STEADY FREE SURFACE PROFILE OF FLOWS WITH AIR-CORE VORTEX AT VERTICAL INTAKE

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Air-core vortex formation at intakes is a significant hydraulic engineering problem in many situations. It occurs typically whenever the submergence is less than a critical value and causes some detrimental effects such as reduction in intake discharge, resulting vibrations and noises as well as operational difficulties.

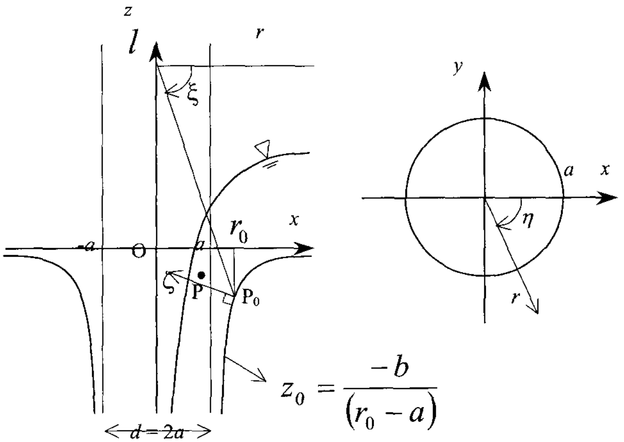

Fig. 1 Definition of coordinate components

Many analytical approaches have been presented in the literature in order to attain a theoretical view of the far-field velocity; the flow representation has not been defined so far by any comprehensive analytical analysis. The concept of simple Rankine vortex normally used in the basic equations [1,2] could not be applied for the case of air-entraining vortex. Trivellato et al [3] set the water surface equal to the stationary headwater while other experimental works only focus on the critical submergence. Consequently, these approaches could not be used to predict the water surface profile of flow with air core vortex.

In this paper, the water surface profile of a steady air core vortex flow into a vertical intake is derived through out a depth-averaged model of open channel flows over the 3-D curvilinear bottom plane using a generalized and body fitted coordinate system (ξ, η, ζ) defined as in Fig. 1. The depth-averaged equations are obtained by integrating the Navier-Stokes equations in a fixed curvilinear coordinate system [4] with respect to the axis normal to the bottom plane applying shallow water assumption and kinetic boundary condition at water surface. The assumption of fully free air-core vortex in the new coordinate system allows us to use the Kelvin's theorem of the conservation of circulation for the whole flow field and the vortex is assumed axisymmetric and steady. After some manipulations the equation described the water surface profile was derived as:

$$\frac{\partial h}{\partial r_0} = -\frac{f_1(h, r_0)}{f_2(h, r_0)} \tag{1}$$

in which

$$\begin{aligned} f_1(h, r_0) = &\left\{ gb^2 \frac{r_0^2 (r_0 - a)}{\left[(r_0 - a)^4 + b^2\right]^{3/2}} - b\frac{\Gamma^2}{8\pi^2} \frac{3\left[(r_0 - a)^4 + b^2\right] + 2r_0 (r_0 - a)^3}{r_0^2 \left[(r_0 - a)^4 + b^2\right]^{3/2}} \right\} h^4 \\ &+ \left\{ \mathrm{gb} \frac{r_0^2}{(r_0 - a)^2} - \frac{\Gamma^2}{4\pi^2} \frac{1}{r_0} \right\} h^3 + \left\{ Q_0^2 b \frac{2(r_0 - a)^3}{r_0 \left[(r_0 - a)^4 + b^2\right]^{3/2}} + Q_0^2 b \frac{3(r_0 - a)^2 \left[(r_0 - a)^4 - b^2\right]}{\left[(r_0 - a)^4 + b^2\right]^{5/2}} \right\} h^2 \\ &- \frac{Q_0^2}{r_0} h - f.Q_0^2 \frac{\left[(r_0 - a)^4 + b^2\right]^{1/2}}{(r_0 - a)^2} \end{aligned} \tag{2}$$

and

$$f_2(h, r_0) = \frac{r_0^2 (r_0 - a)^2}{\left[(r_0 - a)^4 + b^2\right]^{1/2}} gh^3 + \frac{\Gamma^2}{4\pi^2} \frac{1}{r_0 \left[(r_0 - a)^4 + b^2\right]^{1/2}} bh^3 - Q_0^2 \tag{3}$$

where a, b are shape parameter of the bottom plane, r_0 is the distance from projection of a point in bottom plane to the center of intake, Γ is circulation, Q_0 is drainage discharge.

The water surface profile is then obtained by solving equation (1) using fourth-order Runge-Kutta scheme from the singular point. The calculated results of the herein derived equations were compared with an empirical formula introduced by Odgaard [1] as in Fig 2, with the perfect fit between model and Odgaard's equation is on the 45° line.

The results showed that, the proposed model yields reliable results in predicting the critical submergence of the intake without any limitation of Froude number - a problem that most of existing model cannot escape, and that, the model can be improved to simulate the flow structure of an air-core vortex.

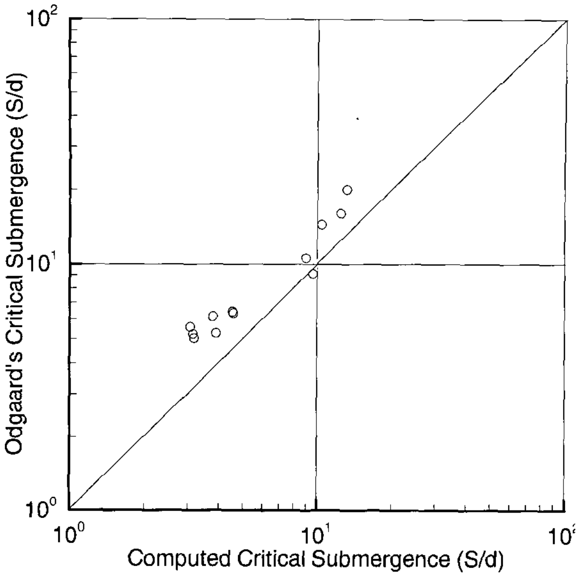

Fig. 2 Comparison of computed critical submergence by model and by Odgaard's equation

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