ON THE ENERGY AND MOMENTUM PRINCIPLES IN HYDRAULICS

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In the application of the energy and momentum principles in pipes and channels, standard hydraulic practice is to assume that the velocity is constant across a section and to ignore the fact that there are boundary layers on all solid surfaces. A simple and approximate method of simulating the real flow is to introduce energy (Coriolis) and momentum (Boussinesq) correction factors as fundamental to the process of describing real fluid flow. They are usually presented in textbooks, however, as a brief afterthought rather than as fundamental, and thereafter are almost completely ignored. To quote Shakespeare's *Hamlet*, they are "... a custom more honoured in the breach than the observance".

Most presentations of the energy principle in hydraulics use Bernoulli's theorem, valid along a streamline. In general the Bernoulli "constant" varies across streamlines, a point which is not always emphasised in lectures, books, or understood by students; and, for real fluids it varies along the streamline, but by different amounts for different streamlines. Many textbooks include a consideration of the energy equation in an integral sense, but only in passing. Similarly, the momentum principle has been treated inadequately, although it has essentially always been an integral principle. The neglect of the proper consideration of energy and momentum in practical flow problems means that there can be up to a 5-10% error even in simple flow calculations. It would seem to be important to use the integral form of the conservation of energy principle and the corrected momentum principle in introducing hydraulics to students both pedagogically and practically.

In this paper, generalised energy and momentum coefficients are derived based on integral conservation principles. Their values are larger than previously recognised, due to the inclusion of secondary velocities and turbulence. The traditional and generalised versions are shown in the Table:

$\alpha = \frac{1}{U^3 A}\int_A u^3\, dA$	Traditional Coriolis coefficient
$\bar{\alpha}_1 = \frac{1}{U^3 A}\overline{\int_A \left(u^2+v^2+w^2\right) u\, dA}$	Generalised Coriolis coefficient for calculating mean transport of kinetic energy over a section, using all velocity components and a time average for turbulent contributions
$\beta = \frac{1}{U^2 A}\int_A u^2\, dA$	Traditional Boussinesq coefficient
$\bar{\beta} = \frac{1}{U^2 A}\overline{\int_A u^2\, dA} = \frac{1}{U^2 A}\int_A \left(\bar{u}^2 + \overline{u'^2}\right) dA$	Generalised Boussinesq coefficient with allowance for turbulence

These coefficients can be used in steady and unsteady flows in pipes and channels. The equations for steady flow which result for a control volume with a number of planar faces through which fluid flows are:

$$\text{Mass}: \sum_j Q_j = 0, \quad \text{(Continuity equation)} \tag{1a}$$

$$\text{Momentum}: \sum_j \left(\overline{p}A + \rho\overline{\beta}\frac{Q^2}{A} \right)_j \hat{\mathbf{n}}_j + \mathbf{P} = \rho \mathbf{g} V, \tag{1b}$$

$$\text{Energy}: \sum_j H_j Q_j = \frac{-\dot{E}}{\rho g}, \quad \text{where} \quad H = \frac{p}{\rho g} + z + \frac{\overline{\alpha}_1}{2g}\frac{Q^2}{A^2} \tag{1c}$$

The summations in j are over all parts of the control surface through which fluid passes. Usually the loss term $-\dot{E}$ is assumed to be zero for arbitrary control volumes, but it is possible to calculate this for pipes and channels using the Weisbach equation. Other symbols are: Q is discharge, $\overline{p}$ is the mean pressure, ρ is density, A is cross-sectional area, $\mathbf{g} = (0, 0, -g)$ where g is gravitational acceleration, z is the vertical co-ordinate, directed upwards, $\hat{\mathbf{n}}$ is a unit vector with direction normal to and directed outwards from the control surface, and $\mathbf{P}$ is the force exerted by the fluid on the solid part of the control surface, while V is the volume enclosed. These are all commonly used terms. What is different in the above equations is that H is the *mean* total head across a section, as defined in equation (1c), which contains the energy correction factor $\overline{\alpha}_1$, while $\overline{\beta}$ in equation (1b) is the momentum correction factor. Both factors are defined in the Table above, and corrected for turbulence and the effects of secondary flows.

The energy equation (1c) is derived from integral expressions, and is not Bernoulli's equation, as it calculates the integrated energy flow through parts of the control surface. For theoretical velocity distributions typical values of α are $1.05 - 1.1$, while β varies from about 1.015 to 1.05. However, previously laboratory measurements of α over a smooth concrete bed give 1.035-1.064, while for earth channels, larger values have been found, such as 1.25 for irrigation canals and 1.35 in the Rhine River. Secondary flow velocity contributions to the magnitude of $\overline{\alpha}_1$ might be 0.01 to 0.05, especially downstream of a pipe bend or in a meandering river. The contribution of turbulence to α is roughly 0.01 to 0.02. Generally, the values of both energy and momentum correction factors are larger than hitherto considered.

Finally, several problems from elementary hydraulics are presented using the generalised energy and momentum correction factors. The solution of each is enhanced by including momentum or energy coefficients where appropriate. Routine inclusion of the coefficients in the derivations and in problem solving might be considered for undergraduate teaching. The problems include the force due to a jet of water, the siphon, and the Venturi meter. In the latter, the result obtained using Bernoulli's equation is usually modified by a "velocity coefficient" due to "losses in the energy equation". Using a Coriolis coefficient explains that the coefficient is actually necessary to allow for the variation of velocity across the flow, rather than energy along streamlines, which is not allowed for in the Bernoulli approach.

The conclusions of the paper are that for flows in pipes and channels, it is more physically meaningful to use the integral form of the conservation of energy principle, rather than Bernoulli's theorem, which is valid only along a streamline. Equations for integral momentum and energy principles have been formulated and a simpler expression of the momentum principle has been found. Traditional Coriolis and Boussinesq coefficients have been found to be defective, as they neglect the effects of turbulence and secondary currents. It is asserted that these factors and the integral form of the energy principle should be included both in teaching hydraulics and in applications in hydraulic practice.