*KGS Spring Conference 2004 / March 25~26, 2004 / Seoul / Korea*

# Experimental Investigation of Wheeler's Hardening Model for Pusan Clays
# 부산점토의 실험결과에 적용된 Wheeler의 경화모형

**M. Suneel[1], 정성교[2], S.G. Chung, K.N. Prasad[3], and K.G. Rao[1]**

**1) 동아대학교 토목공학과 박사과정, Doctorate student, Dept of Civil Engineering, Dong-A University**
**2) 동아대학교 토목공학과 교수, Professor, Dept of Civil Engineering, Dong-A University**
**3) S.V대학교 토목공학과 부교수, Associate Professor, Dept of Civil Engineering, S. V. University, Tirupati, India**

**개요** : 부산점토에 대하여 Wheeler의 경화모형의 적용성을 평가하였다. 이를 위하여 양산지역의 부산점토에 대한 삼축 및 압밀시험 결과가 이용되었으며, 그 모형에 적합한 매개정수들이 결정되었다. 적용결과 예측된 거동은 실험결과와 정량적으로 잘 일치하는 것으로 나타났으나, 향후 압밀 및 장기거동 해석 등을 위하여 더 많은 연구가 요구된다.

**Key words** : anisotropy, natural clay, strength, yielding, modeling

## 1.INTRODUCTION

Accurate description of the yielding of soft clays is essential for safe and economic design of structures on soft clays. The elasto plastic models used in numerical simulations are generally based on models such as Modified Cam clay (Roscoe and Burland, 1968) do not take into account the anisotropy of natural clay deposits. Neglecting the anisotropy of soil behaviour can lead to inaccurate predictions as regards pore pressure development and displacements (Bowey, 1996; Naatanen et al., 1998).

The earliest elasto-plastic models used to describe the constitutive behaviour of soils assumed that the material displayed structural isotropy (Schofield and Wroth, 1968). The effects of the anisotropic structural fabric of natural clays were considered secondary to a fundamental description of the yield behaviour. However the natural formation of clay in horizontal layers of large lateral extent, results in a material displaying an inherent anisotropic structural fabric. A particular fabric arrangement is developed and preferred orientations of particles occur, affecting the stress-strain behaviour in different directions.

Laboratory investigations by many groups of researchers have revealed the significant influence of anisotropic formation stresses and subsequent 'induced' anisotropic stresses on clay stress-strain behaviour. Yield envelopes for many soft clays have been found to be similar in shape, with their location dependent on the composition, anisotropy and stress history of the material (Newson, 1997).

It has been found that the yield envelopes for these soils tend to display more symmetry around the Ko consolidation line in plane stress space (s: t′), rather than axisymmetric stress space (q: p′). However, the development of constitutive models for the description of the yielding behaviour of anisotropic clays has tended to adhere to the notion that symmetry in axisymmetric stress space exists (e.g. Banerjee and Stipho, 1978; Whittle, 1993; Dafalias, 1987). Consequently, the yield envelopes used to describe the yield behaviour of natural clays are not necessarily the most appropriate (particularly at low stress ratios).

The purpose of this paper is to propose a model based on critical state concepts that more accurately describes the yield behaviour of natural clays. In common with the original family of critical state models, the proposed constitutive relationship requires only a minimum number of parameters, which can all be found using standard laboratory tests.

## 2. MODEL FEATURES

This model is extension of the critical state models, with anisotropy of plastic behaviour represented through a rotational component of hardening. The model is applicable to normally consolidated soft soils, where plastic deformations dominate.

### 2.1 Yield and Plasticity

The yield curve is a sheared ellipse, as proposed by Dafalias (1987) and Korhonen & Lojander (1987), defined by

$$f = (q - \alpha p')^2 - (M^2 - \alpha^2)(p'_0 - p')p' = 0 \qquad (1)$$

Where M is the critical state value of stress ratio (where $\eta$ = q/p) and the parameters $p'_0$ and define the size and $\alpha$ the inclination of the yield curve respectively. The parameter $\alpha$ is a measure of the degree of plastic anisotropy of the soil. For the case of isotropy ($\alpha$= 0), Eq (1) reduces to the Modified Cam Clay yield curve.

In the interests of simplicity, an associated flow rule is assumed, and hence:

$$\frac{d\epsilon_d^p}{d\epsilon_v^p} = \frac{2(\eta - \alpha)}{M^2 - \eta^2} \qquad (2)$$

The greatest advantage of assuming an associated flow rule is numerical implementation of the model is far simpler than with a non-associated flow rule. Experimental evidence by Graham et al. (1983) and Korhonen & Lojander (1987) suggests that this assumption is reasonable for many types of soft clays.

The model incorporates two hardening laws. The first one describes changes in size of the yield curve and it is similar to that of Modified Cam Clay.

$$dp'_0 = \frac{vp'_0 d\varepsilon_v^p}{\lambda - \kappa} \qquad (3)$$

The hardening law for change of inclination of the yield curve assumes that plastic volumetric strains, which dominate during anisotropic compression, have the effect of aligning the yield curve about the current stress point, whereas the plastic shear strains, which dominate as the critical state is approached, have the effect of rotating the yield curve back towards an isotropic orientation.

The second hardening rule predicts the change of inclination of the yield curve produced by plastic straining, representing the development or erasure of anisotropy with plastic strains. It is assumed that plastic volumetric strain attempts to drag the value of $\alpha$ towards an instantaneous target value $\chi_v(\eta)$that is dependent on the current value of $\eta$ , whereas plastic shear strain is simultaneously attempting to drag towards a different instantaneous target value $\chi_d(\eta)$(also dependent on $\eta$ ). The rotational hardening law is therefore (Naatanen et al., 1999).

$$d\alpha = \mu[(\chi_v(\eta) - \alpha)d\varepsilon v^p + \beta(\chi_d(\eta) - \alpha)\,|\,d\varepsilon d^p\,|\,] \qquad (4)$$

The overall current target value for will lie between $\chi_v(\eta)$ and $\chi_d(\eta)$. Constants and control, respectively, the absolute rate at which heads towards its current target value and the relative effectiveness of plastic shear strains and plastic volumetric strains in determining the current target value.

### 2.2 Elastic Behaviour

For the stress paths remaining inside the yield curve, the elastic behaviour is likely, in practice, to be anisotropic. At this stage, however, isotropic elastic behaviour is assumed.

$$d\varepsilon_v^e = \frac{\kappa dp'}{vp'}, \qquad d\varepsilon_s^e = \frac{dq}{3G} \qquad (5)$$

## 3. EXPERIMENTAL STUDY

### 3.1 Site description

The Yangsan site is situated in the Southeast of the Korean peninsula and the site is near Pusan. The site consists of the sedimented fine soils near Nakdong River. Some physical properties are summarized in table 1 and figure 1 shows soil profile of site.

**Table 1. Index properties of Pusan clay**

| Class | Properties |
|---|---|
| Moisture content (%) | 40 ~ 62 |

| | |
|---|---|
| Liquid limit (%) | 25 ~ 52 |
| Plastic index (%) | 17 ~ 26 |
| Clay fraction (%) | 63 ~ 75 |

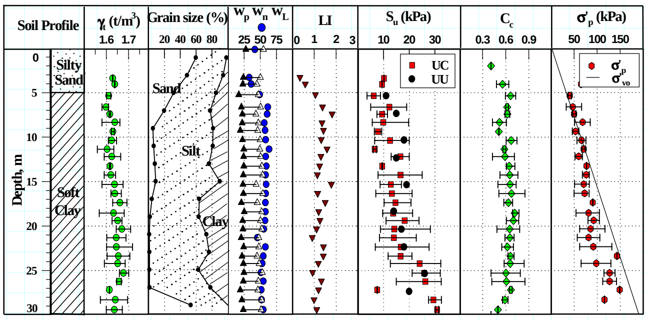

Fig.1. Geotechnical profile of Pusan clay, Yangsan site.

### 3.2 Experiments conducted

The design philosophy of the apparatus was to produce a computer controlled testing facility based on GDS (Geotechnical Digital System). For the present study several undisturbed Thin-wall samples were taken at depths of 21.0 to 21.8m. A site near by was a part of a large geological study (Tanaka, et al., 2001) and comparison of the present data with that study suggests that the main clay mineral in the sediments is illite. The primary objective of the experiments was to observe the yielding behaviour of the Pusan clay. Following onedimensional consolidation of the specimens, Stress path tests were conducted on 50 mm triaxial samples with height of 2D from either normally consolidated or over-consolidated stress states of 10 CIU tests, four anisotropic consolidation tests and $K_0$ Test.

## 4. EVALUATION OF MODEL PARAMETERS

As mentioned earlier the model involves seven soil constants from modified Cam Clay ($\lambda, \kappa, \Gamma, M$ and G) and two additional parameters relating to rotational hardening ($\mu$ and $\beta$). In addition, the initial state of the soil is defined by the stress state and the initial values of the parameters $p_0$ and $\alpha$ defining the initial size and inclination of the yield curve. Values of soil constants $\lambda, \kappa, \Gamma, M$ and G can be measured in laboratory tests using relatively standard procedures. The following section brings out procedures for evaluating the remaining soil constants ($\mu$and $\beta$) and the initial values of the parameters $p_0$ and $\alpha$.

### 4.1 Initial inclination of yield surface($\alpha$)

The inclination of the yield curve resulting from the previous stress and strain history of the soil deposit, should ideally be determined by conducting triaxial stress probes on several identical soil samples along different stress paths, in order to identify a number of points on the yield curve. Unfortunately, in practice, this would often be unfeasible or unduly time-consuming. There is, however, a simpler method of estimating an initial value of , if it can be assumed that the previous history of the soil deposit is restricted to simple one-dimensional ($K_0$) loading, and possible unloading, to a normally consolidated or lightly over consolidated state.

If the normally consolidated value of $K_0$ can be measured or estimated, perhaps by using Jaky's simplified formula ($K_0 = 1-\sin\phi'$ ), this can be used to calculate a corresponding value of stress ratio $k_0$. The model predicts that only one value of yield curve inclination  would produce one dimensional straining for continuous loading at this stress ratio $k_0$, and this therefore provides an estimate for the initial value of $\alpha$.

The Yield curve inclination corresponding to one-dimensional consolidation $\alpha_{k0}$ is given by

$$\alpha_{k0} = \frac{\eta^2{}_{ko} + 3\eta_{ko} - M^2}{3} \qquad (6).$$

Fig.1 shows experimental yield points and theoretical yield curve corresponding to Wheeler's equation, with the value of $\alpha$ calculated from above equation. In calculating $\alpha_{k0}$, a value of $K_0 = 0.47$ was taken from $K_0$ test result and $M = 1.4$ measured in triaxial compression, leading to $\eta_{k0} = 0.82$ and hence $\alpha_{k0} = 0.40$. The resulting inclination of the yield curve agrees reasonably well with the test data, overall, however, the proposed method of estimating an initial value for  appears sufficiently accurate for practical purposes.

## 4.2 Initial size of yield curve (p'$_0$)

The $p'_0$ parameter ideally single yield point would be identified by either isotropic or $K_0$ consolidation in a triaxial apparatus. Alternatively, one-dimensional consolidation in an oedometer would be possible, but this would require either measurement or estimation of radial stress, in order to fully define the stress state. Fig. 1 shows that the initial yield points from the tests on Pusan clay can be fitted by a size of yield curve defined by $p'_0$= 171 kPa. Fig. 2 shows the yield envelopes predicted by Wheeler's model for Osaka clay.

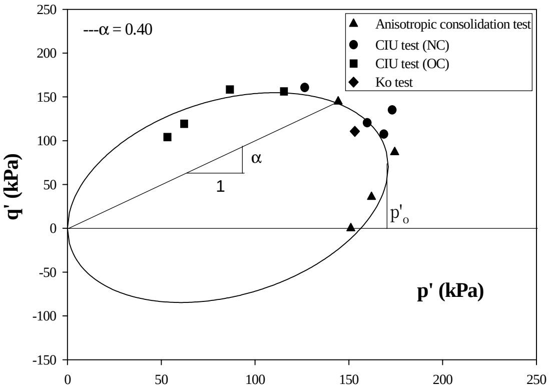

Fig. 2 Initial yield curve for Pusan clay

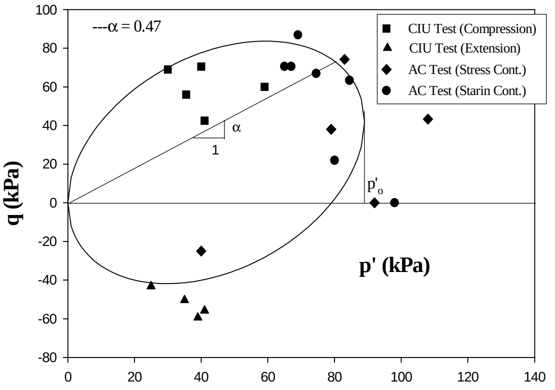

Fig. 3 Initial yield curve for Osaka clay

## 4.3 Soil constant (β)

The model parameter $\beta$ defines the relative effectiveness of plastic shear strains and plastic volumetric strains in rotating the yield curve. For plastic loading along any constant $\beta$ stress path, $\alpha$ will ultimately tend to a final equilibrium value, which can be found by setting $d\alpha=0$ in Eq. (4) and combining with Eq. (2). This leads to quadratic equation for $\alpha$ for a stress path with any constant value of $\eta$ (see Naatanen et al., 1999).

$$\left(\frac{3}{4}\eta-\alpha\right)(M^2-\alpha^2)=-2\beta\left(\frac{1}{3}\eta-\alpha\right)(\eta-\alpha) \tag{7}$$

For loading at the $K_0$ stress ratio $\eta_{K0}$, Eq. (7) shows that only one value of  will result in a value of $\alpha$ corresponding to $\alpha_{K0}$ from Eq. (6). Combining Eqs. (6) And (7) gives the following expression for the required value of $\beta$:

$$\beta=\frac{3\left(M^2-\eta_{ko}^2-\frac{3\eta_{ko}}{4}\right)}{2(\eta_{ko}^2-M^2+2\eta_{ko})} \tag{8}$$

Knowledge of the value of $K_0$ (and hence $\eta_{ko}$) can therefore be used to determine an appropriate value for $\beta$, using Eq. (8). Inserting M = 1.4 and $\alpha_{ko}$= 0.40 in Eq. (8) leads to an estimate of $\beta$ = 2.89 for Pusan clay.

### 4.4 Soil constant (μ)

The model parameter '$\mu$' controls the rate at which '$\alpha$' tends towards its current target value. It is difficult to devise a simple and direct method for experimentally determining the value of $\mu$ for a given soil (Wheeler et al., 1999). The only solution appears to be to conduct model simulations with several different values of $\mu$ and then to compare these simulations with the observed behavior in order to select the most appropriate value for $\mu$. Accordingly, a value of 10 is chosen for Pusan clay.

**Table 2. Wheeler's model parameters**

| Name | $\lambda$ | $\kappa$ | M | $\alpha$ | $\beta$ | $\mu$ | $\upsilon$ |
|---|---|---|---|---|---|---|---|
| Pusan clay | 0.247 | 0.081 | 1.40 | 0.40 | 2.89 | 10 | 0.22 |
| Osaka clay | 0.355 | 0.047 | 1.41 | 0.47 | 1.30 | 80 | 0.22 |

## 5. MODEL PREDICTIONS FOR PUSAN CLAY

Experimental results and model simulations are presented here. The model constants for Pusan clay have been experimentally determined as $\lambda$= 0.247, $\kappa$= 0.081, M = 1.4, $\upsilon$= 0.22, $\beta$ = 2.89 and $\mu$ = 10, with the initial state of the soil represented by $p'_o$= 171kPa and $\alpha$= 0.40. Simulations were performed to find the stress strain response of the soil.

It may be observed that the overall pattern of behavior illustrated in Figures 4-6 is qualitatively consistent with observations of experimental test results. The discrepancies could be ascribed to the degree of anisotropy simulated by rotating hardening model, which may not be truly representing the degree of erasure of fabric anisotropy during shearing.

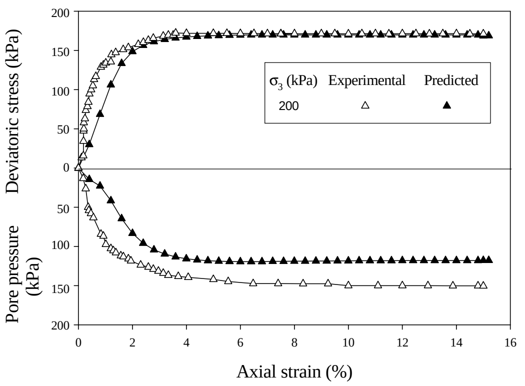

Fig. 4 Deviator stress plotted against strain

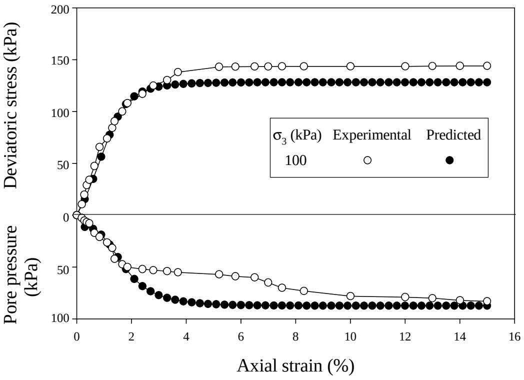

Fig. 5 Deviator stress plotted against strain

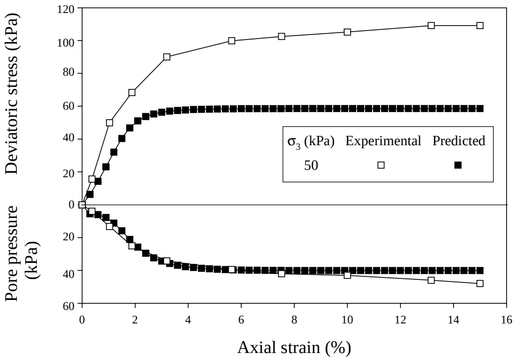

Fig. 6 Deviator stress plotted against strain

## 6. CONCLUDING REMARKS

A programme of triaxial tests on Pusan clay was used to investigate the validity of elasto-plastic constitutive model incorporating a rotational component of hardening. Model predictions are shown to be qualitatively consistent with experimental observations. The discrepancies could be ascribed to the degree of anisotropy simulated by rotating hardening model, which may not be truly representing the degree of erasure of fabric anisotropy during shearing. Further choice of the soil constant which is arbitrary, which should be determined based on appropriate method. In the light of these predictions, refinement of hardening law together with suitable method of determining the soil constant seems to be required to obtain much better agreement between experimental results and model prediction.

## ACKNOWLEDGEMENTS

This work was supported by grant No. R01-2003-000-10375-0 from the Basic Research Program of the Korea Science and Engineering Foundation.

## REFERENCE

1. Allman, M. A. and Atkinson, J.H. (1992). Mechanical properties of reconstituted Bothkennar soil, Geotechnique, Vol. 42 (2), pp. 289~ 301.
2. Atkinson, J. H., Richardson, D. and Robinson, P.J. (1987). Compression and extension of $K_o$ Normally consolidated kaolin clay. Journal of Geotechnical Eng., ASCE, Vol. 113(12), pp. 1468~1484
3. Banerjee, P.K. and Stipho, A.S. (1978). associated and a non-associated constitutive relations for undrained behaviour of isotropic soft clays. Int. Jour. Num. Anal. Meth. Geom, Vol. 2, pp. 35~36.
4. Dafalias, Y.F. (1987). An anisotropic critical state clay plasticity model, Constitutive laws for engineering materials theory and applications, (Eds) C.S. Desai et al., Elsevier Science, pp.513~521.
5. Diaz-Rodriguez, J.A., Leroueil, S. and Aleman, J.D. (1992). Yielding of Mexico City clay and natural clays. Journal of Geotech. Eng., ASCE, Vol. 118 (7), pp. 981~995.
6. Graham, J., Crooks, J.H. and Lau, S.L.K. (1988). Yield envelopes: identification and geometric properties. Geotechnique, Vol.38 (1), pp. 125~134.
7. Korhonen, K.H & Lojander. M. (1987). Yielding of Perno clay. Proc. of the 2nd Int. Conf. on Constitutive laws for Engineering Materials, Tucson, Arizona. Vol.2, pp. 1249~1255.
8. Newson, T.A. 1997. Modelling the yielding behaviour of natural clays. Proc. of the XIV ICSMFE, hamburg, Rotterdam: A. A. Balkema. Vol. 1, pp.381~386.
9. Naatanen, A., Wheeler, S., Karstunen, M and Lojander, M (1999). Experimental investigation of an anisotropic hardening model for soft clays. Proc. of the 2nd Int. Symp. on Pre-failure Deformation Characteristics of Geomaterials, Torino, Italy.
10. Schofield, A.N and Wroth, C.P. (1968). Critical State Soil Mechanics London, Mc Graw-Hill.
11. Tanaka, H., Mishima, O., Tanaka, M., Park, S.Z., Jeong, G.H and Locat, J (2001). Characterization of Yangsan clay, Pusan, Korea. Soil and Foundations Vol. 1, pp.431~434
12. Wheeler, S. (1997). A rotational hardening elasto-plastic model for clays. Proc. of the XIV ICSMFE, Hamburg, Rotterdam, A. A Balkema. Vol. 1, pp. 431~434.
13. Whittle, A.J. (1993). Evaluation of a constitutive model for overconsolidated clay. Geotechnique. Vol. 43, pp.289~313.