NONLINER ANALYSIS ON SHALLOW TURBULENT WAKE FLOWS

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Experimental and theoretical researches have shown that high Reynolds number shallow shear flows such as shallow wakes and shallow jets are reminiscent to their counterparts in low Reynolds number unbounded flows (e.g. Chu & Babarutsi 1988 and Chen & Jirka 1995). The large-scale two-dimensional coherent structures in shallow flows are the end-products of hydrodynamic instabilities. Stability analyses are found to produce results that are generally consistent with experimental data (such as Chen & Jirka 1997, Kolyshkin & Ghidaoui 2003). This paper uses a nonlinear shallow water-based model to investigate the growth of instabilities near the transition point and assess the validity of weakly nonlinear analysis. The results support the use of Landau's equation to describe the growth of flow instabilities near the critical conditions. A plot of square of perturbation amplitude A^2 against Stability parameter S is shown in Fig. 1. The relation of $|A| \propto \sqrt{S_c - S}$, which is derived from the Landau equation, is revealed, where S_c is the critical value marking the onset of convective instability in shallow wakes. In addition, the analysis shows that the oscillation frequency in the island wake appears to be governed by the period-doubling mechanism (Fig. 2). The period-doubling bifurcation evidenced in the numerical result marks the transition of wakes from primary instability to secondary instability (Landau & Lifshitz 1959). Moreover, we solve for the wake in two ways: (i) by solving for the flow around a cylinder and (ii) by replacing the cylinder by the velocity at aft of the cylinder which we obtained in (i) (i.e., the velocity is used as an upstream boundary condition). The wake structures obtained with and without cylinder are similar but the oscillation frequency and amplitude are different. The nonlinear model indicates that the vortex shedding regime in the shallow wakes is linked to flow separation. Although this result is preliminary and needs further scrutiny, it is potentially significant since it appears to challenge the well accepted approach that one can investigate the stability of shallow wakes by taking a base flow which implicitly reflects the presence of a blunt body through a velocity defect law, but does not reflect the details of the separation point.

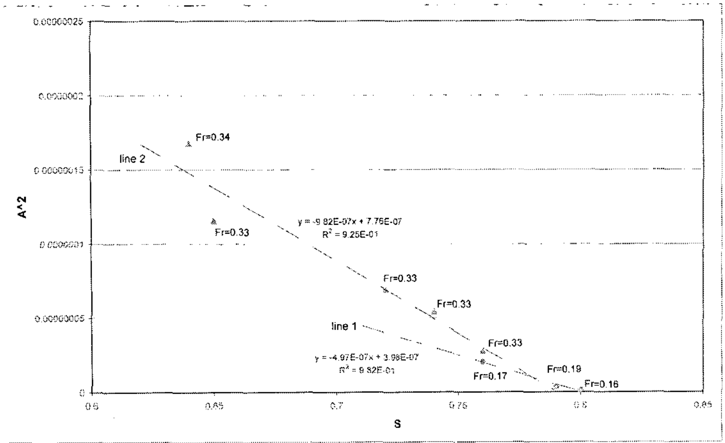

Fig. 1 Shallow wake cases: square of amplitude A^2 against Stability parameter S, for S close to S_c.

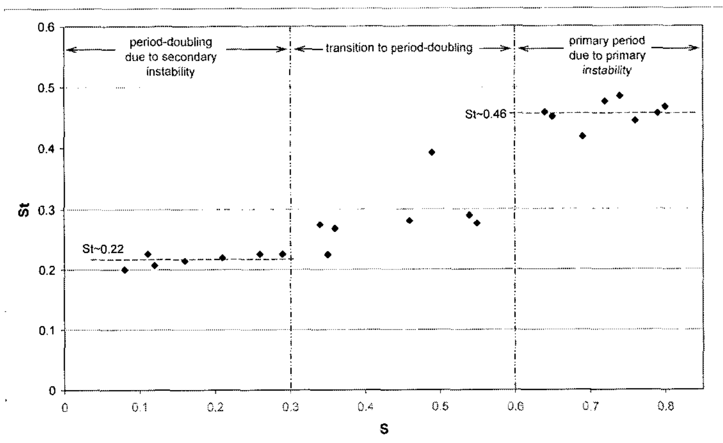

Fig. 2 Cylinder cases: Strouhal number St as a function of Stability parameter S

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