MODELLING OF STEADY TRANSCRITICAL FLOW IN OPEN CHANNELS

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Transcritical flow commonly encountered in open channel discharge measuring structures cannot be simulated by most of the common flow models which assume hydrostatic pressure distribution. This assumption restricts the application of the models to flow situations where the curvature of the streamlines is insignificant. In this study, two Boussinesq-type momentum equation models are employed for the numerical simulation of free flow in such types of structures. These models incorporate different degrees of dynamic pressure corrections as a result of the pre-assumed uniform and linear variation of centrifugal term at a vertical section. The effect of the pressure correction factors on the simulation results of the models is also examined.

Fenton (1996) presented a simple method based on the assumption of a constant centrifugal term at a vertical section that includes the possible variation of the channel width. For steady flow in a rectangular channel, this method yields the following equations:

$$\beta\frac{Q^2H}{4A}\frac{d^3H}{dx^3}+\beta\frac{Z_b'Q^2}{2A}\frac{d^2H}{dx^2}+\left(1+Z_b'^2\right)\left(\left(gA-\beta\frac{Q^2B}{A^2}\right)\frac{dH}{dx}+gA\left(Z_b'+S_f\right)\right)$$
$$+\omega_0\beta\frac{HQ^2}{A}\left(\frac{Z_b'''}{2}+\frac{Z_b'Z_b''}{H}\right)-\beta\frac{HQ^2}{A^2}\left(\left(1+Z_b'^2\right)+H\left(\frac{1}{2}\frac{d^2H}{dx^2}+\omega_0Z_b''\right)\right)\frac{dB}{dx}=0, \quad (1)$$

$$p=\rho(\eta-z)\left(g+\frac{\beta Q^2}{A^2\left(1+Z_b'^2\right)}\left(\omega_0Z_b''+\frac{1}{2}\frac{d^2H}{dx^2}\right)\right), \quad (2)$$

in which H is the depth of flow; Z_b', Z_b'' and Z_b''' are the first, second and third derivatives of the bed profile respectively; S_f denotes the friction slope; Q is the discharge; A is flow cross-sectional area; B is the width of the channel; β refers to the Boussinesq coefficient; g is gravitational acceleration; ρ is the density of the fluid; η is the mean elevation of the free surface; z is the vertical coordinate of a point in the flow field; p is the pressure; and ω_0 is a weighting factor. These equations along with the modified version, which are modelled based on the assumption of a linear variation of centrifugal term with depth, are investigated for simulating transcritical flows over curved beds with and without lateral contractions. The modified Boussinesq-type equations read as

$$\frac{Q^2H}{3A}\frac{d^3H}{dx^3}+\frac{Q^2Z_b'}{2A}\frac{d^2H}{dx^2}+\left(gA-\beta\frac{Q^2B}{A^2}\right)\frac{dH}{dx}+gA\left(Z_b'+S_f\right)+\frac{HQ^2}{A}\left(\frac{Z_b'''}{2}+\frac{Z_b'Z_b''}{H}\right) \tag{3}$$

$$-\frac{H^2Q^2}{A^2}\left(\frac{\beta}{H}+Z_b''+\frac{2}{3}\frac{d^2H}{dx^2}\right)\frac{dB}{dx}=0,$$

$$p=\rho\left(g(\eta-z)+\frac{Q^2}{A^2}\left(Z_b''(\eta-z)+\frac{d^2H}{dx^2}\left(\frac{\eta-z}{\eta-Z_b}\right)\left(\frac{1}{2}(\eta+z)-Z_b\right)\right)\right) \tag{4}$$

In order to achieve the objectives of the study, a finite difference discretisation scheme is employed to discretise the nonlinear flow equations, Eqs. (1)-(4), after formulated the flow problems as a boundary value problem. The resulting nonlinear finite difference equations with the specified boundary conditions at the appropriate flow sections are solved using the Newton-Raphson iterative method with a numerical Jacobian matrix. The reliability of the predictions of the models is verified using test data from experiments conducted on flow over trapezoidal profile weirs as well as from the literature.

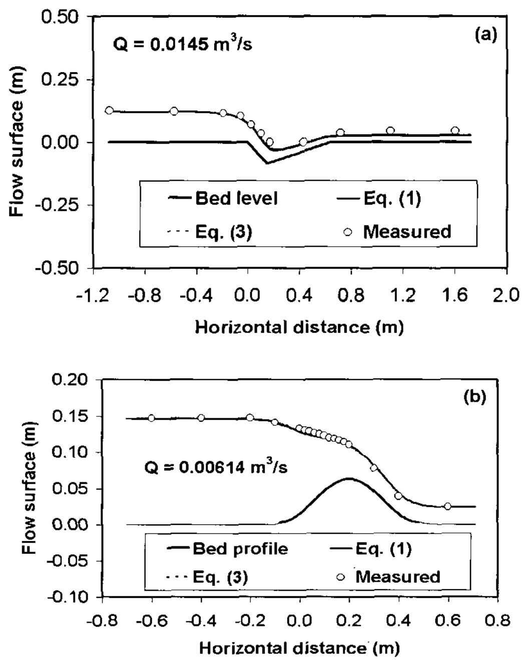

Fig. 1 Comparison of computed and measured results: a) flow in a Parshall flume; b) flow over a curved bed combined with sidewall curvature.

Computed results for flow surface and bed pressure profiles, and pressure distributions at different sections in the solution domain are compared to the experimental data. Good agreement is attained between the computed and measured results. Some of the results are shown in Fig.1. The comparison results suggest that the 2-D flow structures for such type of flow situation are better described by the proposed model which incorporates a higher-order centrifugal correction. This study recommends that higher-order pressure equation should be used when accurate simulation of the pressure profile of a flow with pronounced streamline curvatures is sought.

REFERENCES

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