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MUSIC 알고리즘에 의한 레이더 반사단면적 계산법에 관한 연구

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A Study on Calculation of RCS Using MUSIC Algorithm

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요 약 : 레이더 물표에 대한 반사성능은 RCS(Radar Cross Section)에 의하여 결정된다. 형상이 비교적 간단한 물체에 대해서는 고유함수 접근법에 의하여 RCS를 정확하게 예측할 수 있으나, 해상물표와 같은 복잡한 물표에 대해서는 저주파 및 고주파 산란해석 기법 등을 이용하여 근사적인 해를 찾을 수 밖에 없고 계산적으로도 복잡하다. 본 연구에서는 수신신호의 파라미터를 정도 높게 추정할 수 있는 MUSIC 알고리즘을 이용하여 RCS 값을 근사적으로 구할 수 있는 알고리즘을 제안한다. 제안된 방법에서는 레이더 물표는 산란체의 링으로서 가정되고 레이더 물표로부터 반사된 신호들의 진폭은 통계적 성질을 지니고 분포하게 된다. 결과적으로 제안된 레이더 신호모델에 MUSIC 알고리즘이 적용되고, 레이더 물표의 RCS는 간단한 대수적인 방법으로 계산된다.

핵심용어 : 레이더 반사 단면적, 초고분해 알고리즘, 뮤직, 레이더 신호처리, 레이더 물표

ABSTRACT : *The detectability of radar depends on RCS(radar cross section). The RCS for complex radar targets may be only approximately calculated by using low-frequency or high-frequency scattering methods, while the RCS for simple radar targets can be exactly obtained by applying an eigen-function method. However, the conventional methods for calculation of RCS are computationally complex. We propose an approximation method for RCS calculation by MUSIC algorithm. In this research, it is assumed that the radar target is considered as a ring of scatterers. The amplitudes of scatterers may be statistically distributed. As the result, the radar signal model is proposed to use MUSIC, and the RCS is calculated by a simple linear algebraic method.*

KEY WORDS : *radar cross section, superresolution algorithm, MUSIC, radar signal processing, radar target*

1. Introduction

The multiple signal classification (MUSIC) algorithm has been well known as superresolution technique for estimation of spatial parameters of receive signal in array signal processing(R. O. Schmidt, 1986). It estimates the DOA(direction of arrival) of the incoming signal and analyses the distribution of the signal power. Especially, the MUSIC has been effectively used to enhance performance of a radar signal processor(Hamid Krim and Mats Viberg, 1996). In radar system, the detectability of a target depends on the RCS(radar cross section) which denotes strength or weakness of the target scatterers(M. I. Skolnik, 2001).

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In general, when the radar target is irradiated by the radar transmitted signal, the energy will reflect and scatter to all sides of the target. The RCS is an important measure for identification and classification of a radar target. The RCS for complex radar targets may be only approximately calculated by using low-frequency or high-frequency scattering methods, while the RCS for simple radar targets can be exactly obtained by applying an eigen-function method(W.L. Stutzman and G. A. Thiele, 1998). In practice, the conventional methods for calculation of RCS are computationally complex. The MUSIC algorithm for RCS measurement has been proposed by Horiuchi et. al., as shown in (Horiuchi et.al, 1998). However, the technique uses GTD(geometrical theory of diffraction) to obtain reflection coefficients of scatterers, and then the RCS is calculated by

MUSIC.

This paper proposes an algorithm for complete calculation of RCS by MUSIC. In our research, a radar target is assumed as a ring of scatterers which is locally distributed and has statistically distribution. A statistical nature of the scatterers depends on a radar target(Bassem R. Mahafza, 2000). The scattered signals consist of many elementary plane waves. In radar receiver, all the scattered waves are spatially distributed with angular spread. Under this assumption, the radar signal model is established. For the proposed signal model, the MUSIC algorithm is applied to calculate the RCS of the radar target.

The radar signal model and the MUSIC algorithm are described in section 2 and section 3, respectively. In section 4, the method for RCS calculation is suggested by using the result of MUSIC. Finally, we have conclusions in section 5.

2. Radar Signal Model

In radar system, the radar sends a signal toward targets. When the transmitted signal reaches the target, it will reflects back to radar, as shown in Fig. 1.

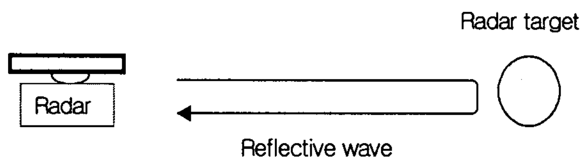

Figure. 1 Reflection of Radar Signal

However, the transmitted signal is scattered at the surface of the target and the reflected signal consists of many scattered waves. When the reflected signal returns radar, the receiving signal will be summation of many scattered elementary waves. Herein, consider the case that the narrowband signal $s(t)$ is transmitted and its scattered waves are received at radar with N antenna elements. The received signals may be modeled as a superposition of a large number of independent and identically distributed (i.i.d.) plane waves, which consist of coherent waves. A statistical nature for amplitudes of the scatterers depends on a radar target. With respect to the received signal, the spatial signature characterizing radio channel is given by the linear combination of the array response vector. Each scattered wave has a different DOA each other, as shown in Fig. 2.

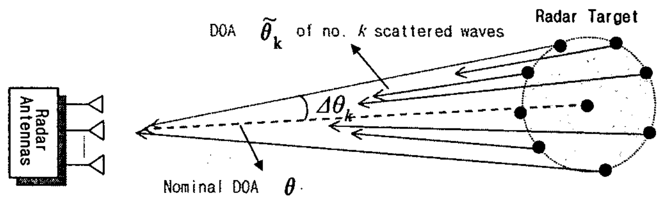

Figure 2. Radar Signal Model

3. RCS Calculation by MUSIC

The received signal vector, $\boldsymbol{x}(t) \in \mathbb{C}^{N\times 1}$, which is observed at array antenna, is modeled by

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \beta_k \boldsymbol{a}(\theta + \tilde{\theta}_k) s(t - \tau_k) + \boldsymbol{n}(t) \tag{1}$$

where

$\boldsymbol{a}(\theta_m) = [1 e^{-j\frac{2\pi}{\lambda} d\cos\theta} \cdots e^{-j\frac{2\pi}{\lambda} d(N-1)\cos\theta}]^T$,

β_k : reflection coefficient with complex amplitude of the k-th scattered signal.

K : total number of scatterers around radar target.

$\boldsymbol{a}(\theta+\tilde{\theta}_k) \in \mathbb{C}^{N\times 1}$: the array response vector with DOA $\theta+\tilde{\theta}_k$.

$\theta+\tilde{\theta}_k$: arrival angle of the k-th scattered signal.

$\tilde{\theta}_k$: the angle deviated from the DOA θ.

τ_k : the time delay of the k-th scattered signal.

$\boldsymbol{n}(t) \in \mathbb{C}^{N\times 1}$: noise vector.

If the time delays of all the scattered signals are negligible in comparison with the reciprocal of the bandwidth of the signal, the approximation $s(t-\tau_k) \approx s(t)$, $k = 1, \cdots, K$ can be applied to Eq. (1), which can be rewritten as

$$\begin{aligned} \boldsymbol{x}(t) &= \sum_{k=1}^{K} \beta_k \boldsymbol{a}(\theta + \tilde{\theta}_k) s(t) + \boldsymbol{n}(t) \\ &= \boldsymbol{v} r(t) + \boldsymbol{n}(t) \end{aligned} \tag{2}$$

where $\boldsymbol{v} \in \mathbb{C}^{N\times 1}$ denotes the spatial signature characterizing the radio channel with relation to $s(t)$, and

$$v = \sum_{k=1}^{K} a(\theta+\tilde{\theta}_k), \tag{3}$$

$$r(t) = \sum_{k=1}^{K} \beta_k s(t). \tag{4}$$

It may be observed from Eq. (4) that the received signal power will be determined by summation of K scattered waves and varied with the instantaneous realization of the spatial signature. Assuming that the noise for each array element is uncorrelated with the signals, spatially and temporally white, and has its variance σ_n^2, the ensemble averaged covariance matrix $\boldsymbol{R} \in \mathbb{C}^{N\times N}$ of $\boldsymbol{x}(t)$ is given by

$$\boldsymbol{R} = E[\boldsymbol{x}(t)\boldsymbol{x}^{\dagger}(t)] = \sigma_s^2 \boldsymbol{v}\boldsymbol{v}^{\dagger} + \sigma_n^2 \boldsymbol{I} \tag{5}$$

where $(\cdot)^{\dagger}$ denotes transpose and complex conjugate operation, $\sigma_s^2 = E[\,|\,r(t)^2\,|\,]$ denotes the average received signal power, and $\boldsymbol{I} \in \mathbb{R}^{N\times N}$ is an identity matrix. From M observed samples, $\boldsymbol{R}$ can be estimated by

$$\hat{\boldsymbol{R}} = \frac{1}{M}\sum_{m=1}^{M} \boldsymbol{x}(t_m)\boldsymbol{x}^{\dagger}(t_m). \tag{6}$$

Here, note that if $M \to \infty$, $\hat{\boldsymbol{R}} \cong \boldsymbol{R}$. Provided that the matrix $\boldsymbol{R}$ is available, the MUSIC algorithm is described as follows. The eigen-decomposition of $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \sum_{i=1}^{N} \lambda_i \boldsymbol{e}_i \boldsymbol{e}_i^{\dagger} = \lambda_1 e_1 e_1^{\dagger} + \sigma_n^2 \boldsymbol{E}_N \boldsymbol{E}_N^{\dagger} \tag{7}$$

where $\boldsymbol{E}_N = [\boldsymbol{e}_2, \cdots, \boldsymbol{e}_N]$, $\lambda_1 > \lambda_2 = \lambda_3 = \cdots = \lambda_N = \sigma_n^2$, and λ_i and $\boldsymbol{e}_i$ are the i-th eigenvalue and the i-th normalized eigenvector of $\boldsymbol{R}$, respectively. The matrix, $\boldsymbol{E} = [e_1\ \boldsymbol{E}_N] \in \mathbb{C}^{N\times N}$, is composed of N column vectors and forms an orthonormal basis, i.e., $\boldsymbol{E}^{\dagger}\boldsymbol{E} = \boldsymbol{I}$. The span of $\boldsymbol{E}_S$ which is known as the signal subspace is orthogonal to the noise subspace spanned by $\boldsymbol{E}_N$. It is clear that the signal subspace is also spanned by $\boldsymbol{v}$. Based on this concept, MUSIC finds the noise subspace spanned by $\boldsymbol{E}_N$. In the absence of noise, MUSIC finds $\boldsymbol{v}$ that has intersection to the signal subspace, and then extracts the DOA. In the presence of noise, however, there exists no spatial signature that maps completely on the signal subspace, or which is orthogonal to the noise subspace. For this case, MUSIC can only find the closest to the signal subspace. Now, the cost function of MUSIC can be defined as

$$f(\theta+\tilde{\theta}_k) = \frac{\|\boldsymbol{E}_N^{\dagger}\boldsymbol{v}\|^2}{\|\boldsymbol{v}\|^2} \tag{8}$$

where $\boldsymbol{E}_N$ denotes the noise subspace, and $|\cdot|^2$ denotes the Euclidean norm. Thus, MUSIC finds $\theta+\tilde{\theta}_k$, which locally minimize $f(\theta+\tilde{\theta}_k)$.

4. RCS Calculation

With the estimated DOA, $\theta+\tilde{\theta}_k$, the spatial signature may be calculated. From Eq. (5), we have

$$\hat{\sigma}_s^2 = (\boldsymbol{R} - \sigma_n^2\boldsymbol{I})(\boldsymbol{v}\boldsymbol{v}^{\dagger})^{-1}. \tag{9}$$

In Eq.(9), the calculated $\hat{\sigma}_s^2$ corresponds to the reflection strength the of a radar target. If the number of radar targets is P, Eq. (2) can be reformulated as follows.

$$\begin{aligned}\boldsymbol{x}(t) &= \sum_{p=1}^{P}\sum_{k=1}^{K} \beta_{pk} \boldsymbol{a}(\theta_p+\tilde{\theta}_{pk}) s_p(t) + \boldsymbol{n}(t) \\ &= \boldsymbol{V}\boldsymbol{r}(t) + \boldsymbol{n}(t)\end{aligned} \tag{10}$$

where $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_P]$, $\boldsymbol{r}(t) = [r_1(t), \cdots r_P(t)]^T$. Therefore, the covariance matrix of $\boldsymbol{x}(t)$ in Eq. (10) may be used to estimate the DOA of targets. Finally, the RCS for all radar targets of interest may be calculated by

$$\boldsymbol{R}_s = (\boldsymbol{V}^{\dagger}\boldsymbol{V})^{-1}\boldsymbol{V}^{\dagger}(\boldsymbol{R} - \sigma_n^2\boldsymbol{I})\,\boldsymbol{V}(\boldsymbol{V}^{\dagger}\boldsymbol{V})^{-1} \tag{11}$$

where $\boldsymbol{R}_s = E[\boldsymbol{r}(t)\boldsymbol{r}(t)^{\dagger}] \in \mathbb{C}^{P\times P}$. The diagonal components of $\boldsymbol{R}_s$ correspond to the received signal powers which are scattered from all targets around radar.

5. Conclusions

RCS is an important measure for identification and classification of a radar target. An accurate prediction of target RCS is critical to design and develop robust target discrimination algorithm. Traditional methods to calculate the RCS require very complex computation. In our approach, MUSIC algorithm for parameter estimation of the received signal was used to calculate RCS. For this, a radar target is assumed as a ring of scatterers, which may be statistically distributed. Under this assumption, a radar signal model is proposed. The spatial signature of the received signal is estimated by using MUSIC.

Finally, we provide the algorithm for RCS calculation by using linear algebraic method. Further study should be done to evaluate the validity of our algorithm and compare the proposed method with conventional techniques for RCS calculation.

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