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RHC를 이용한 비선형 Backlash 시스템 제어

유 경 상
두원공과대학 전기과

RHC for Nonlinear backlash system control

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Abstract -We present a receding horizon control [RHC] algorithm for compensation of backlash at the input of a stable linear system under control rate constraints. The problem is posed as a receding horizon optimal control [RHOptC] problem for a piecewise affine [PWA] system by modelling the backlash nonlinearity as a PWA system with a state space partition consisting of three regions. The RHC problem involves solving, at each step, 3^N quadratic programmes[QP], where N is the optimization horizon. This strategy leads, at the cost of some performance degradation, to much smaller computational load since a feasible rather than optimal solution has to be obtained at each step.

1. Introduction

Backlash is a common nonlinearity that limits control performance in many industrial applications, Notably mechanical and hydraulic systems. According to the survey paper Nordin and Gutman [1], Few control innovations aimed at this problem have been presented since the early strategies based on describing function analysis [2]. A novel scheme was introduced in Tao and Kokotovic [3], Tao and Kokotovi´c [4] based on adaptive inversion of the backlash nonlinearity. Other nonlinear techniques such as dynamic inversion using neural networks and backstepping have also recently been proposed [5]. More recently, the idea of using the model predictive control [MPC] or receding horizon control suggested in Zabiri and Samyudia [6]. The proposed MPC controller incorporates an inverse model of the backlash function and logic variables are introduced that permit the use of mixed integer quadratic programming for the computations. The resulting system falls into the general class of mixed logical dynamical [MLD] systems introduced by Bemporad and Morari [7]. MLD systems have been shown to be equivalent to piecewise a.ne [PWA] systems in Bemporad et al. [8]. RHC of PWA systems is a subject of current research and several algorithms have been proposed in recent literature [9,10]. A key issue in controlling these systems is the inherent computational complexity of controller synthesis and analysis [11]. In this paper we consider backlash compensation under the RHC framework. By modelling the backlash nonlinearity as a PWA system with three regions, the receding horizon control [RHC] algorithm with horizon N involves the solution of 3^N quadratic programmes[QP] [12]. To circumvent the complexity issue, we did solve these QPs. The remainder of the paper proceeds as follows. In Section 2 we formulate the MPC problem for backlash compensation under rate constraints. In Section 3 we provide simulations results and finally conclusions are given in Section 4.

2. The receding horizon control problem

We consider the following model of a linear discrete time system with a backlash nonlinearity at the input:

$$\xi_{k+1} = A\xi_k + Bv_k, \quad \xi \in \mathbb{R}^n, v_k \in \mathbb{R}, \qquad (1)$$

$$v_k = B(v_{k-1}, u_k), \quad u_k \in \mathbb{R}. \qquad (2)$$

The backlash nonlinearity is given by

$$B(v_{k-1}, u_k) = \begin{cases} m(u_k - \ell) & \text{if } m(u_k - \ell) \le v_{k-1}, \\ v_{k-1} & \text{if } v_{k-1} + m\ell \le mu_k \le v_{k-1} + mr, \\ m(u_k - r) & \text{if } m(u_k - r) \ge v_{k-1}, \end{cases} \qquad (3)$$

where m > 0, r > 0 and < 0. Figure 1 shows its characteristic. We assume that the eigenvalues of A in (1) are inside the unit circle. The backlash function (3) can be represented as a PWA system with state zk = vk.1 and dynamics given by

$$z_{k+1} = A_i z_k + B_i u_k + G_i, \qquad (4)$$

$$v_k = C_i z_k + D_i u_k + E_i, \qquad (5)$$

$$\text{if} \quad (u_k, z_k) \in R_i \triangleq \{(u, z) : L_i u + J_i z \le W_i\}, \qquad (6)$$

for i = *1, 2, 3, where*

$$A_1 = 0, B_1 = m, G_1 = -m\ell,$$
$$L_1 = m, J_1 = -1, W_1 = m\ell$$
$$A_2 = 1, B_2 = 0, G_2 = 0,$$
$$L_2 = m[1 \ -1]^T, J_2 = [-1 \ 1]^T, W_2 = m[r \ -\ell]^T$$

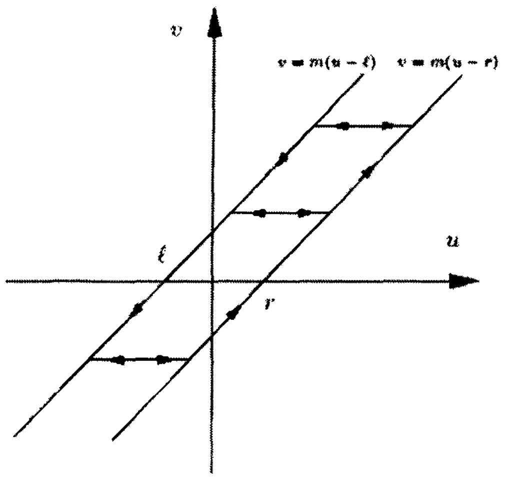

Fig. 1. Backlash characteristic.

$A_3 = 0,\ B_3 = m,\ G_3 = -mr,$
$L_3 = -m,\ J_3 = 1,\ W_1 = -mr,$
and $C_i = A_i$, $D_i = B_i$, $E_i = G_i$ for $i = 1, 2, 3$.

We now define x_k z_k ξ_{kt}, and combine (1) and (4).(6) into a single nonlinear (PWA) equation $x_{k+1} = f(x_k, u_k)$. As the base for the RHC design, we pose, at time k and for the current state $x_k = x$ and the previous input $u_{k-1} = u$, the following finite-horizon optimisation problem:

$$V_N^{OPT}(x, u) \triangleq \min V_N(x_j, u_j), \qquad (7)$$

subject to:

$$x_{j+1} = f(x_j, u_j) \quad \text{for } j = 0, \ldots, N-1, \qquad (8)$$
$$x_0 = x, \qquad (9)$$
$$|u_j - u_{j-1}| \le \Delta \quad \text{for } j = 0, \ldots, N-1, \qquad (10)$$
$$u_{-1} = u, \qquad (11)$$
$$[10\ldots0]x_N = 0. \qquad (12)$$

where

$$V_N(x_j, u_j) \triangleq x_N^T P x_N + \sum_{j=0}^{N-1}(x_j^T Q x_j + u_j^T R u_j). \qquad (13)$$

$$P = \begin{bmatrix} 0 & 0 \\ 0 & \bar{P} \end{bmatrix},\ \bar{P} = A^T \bar{P} A + Q. \qquad (14)$$

Problem (7).(14) is the minimisation of the quadratic objective function (13).(14) for the PWA system (8) under rate constraints (10) and a terminal state constraint (12). Its solution can be found by solving 3^N QPs, which correspond to all the possibilities $(u_j, z_j) \in R_i$ for $i = 1, 2, 3$ and $j = 0, 1, \ldots, N-1$. Some simplifications are possible in certain cases. For example, if the rate limit $\Delta > 0$ is greater than the backlash "deadzone" $r-l$, then we can impose the condition that $(u_j, z_j) \notin R_2$ for $j = 0, 1, \ldots, N-1$, resulting in only 2^N QPs to solve. Also, if we impose the condition $(u_{N-1}, z_{N-1}) \notin R_2$, which we assume, then the terminal state constraint (12) takes the form

$$u_{N-1} = \begin{cases} \ell & \text{if } (u_{N-1}, z_{N-1}) \in R_1, \\ r & \text{if } (u_{N-1}, z_{N-1}) \in R_3. \end{cases} \qquad (15)$$

Equation (15) can be substituted in (7).(14) to obtain QPs having N-1 decision variables. Let $N_{qp} \le 3^N$ be the number of QPs to solve. Each of the QPs has the form

$$\min \mathbf{u}^T H \mathbf{u} + 2\mathbf{u}^T(Fx + a) + b, \qquad (16)$$

subject to:

$$L\mathbf{u} + J\begin{bmatrix} u \\ x \end{bmatrix} \le W, \qquad (17)$$

where $\mathbf{u} = [u_0, \ldots, u_{N-2}]^t \in R^{N-2}$, and H,F, a, b, L, J and W change with each of the N_{qp} possibilities. The vector a and the scalar b are independent of **u**, but a depends on u_{N-1} and b on x and u_{N-1}. Note that b does not affect the minimiser of (16).(17) but it affects the optimal value and hence has to be considered in the evaluation.

Once the Nqp QPs of the form (16).(17) have been solved, the optimal solution to problem (7).(14) is computed as the minimum of the QPs. Let the minimiser be $\mathbf{u}^{OPT} = [u_0^{OPT}, \ldots, u_{N-2}^{OPT}]$. Then the MPC strategy applies the first element of this vector, that is, $u_k = u_{opt}^0$. Time is then stepped forward and the whole procedure is repeated at the next time instant. The configuration for MPC is depicted in Figure 2.

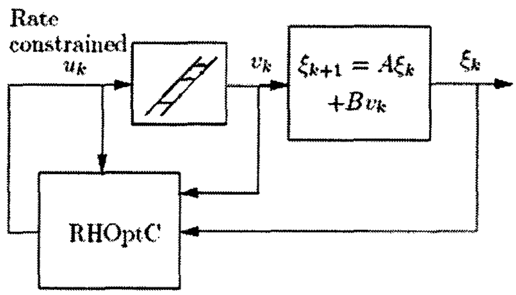

Fig. 2. Configuration for RHC

3. Simulation results

Consider the linear system (1) with matrices

$$A = \begin{bmatrix} 1.70 & -0.72 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and output $y_k = [1\ 0.2]\ \xi_k$. The parameters of the backlash function in (3) are $m = 1$, $r = 0.3$, $l = -0.3$. The rate constraint in (10) is $\Delta = 1$and the matrices in the objective function (13) are selected as in (14) with Q = I and R = 0.01. We first designed a MPC computed for the linear system only under rate constraints, that is, without backlash compensation. Figure 3 shows the resulting output and input responses when the is no backlash in the loop. The same controller was simulated after introducing backlash in the loop as in Figure 2. The resulting output response and the signals at the input and output of the backlash nonlinearity are plotted in Figure 4. We can see that the presence of backlash introduces oscillations in the responses. Secondly, we simulated the closed loop system of Figure 2 under MPC with backlash compensation as described in Section 2. The resulting output response and the signals at the input and output of the backlash nonlinearity are plotted in Figure 5. We can see that the optimal controller compensates the backlash oscillation effect while maintaining the performance close to that without backlash (Figure 3). The resulting output response and the signals at the

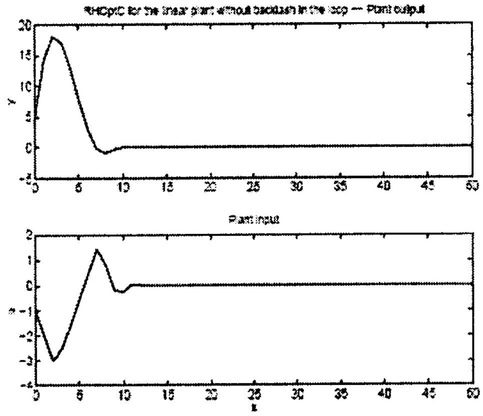

Fig. 3. Linear system output (top) and input (bottom) for RHC for the linear system without backlash in the loop.

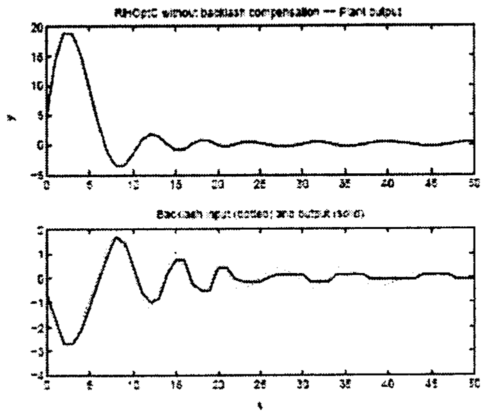

Fig. 4. Linear system output (top) and backlash input and output (bottom) for RHC for the linear system without backlash compensation.

4. Conclusion

We have presented an algorithm for backlash compensation at the input of a stable linear system under rate constraints. The algorithm is based on the traditional QPs arising in the receding horizon optimal control problem for the same system. Simulation examples have shown that performance degradation is small with respect to the optimal solution. In addition, the computational load is smaller since a feasible rather than optimal solution has to be obtained at each step.

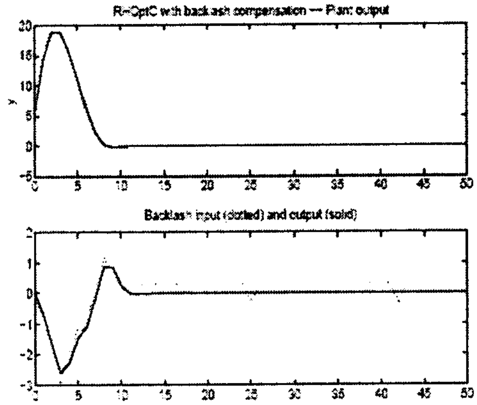

Fig. 5. Linear system output (top) and backlash input and output (bottom) for RHC with backlash compensation.

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