

# Comparison of Cost Effective Measures among Fault-tolerant Multistage Interconnection Networks

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**Abstract.** As the size of a network increases, the network complexity and maintenance cost is high. In literatures, many cost factors of multistage interconnection networks were intensively investigated. However, in some parts, these types of cost definition are not satisfactory enough since they don't relate the cost measure to technology closely and also fail to incorporate exact tradeoffs in designing the network. In this paper, we propose some cost effective measures which combine the cost factors with performance-related reliability measures, and compare several networks for a guidance to select a certain case depending on the principal concerns for use.

## 1 Introduction

In building a network, one of primary concerns is the cost to be incurred. In the case, the "cost" can be defined as various forms. The amount of hardware needed to implement a network is simplest but useful guidance to effectiveness in economic point of view. Namely if a network requires less amount of hardware with same performance and/or fault-tolerance, we can regard it more effective in cost, vice versa. More detailed cost factors were investigated [1-5]. Main cost factors of Multistage Interconnection Networks include: component (SEs,

Muxs, Demuxs) count and complexity, and interconnection cost. Blake and Trivedi [6] defined network complexity as the number of SEs in the MIN and compare it among networks. Tagle and Sharma [4,5] compared the amount of hardware needed to implement their proposed network, in the context of proposing high performance fault-tolerant scheme with self routing, as guidance to cost effectiveness. Hardware cost can be compared in terms of number of stages, number of switches per stage, and/or the size of SEs ( $R \equiv 2^r, r \geq 1$ ) [1-5].

In some parts, these types of cost definition are not satisfactory enough since they don't relate the cost measure to technology closely and also fail to incorporate exact tradeoffs in designing the network. We, however, feel that in many cases even a very approximate analysis can bring about interesting and meaningful results in its generality. Practically, one may be confronted with the situation to choose a moderate cost/fault-tolerance compromise. Here, we are going to carry out cost comparison among networks and to combine the results with Performance Related-Reliability measures for a guidance to select a certain case depending on the principal concerns for use.

We introduced Park et al. [2] in our model and modified it slightly and more close to technology in designing MINs. Hence, finally, it will be shown

why we relate the cost measure to technology and how successful it is to incorporate exact tradeoffs in benchmarking for selecting a good cost/fault-tolerance compromise among MINs.

## 2 Combining Cost with PRR Measures

The major consideration in the design of the network is keeping the switch and link complexity as low as possible. A key issue in fault-tolerant MINs therefore, is the manner in which rerouting or selection of alternate paths is achieved. Irrespective of methods employed in constructing fault-tolerant MINs, there exist two or more paths between a source and a destination pair, which are referred to multi-path MINs. A multi-path MIN consists of switches that have multi-inputs and multi-outputs. The hardware cost is higher than that of unique path MINs in terms of number of stages, number of links, number of switches and/or the size of the SEs. These are some of the principal factors that contribute to system complexity of a MIN.

We incorporate the “*network complexity*” of Blake and Trivedi [6] as the cost to be compared. The concept “*network complexity*” is very simple yet useful for fair comparison among the networks. Here, to make a fair comparison, we use gate counts in the network components to compensate for the differences in network construction. The SEs are considered as  $k \times k$  crossbar switches, denoted  $SE_k$ , so an  $SE_k$  has  $4k(k-1)$  gates [7], and the Muxs/Demuxs have  $2(k-1)$  gates, where  $k$  is the number of input/output links. For example,  $SE_2$  has 8 gates whereas  $SE_3$  has 24, thus  $SE_3$  has three times as large gates as  $SE_2$ . This is different from the assumption adopted in Par et al. [2] and Tzeng et al. [8], which assume that the cost of an SE

is proportional to the number of crosspoints within the SE.

We incorporate the PRR measures of H.J. Kim, S.H. Yoon, and S. Kim [9] as fault-tolerance to be compared. They suggested three PRR measures, mean number of operational I/O pairs, mean number of survivable inputs and mean number of survivable outputs at give time instant.

Let us define cost effectiveness measure of a network as the ratio of its PRR values to cost. Thus we have three cost effectiveness measures as a benchmark to choose a good cost/fault-tolerance compromise. These measures combined with PRR measures give us the normalized and compromised guidance for cost-related issues. The measures are:

- $\gamma(t) = \delta(t)/NC_i$  : mean number of operational I/O pairs given network complexity,
- $\eta(t) = SI(t)/NC_i$  : mean number of survivable inputs given network complexity,
- $\rho(t) = SO(t)/NC_i$  : mean number of survivable outputs given network complexity.

From the definition of networks and combining gate counts, we summarize the network complexity of each MIN as follows:

- $NC_{SEN} = N \cdot \log_2 N / 2$ ,
- $NC_{SEN^*} = N \cdot (1 + \log_2 N) / 2$ ,
- $NC_{ASEN} = 3N \cdot (1 + 2(\log_2 N - 2)) / 4$ ,
- $NC_{ACN} = 3N \cdot (\log_2 N)$ ,
- $NC_{EXN} = N \cdot (2 + (\log_2 N + 1)) / 2$ ,
- $NC_{GIN} = N \cdot (1 + \log_2 N)$ ,
- $NC_{MDN} = 3N \cdot (\log_2 N - 1)$ ,

where  $NC_{GIN}$  is the case with  $R=L=2$  and  $NC_{MDN}$  is the case with  $R=d=2$ , respectively.

### 3 Numerical Results

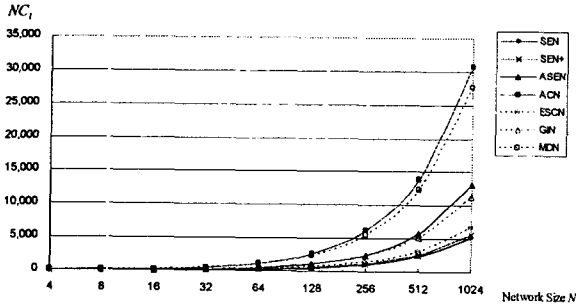


Figure 1: Network complexity as a function of network size

From the equations for network complexity in Section 2 and Figure 1, we observe that  $NC_{ACN}$  is highest for all the network size among MINs to be evaluated. Although ACN provides us multiple redundant paths between input and output pairs, we should trade this benefit for much higher network complexity. Figure 2 illustrates this trade-off clearly. Even in repairable system, ACN is proved to be the worst network in terms of the PRR measures normalized by network complexity, whereas SEN wins the highest irrespective of the presence of repair except for  $\gamma(t)$  with  $\lambda_B t = 0.08$ . Due to the simple structure of network, SEN+ and ESCN also obtain better performance rather than other networks.

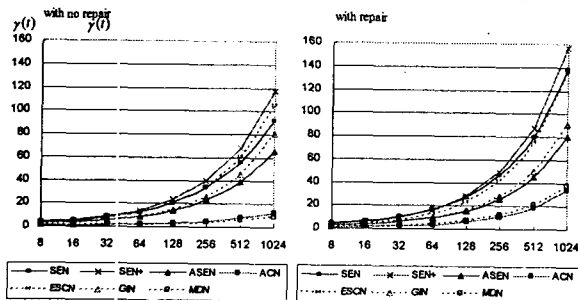


Figure 2:  $\gamma(t)$  as a function of network size with  $\lambda_B t = 0.08$

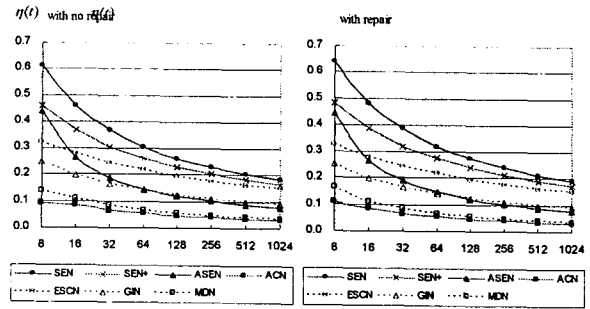


Figure 3:  $\eta(t)$  as a function of network size with  $\lambda_B t = 0.08$

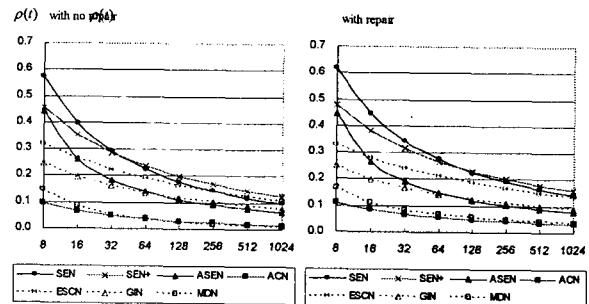


Figure 4:  $\rho(t)$  as a function of network size with  $\lambda_B t = 0.08$

The presence of repairable system makes little difference in mean number of survivable inputs and outputs given network complexity as shown in Figure 3 and 4. It is noteworthy that the performance of ASEN decreases radically with network size partially because its structure grows more complex due to  $3 \times 3$  SEs.

From the above results, we can present final results for a guidance to select a cost/fault-tolerance compromise among MINs depending on the principal purpose and the environment of the system as Table 1.

### 4 Conclusions

In the course of constructing redundant-path MIN, and computing its reliability, we observe that irrespective of methods providing redundancy fault-

Table 1: Benchmark for selection of networks

System Purpose(needs)	System Environment	Recommended Networks
multiple one-to-one communication	poor repair	ESCN, GIN, SEN+
	repair-upon-fault	
more source for broadcasting	poor repair	ESCN, ACN, GIN
	repair-upon-fault	ESCN, SEN+, GIN
more destination to broadcast	poor repair	ESCN, GIN, SEN+
	repair-upon-fault	

tolerant MINs are obtained by paying for more cost than unique-path MINs. Nevertheless, most part of studies skips cost-consideration in constructing MINs and evaluating them. Even in the studies treating cost-related issues, it is rare to combine cost measures with reliability measures synthetically.

In this paper, we proposed some cost effective measures which combine the cost factors with performance-related reliability measures, and compare several networks. We believe that our work will be a guidance to select a certain network type depending on the principal concerns for use.

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