Comparison of the Numerical, Theoretical, and Empirical Scattering Models for Randomly Rough Surfaces

Jin-Young Hong and Yisok Oh
Department of Radio Science and Communication Engineering, Hongik University,
72-1 Sangsu-Dong, Mapo-Gu, Seoul, Korea. 121-791
nir26@hanmail.net, yisokoh@hongik.ac.kr

Abstract: The scattering problem of the randomly rough surface is examined by the method of moments(MoM), small perturbation method (SPM), integral equation method (IEM) and the semi-empirical polarimetic model. To apply the numerical technique of the MoM to microwave scattering from a rough surface, at first, many independent randomly rough surfaces with a rms height and a correlation length are generated with Gaussian random deviate. Then, an efficient Monte Carlo simulation technique is applied to estimate the scattering coefficients of the surfaces.

1. Introduction

An efficient numerical solution (MoM), theoretical models, and the semi-empirical polarimetric model for the scattering problem of a bare soil surface are presented and examined. The semi-empirical polarimetic model, denoted by 'OSU model'[1] is compared with the numerical solution, measurements as well as the theoretical models such as the small perturbation method (SPM), and the integral equation method (IEM). At first a random rough surface is generated with Gaussian correlation function and an efficient Monte Carlo simulation technique is applied to the estimation of the scattering coefficients. Then the semi-empirical polarimetric model (OSU model), numerical solution, the theoretical models, and measurements are compared each other. The probability density function of the phase differences from the averaged Mueller matrix is also examined. It was known that the theoretical models have limitations to predict the two phase parameters; the degree of correlation and the co-polarized phase-difference. Therefore, the OSU model for the two phase parameters is compared with measurements and the theoretical models.

2. The OSU Model

A semi-empirical polarimetric model had been reported by Oh, Satabandi, and Ulaby in 2002 [1] for microwave backscattering from natural bare soil surfaces. The scattering model, denoted by 'OSU model' in this paper, had been developed based on an extensive database that had been obtained by ground-based scatterometer systems and the JPL airborne synthetic aperture radar (AirSAR) system. The input parameters of the scattering model are the volumetric soil moisture content, the rms height, the correlation length, an incident angle and a radar frequency. The outputs of the scattering model are vv-, hh-, hv-polarized backscattering coefficients and the phase parameters such as the degree of correlation and the co-polarized phase-difference. Therefore, the scattering model can provide the Ensemble-averaged Mueller matrix and the polarization synthesis.

The cross-polarized backscattering coefficient, the co- and cross-polarized ratio p and q of the OSU model had been modeled as functions of the volumetric soil moisture content m_v (cm^3/cm^3), incidence angle θ, the rms height s (m), the correlation length l (m), and wave number k (m^{-1}).

$$\sigma_{vh}^0 = 0.11\, m_v^{0.7} (\cos\theta)^{2.2} \{1-\exp[-0.32(ks)^{1.8}]\} \quad (1)$$

$$p = 1-\left(\frac{\theta}{90^0}\right)^{0.35\, m_v^{-0.65}} \cdot e^{-0.4\,(ks)^{1.4}} \quad (2)$$

$$q = 0.10\left[ks/kl + \sin(1.3\theta)\right]^{1.2}\{1-\exp[-0.9(ks)^{0.8}]\} \quad (3)$$

Then, the vv- and hh-polarized backscattering coefficients can be obtained from above equations.

$$\sigma_{vv}^0 = \sigma_{vh}^0/q, \text{ and } \sigma_{hh}^0 = p\,\sigma_{vv}^0, \quad (4)$$

The probability density function (PDF) of the co-polarized phase angle $\phi_c = \phi_{hh} - \phi_{vv}$ can be completely specified by the degree of correlation α and the co-polarized phase-difference ζ, which are measures of the width and the mean of the PDF, respectively [2]. The OSU model includes the empirical models for the two phase parameters.

$$\alpha = 1-(0.17+0.01\,kl+0.5\,m_v)\cdot(\sin\theta)^{1.1(ks)^{-0.4}} \quad (5)$$

$$\varsigma = (0.44+0.95\,m_v - ks/kl)\,\theta. \quad (6)$$

Then, the ensemble-averaged differential Mueller matrix elements can be computed from the three backscattering coefficients σ_{vv}^0, σ_{hh}^0, σ_{vh}^0 and the two phase-difference parameters α, ς as follows:

$$M_{11}^0 = 1/4\pi \quad \sigma_{vv}^0, \quad (7)$$

$$M_{22}^0 = 1/4\pi \quad \sigma_{hh}^0, \quad (8)$$

$$M_{12}^0 = M_{21}^0 = 1/4\pi \quad \sigma_{vh}^0, \quad (9)$$

$$M_{33}^0 = 1/4\pi \quad \left(\alpha\cos\varsigma\sqrt{\sigma_{vv}^0\,\sigma_{hh}^0} + \sigma_{vh}^0\right) \quad (10)$$

$$M_{44}^0 = 1/4\pi \quad \left(\alpha\cos\varsigma\sqrt{\sigma_{vv}^0\,\sigma_{hh}^0} - \sigma_{vh}^0\right) \quad (11)$$

$$M_{43}^0 = -M_{34}^0 = 1/4\pi \quad \alpha\sin\varsigma\sqrt{\sigma_{vv}^o\,\sigma_{hh}^o}. \quad (12)$$

The degree of correlation and the co-polarized phase-difference, as well as the backscattering coefficients, can be computed from the modified Mueller matrix elements.

$$\alpha = \frac{1}{2}\sqrt{\frac{\left(M_{33}^o + M_{44}^o\right)^2 + \left(M_{34}^o - M_{43}^o\right)^2}{M_{11}^o M_{22}^o}} \tag{13}$$

$$\varsigma = \tan^{-1}\left(\frac{M_{34}^o - M_{43}^o}{M_{33}^o + M_{44}^o}\right) \tag{14}$$

3. The Numerical Method

The numerical method for electromagnetic wave scattering from a randomly rough surface is based on the formulation of an integral equation. The integral equation can be converted into a matrix equations using the MoM. The surface has one-dimensional height profile specified by z=f(x). In two-dimensional scattering problem $\bar{r} = x\hat{x} + z\hat{z}$, the Green's function obeys the following equation.

$$(\nabla^2 + k^2)G(\bar{r},\bar{r}') = -\delta(\bar{r} - \bar{r}') \tag{15}$$

where, $G(\bar{r},\bar{r}') = \frac{i}{4}H_0^{(1)}(k|\bar{r} - \bar{r}'|)$ (16)

Let the spaces above and below the rough surface be denoted by region V_1 and region V_2

$$E_y^i(\bar{r}) + \int \left[E_y(\bar{r}')\frac{\partial G(\bar{r},\bar{r}')}{\partial n'} - G(\bar{r},\bar{r}')\frac{\partial E_y(\bar{r}')}{\partial n'} \right] dl' = \begin{cases} E_y(\bar{r}) & ,\bar{r} \in V_1 \\ 0 & ,\bar{r} \in V_2 \end{cases} \tag{17}$$

Using the pulse basis function and the point matching technique, the integral equation (17) can be cast into the following matrix equation,

$$\begin{bmatrix} [Z_{11}^{mn}] & [Z_{12}^{mn}] \\ [Z_{21}^{mn}] & [Z_{22}^{mn}] \end{bmatrix} \begin{bmatrix} [g_m] \\ [h_m] \end{bmatrix} = \begin{bmatrix} [V_m] \\ 0 \end{bmatrix} \tag{18a}$$

$$E_y = \sum_{m=1}^{N} g_m P_m, \quad \frac{\partial E_y}{\partial n'} = \sum_{m=1}^{N} h_m P_m \tag{18b}$$

$$Z_{11}^{mn} = \int_{\Delta n} \frac{\partial G_{2d1}(\bar{r}_m,\bar{r}_n')}{\partial n'} dl_n', \quad Z_{12}^{mn} = -\int_{\Delta n} G_{2d1}(\bar{r}_m,\bar{r}_n') dl_n' \tag{18c}$$

$$Z_{21}^{mn} = \int_{\Delta n} \frac{\partial G_{2d2}(\bar{r}_m,\bar{r}_n')}{\partial n'} dl_n', \quad Z_{22}^{mn} = -\int_{\Delta n} \varepsilon_r G_{2d2}(\bar{r}_m,\bar{r}_n') dl_n'$$

where P_m represents the mth pulse basis(expansion) function and E_y, $\partial E_y / \partial n'$ are the unknown constants. Once the elements of the impedance matrix and the excitation vector are calculated, the unknown constant can be found by inverting (18a). Consequently, the scattered field can be obtained from

$$E_y^s(\bar{r}) = \int_c [E_y(\bar{r}')\frac{\partial G(\bar{r},\bar{r}')}{\partial n'} - G(\bar{r},\bar{r}')\frac{\partial E_y(\bar{r}')}{\partial n'}] dl' \tag{19}$$

The backscattering coefficient fields are computed using a Monte Carlo simulation by generating 20 independent samples of the randomly rough surface. The randomly rough surface is generated by Gaussian correlation function. Fig. 1 shows the height profile with a Gaussian spectrum having the roughness parameters of ks= 0.14 and kl=2.46, where k is the wave number, s is the rms height, and l is the correlation length.

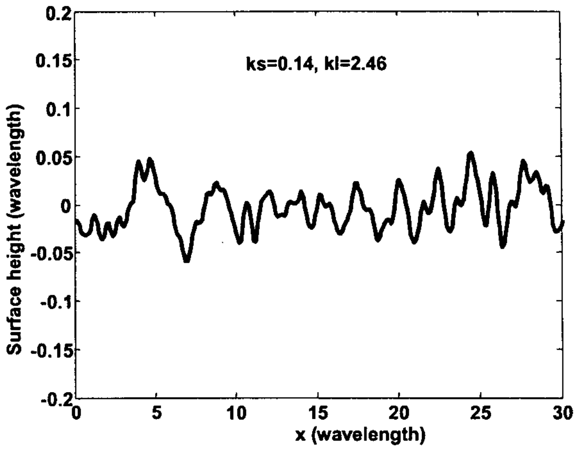


Fig. 1. The height profile with a Gaussian spectrum with ks=0.14, kl=2.46.

4. The Theoretical Models

A couple of theoretical models (the SPM model and the IEM model) for computing the scattering coefficients of randomly rough surface are presented in this section. The formulation of the SPM is obtained as follows [3];

$$\sigma_{pq}^r = 8k^4\sigma_1^2\cos^4\theta|\alpha_{pq}|^2 W(2k\sin\theta,0) \tag{20}$$

where,

$$\alpha_{hh} = R_\perp, \quad \alpha_{vv} = (\varepsilon_r - 1)\frac{\sin^2\theta - \varepsilon_r(1+\sin^2\theta)}{[\varepsilon_r\cos\theta + (\varepsilon_r - \sin^2\theta)^{0.5}]^2}. \tag{21}$$

$W(2k\sin\theta,0)$ is the Fourier transform of the surface correlation function, which is called as the normalized roughness spectrum. The cross-polarized backscattering coefficient is calculated by the 2nd order model.

$$\sigma_{vh}^r = \sigma_{hv}^r = \left[\pi k^4\sigma_1^4\cos^2\theta\frac{|(\varepsilon_r - 1)(R_\parallel - R_\perp)|^2}{2}\right] \times \int\int_{-\infty}^{\infty} \frac{u^2v^2}{|D_0|^2} W(u - k\sin\theta,v)W(u + k\sin\theta,v)dudv, \tag{22}$$

where,

$$D_0 = k_z' + \varepsilon_r k_z, \quad k_z = (k^2 - u^2 - v^2)^{0.5}, \quad k_z' = (k'^2 - u^2 - v^2)^{0.5}. \tag{23}$$

$R_\parallel, R_\perp$ are the Fresnel reflection coefficients for vertical and horizontal polarizations, respectively.

The formulation of the IEM can obtained as the following equations [4]. It is known that two types of terms exist in the scattering coefficients: one representing single scattering and the other multiple scattering.

$$\sigma_{pq}^0 = \sigma_{pq}^s(s) + \sigma_{pq}^M(s) \tag{24}$$

where,

$$\sigma_{pq}^s(s) = \frac{k^2}{2}\exp(-2k_z^2\sigma^2)\sum_{n=1}^{\infty}\sigma^{2n}|I_{pq}^n|^2\frac{W^n(2k_x,0)}{n!} \tag{25}$$

$$\sigma_{pq}^{M}(S)=\frac{k^2}{4\pi}e^{-3k_z^2\sigma^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(2k_z^2\sigma^2)^{n+m}}{n!m!}\int \mathrm{Re}[f_{pq}^{*}F_{pq}(u,v)]$$
$$\cdot W^{n}(u-k_x,v)W^{m}(u+k_x,v)dudv+\frac{k^2}{16\pi}e^{-2k_z^2\sigma^2}$$
$$\cdot\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(2k_z^2\sigma^2)^{n+m}}{n!m!}\cdot\int\left[\left|F_{pq}(u,v)\right|^2+F_{pq}^{*}(u,v)+F_{pq}^{*}(-u,-v)\right]$$
$$\cdot W^{m}(u-k_x,v)\cdot W^{n}(u+k_x,v)dudv \tag{26}$$

with

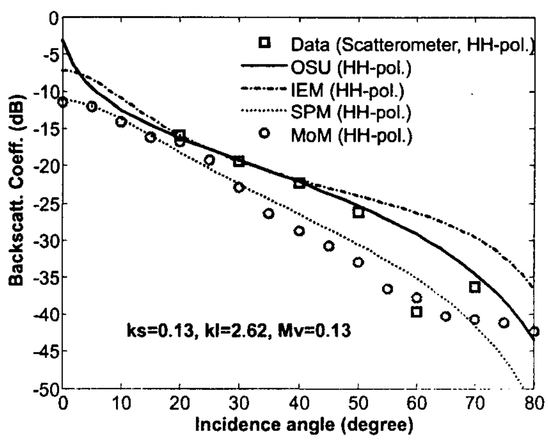
$$I_{pq}^{n}=(2k_z)^2 f_{pa}\exp(-\sigma^2k_z^2)+\frac{k_z^n[F_{pq}(k_x,0)+F_{pq}(-k_x,0)]}{2} \tag{27}$$

Equation (25) represents single scattering because it depends only on one frequency component of the surface roughness spectrum while (26) represents multiple scattering since they show interactions between different frequency components of the surface roughness spectrum through u, v integration. Most natural terrains have a small rms slope. Hence, we expect the single scattering term $\sigma_{pq}^{s}(s)$ dominates over the multiple scattering term $\sigma_{pq}^{M}(s)$ in most situations in polarized scattering calculations. The cross polarized scattering f_{pq} is zero in the backscattering direction and we must evaluate the integrals in (26) numerically.

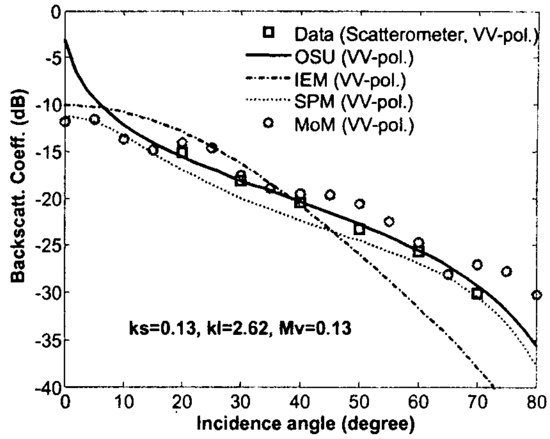
5. Comparison of the models

The backscattering coefficients of the randomly rough surface are compared with field measured data, the OSU model, the MoM and theoretical models such as the SPM and IEM.

Figs. 2 (a) and (b) show comparisons of the field measured data, OSU model, SPM, IEM and MoM for a surface of m_v=0.13, ks=0.13 and kl=2.62 for vv- and hh-polarizations at the validity region of the SPM. Figs. 2 (a) and (b) show that the OSU model and the MoM as well as the theoretical models agree quite well with the measurements for co-polarized backscattering coefficients.



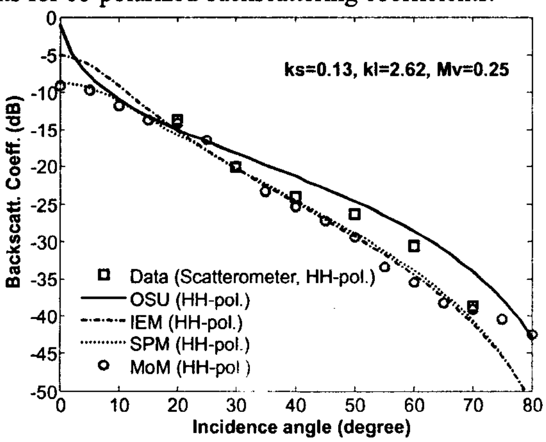
(a)



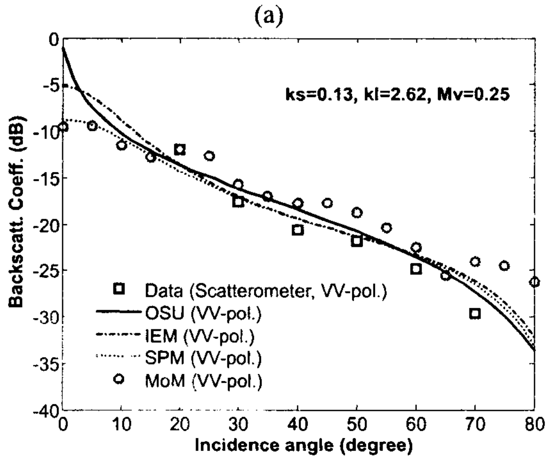
(b)

Fig. 2. Comparisons of the measured data, the OSU model, SPM, IEM and MoM for a surface of m_v=0.13, ks=0.13 and kl=2.62 (a) hh-polarization and (b) vv-polarization at the validity region of the SPM.

Figs. 3 (a) and (b) show comparisons of the field measured data, OSU model, SPM, IEM and MoM for a wet surface of m_v=0.25, ks=0.13 and kl=2.62 for vv- and hh-polarizations at the validity region of the SPM. Figs. 3 (a) and (b) show that the OSU model and the MoM as well as the theoretical models agree quite well with the measurements for co-polarized backscattering coefficients.



(a)



(b)

Fig. 3. Comparisons of the measured data, the OSU model, SPM,

IEM and MoM for a wet surface of m_v=0.25, ks=0.13 and kl=2.62 (a) hh-polarization and (b) vv-polarization at the validity region of the SPM.

Fig. 4 shows comparisons of the measured data, OSU model, and IEM for a surfaces of m_v=0.14, ks=0.127 and kl=2.665 for vv-, hh- and cross-polarizations. Fig. 4 shows that the OSU model as well as the theoretical models agrees quite well with the measurements for vv-and hh-polarized backscattering coefficients, while the theoretical models predict lower values for cross-polarized backscattering coefficients.

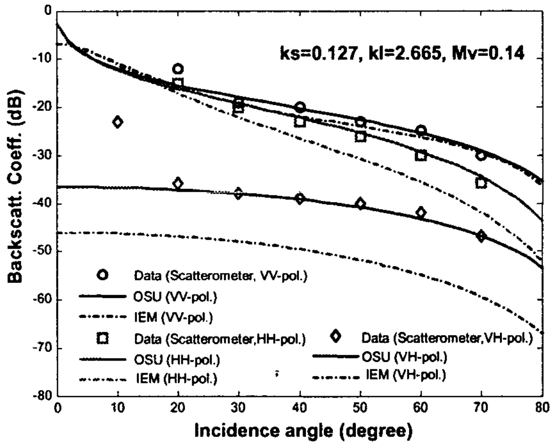


Fig. 4. Comparisons of the measured data, OSU model, and IEM for a surface of m_v=0.14, ks=0.127 and kl=2.665 for vv-, hh- and cross-polarizations.

The degree of correlation is related with the standard deviation of the phase-difference PDF. Therefore, the degree of correlation can be obtained by measurements of many independent surface samples, or by the Monte Carlo simulation with many independent surface samples. Theoretical model gives zero standard deviation; *i.e.*, a delta PDF, for the co-polarized phase difference, which corresponds to α =1. Fig. 5 (a) shows comparison of the degree of correlation for a surface of s=0.94 *cm*, l=6.9 *cm* and m_v =0.09 (cm^3/cm^3) at 1.25 GHz. The measurement usually shows an angular pattern for the degree of correlation, even though the data points are quite scattered. Fig. 5 (b) shows comparisons of the phase difference. It is shown that the SPM model do not agree with the measurements, while the OSU model agrees with the measurements relatively well.

6. Concluding Remarks

An efficient numerical solution (MoM), theoretical models, and the semi-empirical polarimetric model for the scattering problem of a bare soil surface are presented and examined each other. The probability density function of the phase differences from the averaged Mueller matrix is also examined. It was known that the theoretical models have limitations to predict the two phase parameters; the degree of correlation and the co-polarized phase-difference. Therefore, the OSU model for the two phase parameters is compared with measurements and the theoretical models.

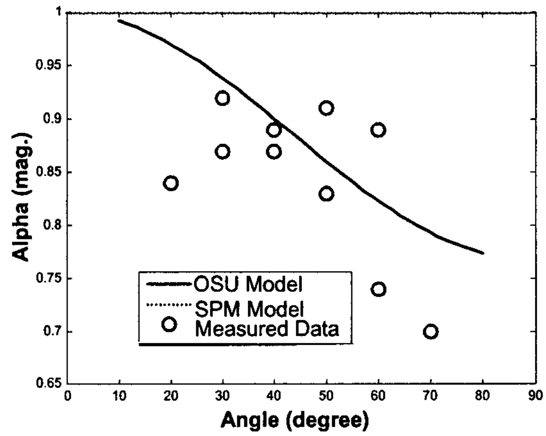
References

[1] Y. Oh, K. Sarabandi, and F. T. Ulaby, "Semi-empirical model of the ensemble-averaged differential Mueller matrix for microwave backscattering from bare soil surfaces", *IEEE Trans. Geosci. Remote Sensing*, vol. 40, pp. 1348-1355, June 2002.

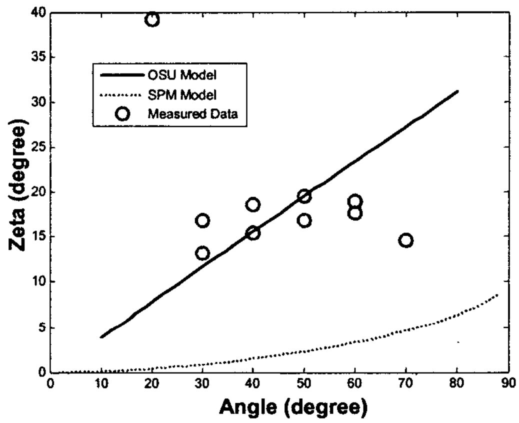
[2] K. Sarabandi, "Derivation of phase statistics of distributed targets from the Mueller matrix," *Radio Sci.*, vol. 27, pp.553-560, Sep.-Oct. 1992.

[3] F.T. Ulaby, M.K. Moore, and A.K. Fung, 1982, *Microwave Remote Sensing, Active and Passive,* vol. 2, Artech House, Norwood, MA, U.S.A.

[4] A.K,Fung. Z.Li and K.S.Chen,"Backscattering from a Randomly Rough Dielectric Surface", *IEEE Trans. Geosci.Remote Sensing*, vol.30, pp. 356-369, March 1992



(a)



(b)

Fig. 5. Comparison of the degree of correlation and the phase difference for a surface of s=0.94 cm, l=6.9 cm and Mv =0.09 at 1.25 GHz. (a)Alpha, (b)Zeta