$\mathcal{H}_\infty$ Fuzzy State-Feedback Control Design for Uncertain Nonlinear Descriptor Systems: An LMI Approach

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Abstract: This paper examines the problem of designing an $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of uncertain nonlinear descriptor systems which is described by a Takagi-Sugeno (TS) fuzzy model. Based on a linear matrix inequality (LMI) approach, we develop an $\mathcal{H}_\infty$ state-feedback controller which guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value for this class of systems. A numerical example is provided to illustrate the design developed in this paper.

Keywords: Fuzzy control, Descriptor systems, $\mathcal{H}_\infty$ Control, LMI

1. INTRODUCTION

Descriptor systems or singularly perturbed systems also known as multiple time-scale dynamic systems normally occur due to the presence of small "parasitic" parameters, typically small time constants, masses, etc. The presence of these "parasitic" parameters can make the dimensionality of a dynamics system prohibitively high. For the past three decades, descriptor systems have been intensively studied by many researchers; see [1]-[8]. In state space, such systems are commonly modelled using the mathematical framework of singular perturbations, with a small parameter, say ε, determining the degree of separation between the "slow" and "fast" modes of the system. However, it is necessary to note that it is possible to solve the singularly perturbed systems without separating between slow and fast mode subsystems. But the requirement is that the "parasitic" parameters must be large enough. In the case of having very small "parasitic" parameters which normally occur in the description of various physical phenomena, a popular approach adopted to handle these systems is based on the so-called reduction technique. According to this technique the fast variables are replaced by their steady states obtained with "frozen" slow variables and controls, and the slow dynamics is approximated by the corresponding reduced order system. This time-scale is asymptotic, that is, exact in the limit, as the ratio of the speeds of the slow versus the fast dynamics tends to zero.

In the last few years, the research on singularly perturbed systems in the $\mathcal{H}_\infty$ sense has been highly recognized in control area due to the great practical importance. $\mathcal{H}_\infty$-optimal control of singularly perturbed linear systems under either perfect state measurements or imperfect state measurements has been investigated via differential game theoretics approach. Although many researchers have studied the $\mathcal{H}_\infty$ control design of linear singularly perturbed systems for many years, the $\mathcal{H}_\infty$ control design of nonlinear singularly perturbed systems remains as an open research area. This is due to, in general, nonlinear singularly perturbed systems can not be decomposed into slow and fast subsystems.

Recently, a great amount of effort has been made on the design of fuzzy $\mathcal{H}_\infty$ for a class of nonlinear systems which can be represented by a Takagi-Sugeno (TS) fuzzy model; see [14]–[19]. Fuzzy system theory enables us to utilize qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. Recent studies [9]-[21] show that a fuzzy linear model can be used to approximate global behaviors of a highly complex nonlinear system. In this fuzzy linear model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" of these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling which uses a single model to describe the global behavior of a system, fuzzy modelling is essentially a multi-model approach in which simple sub-models (linear models) are combined to describe the global behavior of the system. Employing the existing fuzzy results [9]-[21] on the nonlinear descriptor system, one ends up with a family of ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes. In general, ill-conditioned linear matrix inequalities are very difficult to solve.

What we intend to do in this paper is to design an $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of uncertain nonlinear descriptor systems. First, we approximate this class of uncertain nonlinear descriptor systems by a Takagi-Sugeno fuzzy model. Then based on an LMI approach, we develop a technique for designing an $\mathcal{H}_\infty$ fuzzy state-feedback controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value. To alleviate the ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes, these ill-conditioned LMIs are decomposed into ε-independent and ε-dependent LMIs. The ε-independent LMIs are not ill-conditioned and the ε-dependent LMIs tend

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to zero when ε approaches to zero. If ε is sufficiently small, the original ill-conditioned LMIs are solvable if and only if the ε-independent LMIs are solvable. The proposed approach does not involve the separation of states into slow and fast ones, and it can be applied not only to standard, but also to nonstandard nonlinear descriptor systems. Finally, a numerical simulation example is presented to illustrate the theory development.

This paper is organized as follows. In Section 2, system description and definition are presented. In Section 3, based on an LMI approach, we develop a technique for designing an $\mathcal{H}_\infty$ fuzzy state-feedback controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value for the system described in Section 2. The validity of this approach is demonstrated by an example from a literature in Section 4. Finally in Section 5, conclusions are given.

2. SYSTEM DESCRIPTION AND DEFINITION

First, we generalize the TS fuzzy descriptor system to represent a TS fuzzy descriptor system with parametric uncertainties as follows:

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= \textstyle\sum_{i=1}^r \mu_i(\nu(t))\Big[[A_i + \Delta A_i]x(t) \\ &\quad +[B_{1_i} + \Delta B_{1_i}]w(t) \\ &\quad +[B_{2_i} + \Delta B_{2_i}]u(t)\Big], \quad x(0) = 0 \\ z(t) &= \textstyle\sum_{i=1}^r \mu_i(\nu(t))\Big[[C_{1_i} + \Delta C_{1_i}]x(t) \\ &\quad +[D_{12_i} + \Delta D_{12_i}]u(t)\Big] \end{aligned} \tag{1}$$

where $E_\varepsilon = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}$, $\varepsilon > 0$ is the singular perturbation parameter, $\nu(t) = [\nu_1(t) \ \cdots \ \nu_\vartheta(t)]$ is the premise variable that may depend on states in many cases, $\mu_i(\nu(t))$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\mu_i(\nu(t)) \geq 0$ and $\sum_{i=1}^r \mu_i(\nu(t)) = 1$), ϑ is the number of fuzzy sets, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, and $z(t) \in \Re^s$ is the controlled output, the matrices $A_i, B_{1_i}, B_{2_i}, C_{1_i}$ and D_{12_i} are of appropriate dimensions, and the matrices ΔA_i, ΔB_{1_i}, ΔB_{2_i}, ΔC_{1_i} and ΔD_{12_i} represent the uncertainties in the system and satisfy the following assumption.

Assumption 1:

$$\Delta A_i = F(x(t),t)H_{1_i}, \ \ \Delta B_{1_i} = F(x(t),t)H_{2_i},$$

$$\Delta B_{2_i} = F(x(t),t)H_{3_i}, \ \ \Delta C_{1_i} = F(x(t),t)H_{4_i}$$

$$\textit{and} \ \ \Delta D_{12_i} = F(x(t),t)H_{5_i}$$

where H_{j_i}, $j = 1, 2, \cdots, 5$ are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t),t)\| \leq \rho \tag{2}$$

for any known positive constant ρ.

Next, let us recall the following definition.

Definition 1: *Suppose γ is a given positive number. A system (1) is said to have an $\mathcal{L}_2$-gain less than or equal to γ if*

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2 \left[\int_0^{T_f} w^T(t)w(t)dt\right], \ x(0) = 0 \tag{3}$$

for all $T_f \geq 0$ and $w(t) \in \mathcal{L}_2[0, T_f]$.

Note that for the symmetric block matrices, we use $(*)$ as an ellipsis for terms that are induced by symmetry.

3. $\mathcal{H}_\infty$ FUZZY STATE-FEEDBACK CONTROL DESIGN

In this section, we present a new technique for designing a fuzzy state-feedback controller for a TS fuzzy system with parametric uncertainties. Based on an LMI approach, we develop a technique for designing a fuzzy controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value.

Let us design an $\mathcal{H}_\infty$ fuzzy state-feedback controller of the form

$$u(t) = \sum_{j=1}^r \mu_j K_j x(t) \tag{4}$$

such that the inequality (3) holds. Before presenting our next results, the following lemma is recalled.

Lemma 1: *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and a positive constant δ, if there exist a matrix $P_\varepsilon = P_\varepsilon^T$ and matrices Y_j, $j = 1, 2, \cdots, r$, satisfying the following ε-dependent linear matrix inequalities:*

$$P_\varepsilon > 0 \tag{5}$$

$$\Psi_{ii}(\varepsilon) < 0, \quad i = 1, 2, \cdots, r \tag{6}$$

$$\Psi_{ij}(\varepsilon) + \Psi_{ji}(\varepsilon) < 0, \quad i < j \leq r \tag{7}$$

where

$$\Psi_{ij}(\varepsilon) = \begin{pmatrix} \Omega_{ij}(\varepsilon) & (*)^T & (*)^T \\ \tilde{B}_{1_i}^T & -\gamma I & (*)^T \\ \begin{pmatrix} \tilde{C}_{1_i} E_\varepsilon^{-1} P_\varepsilon \\ +\tilde{D}_{12_i} Y_j \end{pmatrix} & 0 & -\gamma I \end{pmatrix} \tag{8}$$

with

$$\Omega_{ij}(\varepsilon) = \begin{pmatrix} A_i E_\varepsilon^{-1} P_\varepsilon + E_\varepsilon^{-1} P_\varepsilon A_i^T \\ +B_{2_i} Y_j + Y_j^T B_{2_i}^T \end{pmatrix},$$

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & B_{1_i} \end{bmatrix},$$

$$\tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta} H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T,$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta} H_{3_i}^T & \sqrt{2}\lambda\rho H_{5_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T,$$

$$\textit{and} \ \ \lambda = \left(1 + \rho^2 \sum_{i=1}^r \sum_{j=1}^r \left[\|H_{2_i}^T H_{2_j}\|\right]\right)^{\frac{1}{2}},$$

then the inequality (3) holds. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r} \mu_j K_j(\varepsilon) x(t) \tag{9}$$

where

$$K_j(\varepsilon) = Y_j P_\varepsilon^{-1} E_\varepsilon. \tag{10}$$

Proof: The desired result can be carried out by a similar technique used in [19] and [20]. The detail of the proof is omitted for brevity. ■

Remark 1: *The LMIs given in Lemma 1 become ill-conditioned when ε is sufficiently small, which is always the case for the descriptor systems. In general, these ill-conditioned LMIs are very difficult to solve. Thus, to alleviate these ill-conditioned LMIs, we have the following theorem which does not depend on ε.* □

Theorem 1: *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and a positive constant δ, if there exist a matrix P and matrices Y_j, $j = 1, 2, \cdots, r$, satisfying the following ε-independent linear matrix inequalities:*

$$EP + PD > 0 \tag{11}$$

$$\Psi_{ii} < 0, \quad i = 1, 2, \cdots, r \tag{12}$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i < j \le r \tag{13}$$

where $EP = P^T E$, $PD = DP$, $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$ *and*

$$\Psi_{ij} = \begin{pmatrix} \begin{pmatrix} A_i P + P^T A_i^T \\ +B_{2_i} Y_j + Y_j^T B_{2_i}^T \end{pmatrix} & (*)^T & (*)^T \\ \tilde{B}_{1_i}^T & -\gamma I & (*)^T \\ \tilde{C}_{1_i} P + \tilde{D}_{12_i} Y_j & 0 & -\gamma I \end{pmatrix} \tag{14}$$

with

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & B_{1_i} \end{bmatrix},$$

$$\tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta} H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T,$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta} H_{3_i}^T & \sqrt{2}\lambda\rho H_{5_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T,$$

$$\text{and } \lambda = \left(1 + \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left[\|H_{2_i}^T H_{2_j}\|\right]\right)^{\frac{1}{2}},$$

then there exists a sufficiently small $\hat{\varepsilon} > 0$ such that the inequality (3) holds for $\varepsilon \in (0, \hat{\varepsilon}]$. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r} \mu_j K_j x(t) \tag{15}$$

where

$$K_j = Y_j P^{-1}. \tag{16}$$

Proof: The detail of the proof is omitted for brevity. ■

4. ILLUSTRATIVE EXAMPLE

Consider a tunnel diode circuit shown in Fig. 1 where the tunnel diode is characterized by

$$i_D(t) = -0.2 v_D(t) - 0.01 v_D^3(t).$$

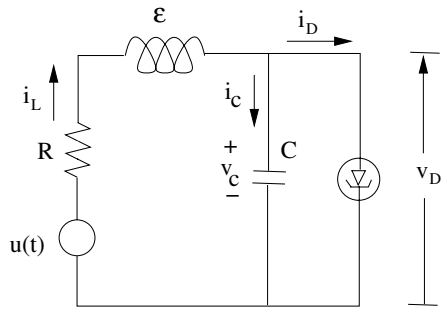

Fig. 1. A tunnel diode circuit.

Assume that ε is a "parasitic" inductance in the network. Let $x_1(t) = v_C(t)$ be the capacitor voltage and $x_2(t) = i_L(t)$ be the inductor current. Then, the circuit shown in Fig. 1 can be modelled by the following state equations:

$$\begin{aligned} C\dot{x}_1(t) &= 0.2x_1(t) + 0.01x_1^3(t) + x_2(t) + 0.01w_1(t) \\ \varepsilon\dot{x}_2(t) &= -x_1(t) - Rx_2(t) + u(t) + 0.1w_2(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{17}$$

where $w_1(t)$ and $w_2(t)$ are the disturbance noise inputs, $z(t)$ is the controlled output and $x(t) = [x_1^T(t) \quad x_2^T(t)]^T$. Note that the variables $x_1(t)$ and $x_2(t)$ are treated as the deviation variables (variables deviate from its desired trajectories). The parameters in the circuit are given by $C = 100\ mF$ and $R = 1 \pm 0.3\%\ \Omega$ with these parameters then (17) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= 2x_1(t) + (0.1x_1^2(t)) \cdot x_1(t) + 10x_2(t) \\ &\quad +0.1w_1(t) \\ \varepsilon\dot{x}_2(t) &= -x_1(t) - (1 \pm \Delta R)x_2(t) + u(t) \\ &\quad +0.1w_2(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \end{aligned} \tag{18}$$

For the sake of simplicity, we will use as few rules as possible. Assuming that $|x_1(t)| \le 3$, the nonlinear network system (18) can be approximated by the following two rules TS model:

Plant Rule 1: IF $x_1(t)$ is $M_1(x_1(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= (A_1 + \Delta A_1)x(t) + B_{2_1}u(t) \\ &\quad +B_1 w(t), \quad x(0) = 0 \\ z(t) &= C_1 x(t). \end{aligned}$$

Plant Rule 2: IF $x_1(t)$ is $M_2(x_1(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= (A_2 + \Delta A_2)x(t) + B_{2_2}u(t) \\ &\quad +B_1 w(t), \quad x(0) = 0 \\ z(t) &= C_1 x(t) \end{aligned}$$

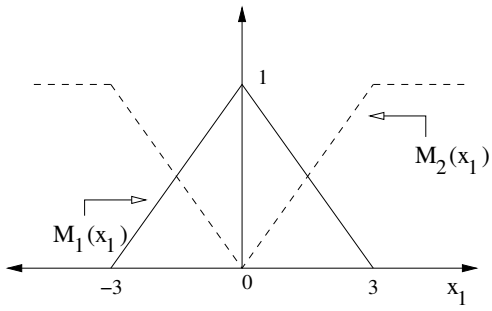

Fig. 2. Membership functions for the two fuzzy set.

where $x(t) = [x_1^T(t) \;\; x_2^T(t)]^T$, $w(t) = [w_1^T(t) \;\; w_2^T(t)]^T$,

$$A_1 = \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix}, \;\; A_2 = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \;\; B_{2_1} = B_{2_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \;\; E_\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix},$$

$$\Delta A_1 = F(x(t),t)H_{1_1}, \;\text{ and }\; \Delta A_2 = F(x(t),t)H_{1_2}.$$

Now, by assuming that $\|F(x(t),t)\| \le \rho = 1$ and since the values of R are uncertain but bounded within 30% of their nominal values given in (17), then we define

$$H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix}$$

$$\text{and } \; H_{2_i} = H_{3_i} = H_{4_i} = H_{5_i} = 0, \quad \text{where } \; i = 1, 2$$

with appropriate dimensions. Fig. 2 shows the plot of the membership function Rules 1 and 2.

Employing results given in [9]-[21] and Matlab LMI solver, it is easy to realize that when $\varepsilon < 0.03$, LMIs become ill-conditioned and Matlab LMI solver yields an error message, "Rank Deficient".

Using the LMI optimization algorithm and Theorem 1 with $\gamma = 1$ and $\delta = 1$, we obtain

$$P = \begin{bmatrix} 0.0943 & 0 \\ -0.7111 & 123.3692 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.0670 & -413.6464 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -0.0447 & -414.8722 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -25.9930 & -3.3529 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -25.8312 & -3.3629 \end{bmatrix}.$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \mu_j K_j x(t) \tag{19}$$

where

$$\mu_1 = M_1(x_1(t)) \;\text{ and }\; \mu_2 = M_2(x_1(t)).$$

Note that Theorem 1 has alleviated the ill-conditioning resulting from the interaction of slow and fast dynamic modes.

Remark 2: *The fuzzy controller guarantees that the $\mathcal{L}_2[0, T_f]$-gain, γ, is less than the prescribed value. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the $\mathcal{H}_\infty$ fuzzy controller with $\varepsilon = 0.01$ is depicted in Fig. 3. The disturbance input signal, $w(t)$, which was used during simulation is given by Fig. 4. Finally, Table I shows the result of the performance index γ of the system with different values of ε. After 5 seconds, the ratio of the regulated output energy to the disturbance input noise energy tends to a constant value which is about 0.032. Thus, $\gamma = \sqrt{0.032} = 0.178$ which is less than the prescribed value 1.* □

Table 1. The performance index γ of the system with different values of ε.

ε	γ
0.01	0.178
0.10	0.279
0.20	0.381
0.30	0.510
0.40	0.679
0.48	0.902
0.49	> 1

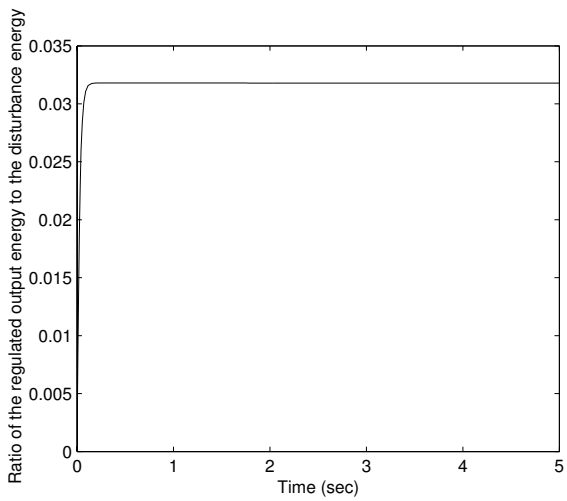

Fig. 3. The ratio of the regulated output energy to the disturbance noise energy: $\frac{\int_0^{T_f} z^T(t)z(t)dt}{\int_0^{T_f} w^T(t)w(t)dt}$.

5. CONCLUSION

This paper has presented an $\mathcal{H}_\infty$ fuzzy state-feedback controller design procedure for a class of uncertain nonlinear descriptor systems described by TS fuzzy model. Based on an LMI approach, we developed a technique for designing an $\mathcal{H}_\infty$ fuzzy controller which guarantees the $\mathcal{L}_2$-gain of the

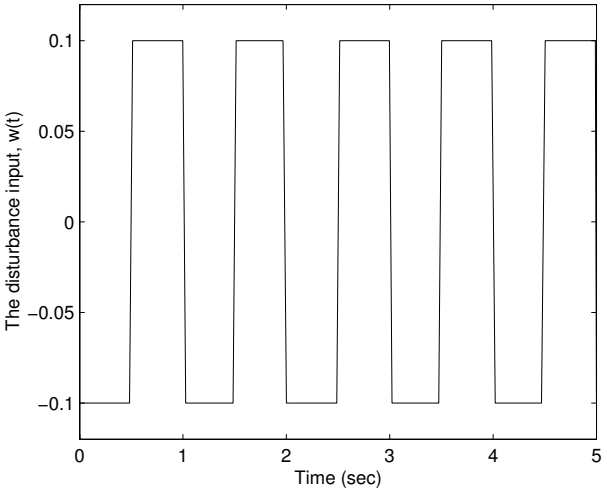

Fig. 4. The disturbance input noise, $w(t)$.

mapping from the exogenous input noise to the regulated output to be less than some prescribed value. To eliminate stiffness difficulties of LMI solvers, solutions to the problem are given in terms of a family of ε-independent linear matrix inequalities. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard, but also to nonstandard nonlinear descriptor systems. A numerical example has been given to show the synthesis procedure developed in this paper.

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