectly is as follows:

$$\begin{aligned} &\underset{y,q,\lambda}{Max}\, U^p = U^p(B(y,q) - py - \lambda yq) \\ &subject\ to, \qquad (1) \\ &U^s(py + \lambda yq - C(y,q)) \geq \underline{U^s}\ (IR) \end{aligned}$$

where $\underline{U^s}$ is the reservation utility for supplier which is the minimum utility level which procurer must guarantee to encourage the supplier to participate in contract. Procurer decides the quantity (y), quality (q), and quality incentive (λ) to procure from the supplier in order to maximize her/his utility. Constraints *(IR: Individual rationality)* mean that procurer should guarantee the reservation utility to supplier maximizing her/his utility. Reservation utility is, generally, the utility level which supplier can obtain when she/he operates independently rather than participating in contract.

Model for information asymmetry case

In information asymmetry case, supplier can get more utility by lying on his/her quality than by revealing their quality truly. Thus, quality of a supplier can b regarded as the type variable of the supplier instead of the decision variable. This lying can reduce the utility of procurer and social welfare seriously. Therefore, the most important point in contract is how procurer leads supplier to reveal her/his quality. In information asymmetry case, though procurer cannot observe the quality of products supplied by supplier perfectly she/he is assumed to know the distribution of quality in market.

The model is as follows.

$$\underset{y,\lambda}{Max}\,U^p = \int_{\underline{q}}^{\bar{q}} U^p(B(y,q) - py - \lambda yq) f(q) dq$$
$$\textit{subject to,} \qquad (2)$$
$$U^s(py + \lambda yq - C(y,q)) \geq \underline{U^s}\ (IR)$$
$$U^s(py + \lambda yq - C(y,q)) \geq U^s(py + \lambda yq' - C(y,q))\ (IC)$$

where q' is the quality level which supplier pretend to produce though she/he produces q and $f(q)$ is the probability density function of quality in market in the range of $[\underline{q}, \bar{q}]$. Two things are different from information symmetry case. One is that q is not a decision variable any more and as the result, the expected utility of procurer with respect to quality distribution is maximized by deciding quantity and quality incentive. Two is that incentive compatibility *(IC)* constraints are added to constraints.

Procurer decides the quantity (y) and quality incentive (λ) to procure from supplier in order to maximize her/his utility based on the quality revelation by supplier. Constraints *(IC: Incentive compatibility)* mean that procurer should decide contracting parameters (y, λ) with which supplier can obtain more utility by revealing her/his true quality than by lying on it.

2. Solutions

In order to examine how imperfect quality observability affects the utility of procurer and supplier and social welfare, the described contract models are solved theoretically here. For simplification of analysis, suppose that supplier is risk neutral, that is, $U^s(\pi^s) = U^s(py + \lambda yq - C(y,q)) = py + \lambda yq - C(y,q)$ and the utility function of procurer is a quasi-linear, which implies that procurer's marginal utility for money is constant, that is, $U^p(B(y,q) - py - \lambda yq) = u^p(y,q) - py - \lambda yq$ where $u^p(y,q)$ represents a utility obtained from y and q replacing $B(y,q)$. These assumptions simplify some technical points and mainly enable us to use surplus analysis. Moreover, supplier is assumed to operate in perfectly competitive market, which implies that the reservation utility should be zero since she/he will obtain zero profit from the independent operation in perfectly competitive market.

<u>Information symmetry case</u>

Equation (1) is modified into equation (3) by the above assumptions.

$$\underset{y,q,\lambda}{Max}\, U^p = u^p(y,q) - py - \lambda yq$$
$$subject\ to, \qquad (3)$$
$$py + \lambda yq - C(y,q) \geq 0\ (IR)$$

Equation (3) can be easily solved by using Lagrangian multiplier method. The detail solution process is in Appendix 1. The results show that procurer decides y^*, q^*, and λ^* which satisfy $\frac{\partial u^p}{\partial y}(y^*,q^*) = \frac{\partial C}{\partial y}(y^*,q^*)$, $\frac{\partial u^p}{\partial q}(y^*,q^*) = \frac{\partial C}{\partial q}(y^*,q^*)$, and $py^* + \lambda^* y^* q^* - C(y^*,q^*) = 0$. These results imply that the optimal solution for information symmetry case is the point where the marginal utility from quantity for procurer is equal to the marginal cost to produce quantity for supplier which is equal to the marginal utility from quantity for supplier in perfectly competitive market and the marginal utility from quality for procurer is equal to the marginal cost to produce quality for supplier which is equal to the marginal utility of quality for supplier in perfectly competitive market. This is consistent with the results of neoclassical production and consumption theory and can be called the first-best optimal result. The first-best social surplus becomes $u^p(y,q) - C(y,q)$, which implies that there is no additional loss in social welfare and the optimal quantity, quality, and quality incentive for both supplier and procurer are determined. Moreover, the price for products supplied by supplier including market price (p) and quality incentive (λ) is equal to the marginal cast of supplier.

<u>Information asymmetry case</u>

Also, equation (2) is modified into equation (4).

$$\underset{y,\lambda}{Max}\, U^p = \int_{\underline{q}}^{\bar{q}} (u^p(y,q) - py - \lambda yq) f(q) dq$$
$$subject\ to, \qquad (4)$$
$$py + \lambda yq - C(y,q) \geq 0\ (IR)$$
$$py + \lambda yq - C(y,q) \geq py + \lambda yq' - C(y,q)\ (IC)$$

Equation (4) should be solved differently from information symmetry, that is, it may not be solved directly by using Lagrangian multiplier method. The below solution process follows the process of Salanie (1998). At first, incentive compatibility constraints should be solved in order for them to be plugged into objective function as follows.

Let $V(q,q')$ be the utility achieved by a supplier of type q who announces her/his type (quality) as q', then the utility is $V(q,q') = py(q') + \lambda(q')y(q')q' - C(y(q'),q)$ where y and λ can be expressed as a function of q because y and λ are decided for each level of q. For (y, λ) to be incentive compatible, it has to be that the following first and second order conditions hold: $\frac{\partial V}{\partial q'}(q,q')|_{q'=q} = 0, \frac{\partial^2 V}{\partial q'^2}(q,q')|_{q'=q} \le 0$ for all q. The first order condition and the second order condition comes down to

$$\frac{\partial V}{\partial q'}(q,q')|_{q'=q} = p\frac{\partial y}{\partial q}(q) + \frac{\partial \lambda}{\partial q}(q)y(q)q + \lambda(q)\frac{\partial y}{\partial q}(q)q + \lambda(q)y(q) - \frac{\partial C}{\partial y}(y(q),q)\frac{\partial y}{\partial q}(q) = 0 \quad (5)$$

and

$$\frac{\partial^2 V}{\partial q'^2}(q,q')|_{q'=q} = p\frac{\partial^2 y}{\partial q^2}(q) + \frac{\partial^2 \lambda}{\partial q^2}(q)y(q)q + \frac{\partial \lambda}{\partial q}(q)\frac{\partial y}{\partial q}(q)q + \frac{\partial \lambda}{\partial q}(q)y(q) + \frac{\partial \lambda}{\partial q}(q)\frac{\partial y}{\partial q}(q)q + \lambda(q)\frac{\partial^2 y}{\partial q^2}(q)q + \lambda(q)\frac{\partial y}{\partial q}(q) + \frac{\partial \lambda}{\partial q}(q)y(q) + \lambda(q)\frac{\partial y}{\partial q}(q) - \frac{\partial^2 C}{\partial y^2}(y(q),q)[\frac{\partial y}{\partial q}(q)]^2 - \frac{\partial C}{\partial y}(y(q),q)\frac{\partial^2 y}{\partial q^2}(q) \le 0 \quad (6).$$

Moreover, differentiating equation (5) with respect to q again gives us

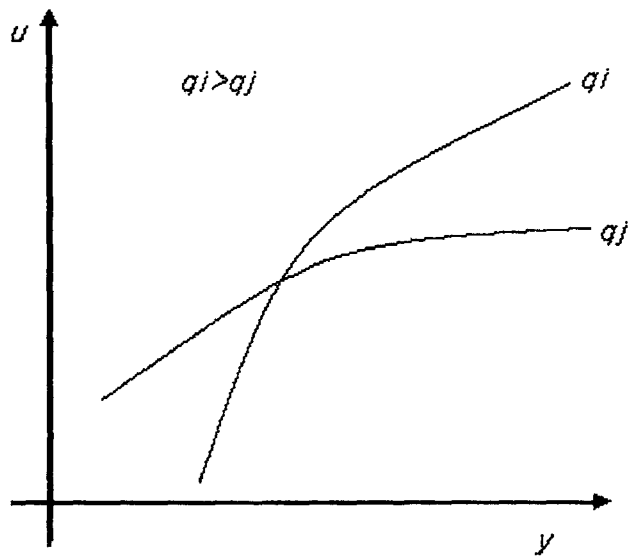
$$p\frac{\partial^2 y}{\partial q^2}(q) + \frac{\partial^2 \lambda}{\partial q^2}(q)y(q)q + \frac{\partial \lambda}{\partial q}(q)\frac{\partial y}{\partial q}(q)q + \frac{\partial \lambda}{\partial q}(q)y(q) + \frac{\partial \lambda}{\partial q}(q)\frac{\partial y}{\partial q}(q)q + \lambda(q)\frac{\partial^2 y}{\partial q^2}(q)q + \lambda(q)\frac{\partial y}{\partial q}(q) + \frac{\partial \lambda}{\partial q}(q)y(q) + \lambda(q)\frac{\partial y}{\partial q}(q) - \frac{\partial^2 C}{\partial y^2}(y(q),q)[\frac{\partial y}{\partial q}(q)]^2 - \frac{\partial^2 C}{\partial y \partial q}(y(q),q)\frac{\partial y}{\partial q}(q) - \frac{\partial C}{\partial y}(y(q),q)\frac{\partial^2 y}{\partial q^2}(q) = 0 \quad (7)$$

and subtracting equation (6) from equation (7) gives us equation (8). $\frac{\partial^2 C}{\partial y \partial q}(y(q),q)\frac{\partial y}{\partial q}(q) \le 0$ (8).

Equation (5) and (8) consist of sufficient and necessary incentive conditions.

Equation (8) includes a very meaningful intuition. Because the second part of equation (8) can be assumed to be non-positive ($\frac{\partial y}{\partial q}(q) \le 0$: Quantity purchased by procurer is decreased in quality[2]), the first of it becomes $\frac{\partial^2 C}{\partial y \partial q}(y,q) \geq 0$ for all y and q which is called the Spence-Mirrlees condition or the single crossing condition. The Spence-Mirrlees condition means that higher quality will require more from procurer for a given increase in quantity than lower quality and thus it is possible for procurer to separate the different types (quality) of suppliers by paying more for higher quality, which allows us to call the Spence-Mirrlees condition as the sorting condition. If the Spence-Mirrlees condition is not satisfied by supplier's utility function, incentive compatibility constraints do not hold because suppliers may not be sorted. Mathematically, $\frac{\partial^2 u^s}{\partial y \partial q} = \lambda - \frac{\partial^2 C}{\partial y \partial q} > 0$ because as noted, $\lambda > 0$ and $\frac{\partial^2 C}{\partial y \partial q} \leq 0$. This implies that the slope of supplier's utility function with respect to y is increasing in quality q (type). Figure 1 illustrates this point. The utility function of supplier with higher quality (q_i) has the steeper slope against y than that with lower quality (q_j).

Figure 1. The Spence-Mirrlees condition



[2]. This can be reasonable in that high quality is more expensive and there are less quantity of high quality in market.

To plug incentive compatibility constraints into objective function (The utility function of procurer), let's estimate the information rent for supplier which procurer should compensate to supplier in order to make supplier reveal her/his quality (type) truly. If $v(q)$ is defined as the utility the supplier with quality q obtains at the optimum of her/his program, it represents the information rent. In order to make supplier reveal her/his quality truly, procurer should give at least $v(q)$ to suppliers with quality q. Mathematically, $v(q) =: V(q,q) = py(q) + \lambda(q)y(q)q - C(y(q),q)$ (9). In equation (9), since all q's except the last q are determined by the last q, equation (9) can be differentiated with respect to the last q and the result is $\frac{\partial v}{\partial q}(q) = -\frac{\partial C}{\partial q}(y(q),q) < 0$ (10), which implies that information rent decreases in quality. This result means that supplier with lower quality thus benefits more from her/his private information and she/he may pretend her/his quality is $q' > q$[3]. Moreover, equation (10) enables us to get rid of $py + \lambda yq$ from $v(q)$. That is, $v(q) = \int_q^{\bar{q}} -\frac{\partial C}{\partial q}(y(t),t)dt$ (11) which can be plugged into objective function representing constraints (*IC*).

In addition to incentive compatibility solution, it is necessary to specify individual rationality constraints. Individual rationality can be expressed by $v(q) \geq 0$ for all q. As noted, because $\frac{\partial v}{\partial q}(q) < 0$ it can come down to $v(\bar{q}) \geq 0$ and it is independent of supplier's type.

With the above notations, it is possible to set up procurer's utility function as follows. From equation (11), we have $\begin{aligned} py(q) + \lambda(q)y(q)q &= C(y(q),q) - v(q) \\ &= C(y(q),q) + \int_q^{\bar{q}} \frac{\partial C}{\partial q}(y(t),t)dt \end{aligned}$ and procurer's utility function becomes $U^P = \int_{\underline{q}}^{\bar{q}} [u^P(y(q),q) - C(y(q),q) - \int_q^{\bar{q}} \frac{\partial C}{\partial q}(y(t),t)dt] f(q)dq$ (12). The third integrand in equation (12) can be changed to $\int_q^{\bar{q}} \frac{\partial C}{\partial q}(y(t),t) f(q)dq = \frac{\partial C}{\partial q}(y,q)[\frac{1}{1-h(q)}]$ where *h(q)* is the hazard rate $(= \frac{f(q)}{1-F(q)})$ (Kamien and Schwartz, 1981; Ross, 1993). The

[3]. By pretending his type is q', supplier with type q can obtain more utility than supplier with type q' revealing her/his type truly (Look at Appendix II).

integrand ($u^p(y,q)-C(y,q)-\frac{\partial C}{\partial q}(y,q)[\frac{1}{1-h(q)}]$) of equation (12) represents the social surplus under information asymmetry (Salanie, 1998). While in information symmetry case, the first-best social surplus was $u^p(y,q)-C(y,q)$, more loss in social welfare happens under information asymmetry by $\frac{\partial C}{\partial q}(y,q)[\frac{1}{1-h(q)}]\geq 0$ which is the impact of imperfect observability of quality on the social welfare and will go to supplier as the compensation to her/his lying. Finally, the optimal solution is obtained by optimizing the integrand of equation (12) in every type of q and therefore, the optimal solution should satisfy $\frac{\partial u^p}{\partial y}(y^*(q),q)=\frac{\partial C}{\partial y}(y^*(q),q)+\frac{\partial^2 C}{\partial y \partial q}(y^*(q),q)[\frac{1}{1-h(q)}]$, which implies that marginal cost for a unit is greater than the price for a unit including both market price (p) and quality incentive (λ). For the results of this paper, it will be very interesting to estimate the size of the loss in social welfare from imperfect quality observability $\frac{\partial C}{\partial q}(y,q)[\frac{1}{1-h(q)}]$ though this paper does not cover it.

V. Conclusion

Quality is important in supply chain because the quality of products supplied by suppliers is the important determinant of the quality of products produced by procurers. Thus, the performance of quality in supply chains will be dependent on how the quality of product produced by suppliers in each stage of the supply chain is coordinated through the supply chain. In other words, the most important issue of this paper is how procurer can obtain the quality and quantity which she/he wants for the reasonable price.

If this quality is perfectly observable along supply chain, it is priced in competitive market reflecting the cost of quality production and results in the first-best optimal quality management in supply chain. However, it is impossible or very difficult for procurers to observe quality perfectly and accurately in many industries. This imperfect observability of quality causes market failure in pricing and supplying quality. As the result, procurers who have less information on quality than suppliers will lose their utility and social welfare will be reduced because of the adverse selection by suppliers.

This paper models and solves contracting schemes for both when quality is perfectly observable and when quality is not perfectly observable. When quality is perfectly observable, the first−best optimal solution which is that the marginal utility of procurer obtained from the quantity and quality supplied by suppliers (the price) is equal to the marginal cost to produce the quantity and quality is obtained. However, when quality is not perfectly observable to procurers the optimal solution cannot be the first−best but the second−best where the price is greater than the marginal cost to produce the quantity and quality and social welfare is less than that of the first−best solution.

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Appendix I

From $L = u^p(y,q) - py - \lambda yq + \mu(py + \lambda yq - C(y,q))$, the first order conditions are $\frac{\partial L}{\partial y} = \frac{\partial u^p}{\partial y} - p - \lambda q + \mu p + \mu\lambda q - \mu\frac{\partial C}{\partial y} = 0$ (A1), $\frac{\partial L}{\partial q} = \frac{\partial u^p}{\partial q} - \lambda y + \mu\lambda y - \mu\frac{\partial C}{\partial q} = 0$ (A2), $\frac{\partial L}{\partial \lambda} = -yq + \mu yq = 0$ (A3), $\frac{\partial L}{\partial \mu} = py + \lambda yq - C(y,q) = 0$ (A4).

From (A3), if y and q are not zero, $\mu = 1$. (A1) and (A2) become $\frac{\partial u^p}{\partial y} = \frac{\partial C}{\partial y}$ and $\frac{\partial u^p}{\partial q} = \frac{\partial C}{\partial q}$, respectively and $MU_y^p = MC_y$ and $MU_q^p = MC_q$.

Appendix II

Utility of supplier with type q pretending her/his type is q' $(>q)$ is
$V(q,q') = py(q') + \lambda(q')y(q')q' - C(y(q'),q)$
$= py(q') + \lambda(q')y(q')q' - C(y(q'),q') - C(y(q'),q)$ because $C(y(q'),q) \leq C(y(q'),q')$
$+C(y(q'),q') > v(q') = V(q',q')$
by $\frac{\partial C}{\partial q} \geq 0$.