A VELOCITY-PROFILE FORMULA VALID IN THE INNER REGION OF TURBULENT FLOW

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A turbulent flow on a smooth bed has a very complex velocity profile. It is known that the general profile can be described by the law of the wall in the bed ($y^+ \le 5$, wall region or viscous sublayer) and by the log-law in the outer part of the layer ($30 \le y^+ \le 300 \sim 700$, overlap region). Some studies insisted that the power law is better than the log-law for the overlap region. Though there were severe debates on which formula is better (Osterlund, 1999; Buschmann and Gad-el-Hak, 2003), generally the log-law has been used to describe the overlap region.

The buffer layer, located between the viscous sublayer and the overlap region, shows a more complex velocity profile. To connect two regions, many researchers, including von Karman (1930), van Driest (1955), and Spalding (1961), suggested a little complex formulas. Most of those formulas have complex functional forms or even integrations.

As pointed by Spalding (1966), mathematically the formula should

(i) pass through the point: at $y^+ = 0$, $u^+ = 0$

(ii) be tangential at this point to: $u^+ = y^+$

(iii) be asymptotic at large y^+ to: $u^+ = \frac{1}{\kappa}\ln y^+ + B$, and

(iv) fit the experimental points at intermediate y^+ values.

If we add one more condition, the equation should satisfy the following conditions.

(v) It should have a simple form so that we can get its parameters easily, and

(vi) Its parameters are not too sensitive. In other words, the parameter value should be invariant to Reynolds number and the velocity profile should not change much due to small change of the parameter.

A new single formula for the inner region was propsed as

$$u^+ = \left(\frac{1}{\kappa}\ln y^+ + B\right)\left\{1 - \exp\left(-D y^+\right)\right\} \tag{1}$$

where $u^+(\equiv u/u^+)$ is the friction velocity, κ is the von Karman constant, $y^+(\equiv yu^*/\nu)$ is the wall unit, B is the integration constant, and D is the damping factor.

Eq. (1) satisfied all the above conditions. It approximates the law of the wall in the vicinity of the wall, and the log-law for larger y^+ values. Moreover, it has a simple form. Using Laufer (1953) and Osterlund (1999)'s data, it was revealed that Eq. (1) fitted the experimental data well, as shown at Figure 1. The damping factor has an average of

$D = 0.14$. The velocity profiles of the formula are not too sensitive to small change of the damping factor. Moreover, this parameter is insensitive to the change of the Reynolds number.

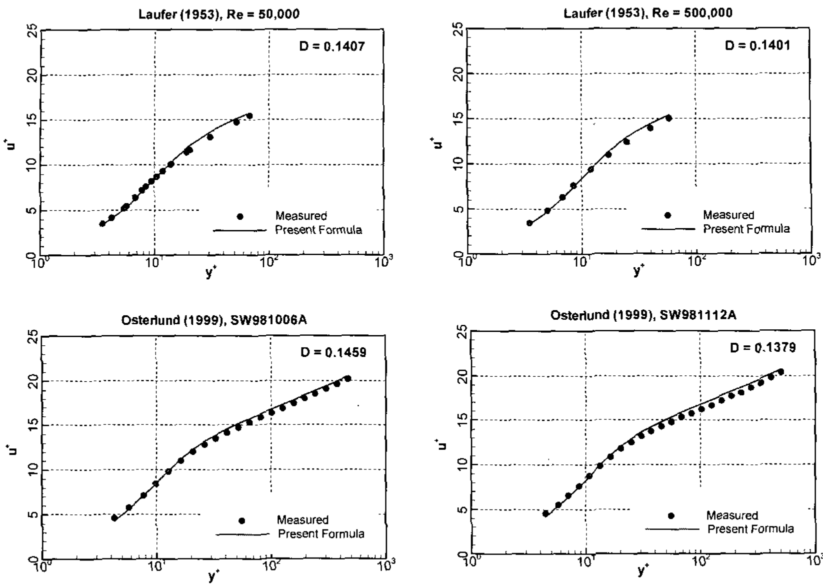

Fig. 1 Results of the present formula

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