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Robust Control of the Robotic Systems Using Self Recurrent Wavelet Neural Network via Backstepping Design Technique

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벡스테핑 기법 기반 자기 회귀 웨이블릿 신경 회로망을 이용한 로봇 시스템의 강인 제어

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Abstract - This paper presents the tracking control method of robotic systems with uncertainties using self recurrent wavelet neural network (SRWNN) via the backstepping design technique. The SRWNN is used as the uncertainty observer of the robotic systems. The adaptation laws for weights of the robotic systems are induced from the Lyapunov stability theorem, which are used for on-line controlling robotic systems. Computer simulations of a three-link robot manipulator with uncertainties verify the validity of the proposed SRWNN controller.

1. Introduction

The adaptive backstepping control is a systematic and recursive design methodology for nonlinear feedback control. Unlike the feedback linearization method having the problems such as precise model and the cancelation of some useful nonlinear term, the adaptive backstepping design offers a choice of design tools for accommodation of uncertainties and nonlinearities and can avoid wasteful cancellations[1]. The key idea of the backstepping design is to select recursively some appropriate state variable as virtual input for lower dimension subsystems of the overall system[1].

On the other hand, wavelet neural networks (WNNs) is applied to deal with the nonlinearties and uncertainties of the control system. But, the WNN is a feedforward structure, that is, a static mapping. So, the SRWNN having the powerful dynamic mapping ability is proposed[2]. In this paper, the SRWNN is employed as the uncertainty observer in the backstepping controller and the error compensator is also used to reduce the approximation error of SRWNN. The adaptation law for weights of the uncertainty observer and the error compensator are induced from the Lyapunov stability theorem, which are used to guarantee the asymptotic stability. Finally, the simulation results for three-link manipulators are provided to demonstrates the effectiveness of our control scheme.

2. Preliminaries

2.1 Model of the robot systems with uncertainties

The dynamics of the n-link robotic system with uncertainties can be expressed in the following Lagrange form[3]:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+F(\dot{q})+\Xi(q,\dot{q},\tau)=\tau \quad (1)$$

where

$$\Xi(q,\dot{q},\tau)=-M(q)\overline{M}^{-1}(q)\tau-\tau_d-\overline{C}(q,\dot{q})-\overline{G}(q)-\overline{F}(\dot{q})+\tau-C(q,\dot{q})-G(q)-F(\dot{q})$$

denotes the uncertainty of the robot system, and $M(q)\in\mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q})\dot{q}\in\mathbb{R}^n$ denotes the Coriolis and centrifugal torques, $G(q)\in\mathbb{R}^n$ is the gravity vector, $F(\dot{q})\in\mathbb{R}^n$ represents the fiction term, and the control input torque is $\tau\in\mathbb{R}^n$. Also, $\overline{M}(q)$, $\overline{C}(q,\dot{q})$, $\overline{G}(q)$, and $\overline{F}(\dot{q})$ are the actual values with uncertainties in the nominal values $M(q)$, $C(q,\dot{q})$, $G(q)$, and $F(\dot{q})$, respectively. τ_d is the external disturbance.

In this paper, it is assumed that the nominal values is only known values for a given robot system. That is, suppose th at the actual values $\overline{M}(q)$, $\overline{C}(q,\dot{q})$, $\overline{G}(q)$, and $\overline{F}(\dot{q})$ a nd the external disturbance τ_d are the unknown values. Acc ordingly, the uncertainty term $\Xi(q,\dot{q},\tau)$ cannot be compu ted.

2.2 Self recurrent wavelet neural network

The SRWNN, a modified model of a wavelet neural network(WNN), has the attractive ability such as dynamic attractor, information storage for later use. Unlike a WNN, since the SRWNN has the mother wavelet layer which is composed of self-feedback neurons, mother wavelet nodes of the SRWNN can store the past information of the network[2]. Thus the SRWNN having the simple structure can be used as a better tool to approximate the complex nonlinear systems than a WNN.

3. Adaptive Backstepping Control System using SRWNN

The dynamics (1) is rewritten by using state variables $X_1=q$ and $X_2=\dot{q}$ as follows:

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=M^{-1}(X_1)\tau-C(X_1,X_2)X_2-G(X_1)\\ &\quad -F(X_2)-\Xi(X_1,X_2,\tau)\end{aligned} \quad (2)$$

The control objective is to design an adaptive backstepping control system based on SRWNN for the state vector X_1 to track the reference trajectory vector q_d. Here, we assume that q_d, $\dot{q}_d$, and $\ddot{q}_d$ is the bounded functions of the time. We now design the adaptive controller using SRWNN via backstepping design technique[1] step by step.

Step 1) ***Design the virtual controller*** X_2

For the tracking control of the state X_1, define the tracking error as

$$Z_1(t) = X_1(t) - q_d(t) \tag{3}$$

and its derivative is

$$\dot{Z}_1(t) = \dot{X}_1(t) - \dot{q}_d(t) = v(t) - \dot{q}_d(t) \tag{4}$$

where $v(t) = \dot{X}_1(t)$ is called the virtual control. Then, the stabilizing function $s(t)$ is defined as follows:

$$s(t) = -K_1 Z_1(t) + \dot{q}_d(t) \tag{5}$$

where the K_1 is a positive definite diagonal matrix.
The first Lyapunov function $V_1(t)$ is chosen as

$$V_1(t) = \frac{1}{2} Z_1^T Z_1. \tag{6}$$

Then, its derivative is

$$\begin{aligned} \dot{V}(t) &= Z_1^T \dot{Z}_1 \\ &= Z_1^T(\dot{X}_1(t) - \dot{q}_d(t)) \\ &= Z_1^T(v(t) - s(t) - K_1 Z_1(t)). \end{aligned} \tag{7}$$

Here, if the virtual control $v(t)$ is chosen as the stabilizing function $s(t)$, the Lyapunov stability condition $\dot{V}_1(t) < 0$ is satisfied. Thus, the asymptotic convergence of the position tracking error $Z_1(t)$ can be guaranteed.

Step 2) ***Design the actual controller*** τ ***using SRWNN***

To design the actual controller τ, we define Z_2 as $Z_2 = v(t) - s(t)$. And then, its derivative of the Z_2 is expressed as

$$\begin{aligned} \dot{Z}_2 &= \dot{v}(t) - \dot{s}(t) \\ &= \dot{X}_2(t) - K_1 \dot{Z}_1(t) + \ddot{q}_d(t) \\ &= M^{-1}(X_1)\{\tau - C(X_1, X_2)X_2 - G(X_1) \\ &\quad - F(X_2) - \Xi(X_1, X_2, \tau)\} - K_1 \dot{Z}_1(t) + \ddot{q}_d(t) \end{aligned} \tag{8}$$

where, τ is a function of X_1, X_2, and $Q_d = (q_d, \dot{q}_d, \ddot{q}_d)$ which denotes the reference position, velocity, and acceleration. Accordingly, the uncertainty term can be represented as $\Xi(X_1, X_2, \tau) = \Xi(X_1, X_2, Q_d)$.

To design the backstepping control system using SRWNN, the following Lyapunov function:

$$V_2(Z_1(t), Z_2(t)) = V_1 + \frac{1}{2} Z_2^T Z_2 \tag{9}$$

And its derivative can be derived as follows:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + Z_2^T \dot{Z}_2 \\ &= Z_1^T(Z_2 - K_1 Z_1(t)) + Z_2^T[M^{-1}(X_1)\{\tau \\ &\quad - C(X_1, X_2)X_2 - G(X_1) - F(X_2) \\ &\quad - \Xi(X_1, X_2, \tau)\} - K_1 \dot{Z}_1(t) + \ddot{q}_d(t)] \end{aligned} \tag{10}$$

From (10), if the backstepping control law τ is designed as

$$\begin{aligned} \tau &= C(X_1, X_2)X_2 + G(X_1) + F(X_2) + \Xi(X_1, X_2, Q_d) \\ &\quad + M(X_1)[K_1 \dot{Z}_1(t) - \ddot{q}_d(t) - K_2 Z_2(t) - Z_1(t)], \end{aligned}$$

where K_2 is a positive definite diagonal matrix, from (10), the backstepping control system is the asymptotic stable. However, since the uncertainty term $\Xi(X_1, X_2, Q_d)$ is the unknown value, τ cannot be evaluated. Accordingly, in this paper, we employ the SRWNN to approximate the nonlinear uncertainty term to a sufficient degree of accuracy. The inputs of the SRWNN are the states X_1 and X_2, and its output is $\hat{\Xi}$. Thus the uncertainty term $\Xi(X_1, X_2, Q_d)$ can be described by the optimal SRWNN plus a reconstruction error vector ϵ_1 as follows:

$$\begin{aligned} \Xi(X) &= \Xi^*(X| A^*) + \epsilon_1 \\ &= \hat{\Xi}(X| \hat{A}) + [\Xi^*(X| A^*) - \hat{\Xi}(X| \hat{A})] + \epsilon_1, \end{aligned} \tag{11}$$

where $X = (X_1, X_2)$, $\hat{A} = diag[\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_n]$; $\hat{A}_i$ $(i = 1, 2, \ldots, n)$ is the collections of the estimated weighting parameters of the SRWNN defined in [2], and A^* is the optimal weighting matrix that achieves the minimum reconstruction error. Then, taking the Taylor series expansion of $\Xi^*(X| A^*)$ around $\hat{A}$ and substituting it into (11), (11) can be obtained that

$$\begin{aligned} \Xi(X) &= \Xi^*(X| A^*) + \epsilon_1 \\ &= \hat{\Xi}(X| \hat{A}) + \tilde{A}^T\left[\frac{\partial \hat{\Omega}(X| \hat{A})}{\partial \hat{A}}\right] + \alpha, \end{aligned} \tag{12}$$

where $\tilde{A} = A^* - \hat{A}$, and $\alpha(t, X) = H(A^*, \hat{A}) + \epsilon_1$, here, H is a high-order term.

Assumption 1: It is assumed that the reconstruction term plus high-order term is bounded as

$$\|\alpha(t, X)\| \le \delta_u = \theta_u^T \Lambda_u(t, X)$$

where $\theta_u \in \mathbb{R}^3$ is an unknown vector and $\Lambda_u(t, X) = [1, \|q(t)\|, \|\dot{q}(t)\|]^T$ is a chosen regressor vector.

Then, we propose the backstepping control law using SRWNN as follows:

$$\begin{aligned} \tau &= C(X_1, X_2)X_2 + G(X_1) + F(X_2) + \hat{\Xi}(X| \hat{A}) + \hat{\delta}_u \frac{Z_2}{\|Z_2\|} \\ &\quad + M(X_1)[K_1 \dot{Z}_1(t) - \ddot{q}_d(t) - K_2 Z_2(t) - Z_1(t)], \end{aligned} \tag{13}$$

Theorem 1: *Assume that the robotic system (1) with unknown model uncertainty is controlled by the SRWNN based backstepping control law (13). Then if the tuning parameters of the SRWNN and the error compensator* $\hat{\delta}_u$

are trained by the following adaptation rules:

$$\hat{A}_i = \lambda_{1,i}\left[\frac{\partial \hat{\Omega}_i(X_i|\hat{A}_i)}{\partial \hat{A}_i}\right]Z_{2,i}(t)$$

$$\hat{\theta}_u = \|Z_2(t)\|\lambda_2 \Lambda_u(t,X)$$

where $i=1,\dots,n$, $\lambda_1 = diag[\lambda_{1,1},\lambda_{1,2},\dots,\lambda_{1,n}]$, *and* $\lambda_2 = diag[\lambda_{2,1},\lambda_{2,2},\lambda_{2,3}]$ *are positive tuning gain matrices, The asymptotic stability of the SRWNN based backstepping system can be guaranteed.*

4. Computer Simulations

In this section, we simulate the three-link manipulator to show the effectiveness and feasibility of the proposed backstepping control system. The dynamics of the three-link manipulator given by [3] is used in this paper. To prove the robustness of the uncertainties, we assume that the link masses m_is and the link lengths a_is are uncertain. Table 1 shows the parameters of the robotic system used in this simulation. In addition, the external disturbance $\tau_d = [0.4sin(2t)\ 0.3cos(3t)\ 0.2sin(2t)]$ is assumed to influence the robot as in (1). The used SRWNN is a simple structure composed of one product layer and the initial value of its weights is chosen randomly in the range of [-1,1]. The inaccurate initial tuning parameters of the SRWNNs are trained optimally by the on-line parameter tuning methodology. The reference trajectory $q_d(t) = [0.3cos(1.5t+(\pi/3))\ 0.1cos(1.5t)\ 0.2cos(1.5)t]$ is considered in this simulation. The initial positions are set to $q_1(0) = q_2(0) = q_3(0) = 0$ and the parameters of the proposed control system are chosen as follows:

$$K_1 = diag[150, 200, 180]$$
$$K_2 = diag[50, 50, 50]$$
$$\lambda_1 = diag[0.01, 0.01, 0.01]$$
$$\lambda_2 = diag[0.01, 0.02, 0.01]$$

The tracking results as shown in Fig. 1 indicate that the proposed control method can overcome unknown model uncertainties and external disturbances. Figs. 2 show the tracking errors of the joint 1, 2, and 3. In Fig. 2, we can see that the tracking errors which occur at the starting point converge to zero in less than a few seconds.

5. Conclusions

In this paper, we has developed the SRWNN based backstepping control system for robotic systems with unknown uncertainties. The SRWNN having simple structures have been used to approximate the unknown uncertainties. The adaptation laws for training weights of the SRWNNs and its error compensator have been induced from the Lyapunov stability theorem, which have been used to guarantee the asymptotic stability of our control system. Finally, the simulation has been performed to show that the or control system is applied for robotic systems without the knowledge of uncertainty.

Table 1. The simulation parameters

	Mass (m_i, kg)		Link(a_i, m)		M.o. Inertia (I_{oi}, kgm^2)
	Nominal	Actual	Nominal	Actual	
Joint 1	1	3.5	0.5	0.6	43.33×10^{-3}
Joint 2	0.7	2.3	0.4	0.5	25.08×10^{-3}
Joint 3	1.4	4.6	0.3	0.4	32.67×10^{-3}

[참 고 문 헌]

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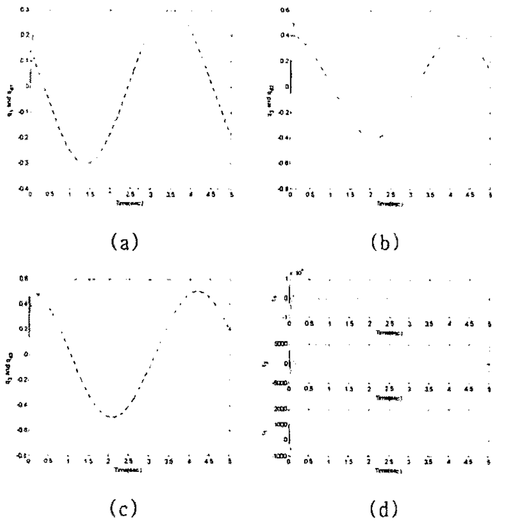

Figure 1. Tracking results and control inputs. (a) Joint 1, (b) Joint 2, (c) Joint 3, (d) control input (solid line: actual output, dotted line: reference output)

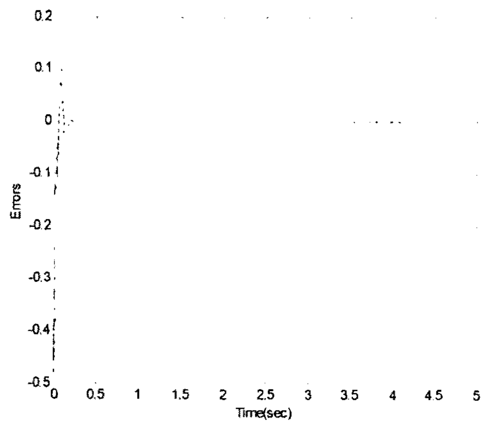

Figure 2. Control errors. (solid line: Joint 1, dotted line: Joint 2, dash-dotted line: joint 3)