DEVELOPMENT OF THE UPWIND MCCORMACK SCHEME

WON KIM

Research Fellow, Korea Institute of Construction Technology,
2311, Daehwa-dong, Goyang, Gyunggi-do, 411-712, Korea
(Tel: +82-31-910-0265, Fax: +82-31-910-0251, e-mail: wonkim@kict.re.kr)

The two-step predictor-corrector scheme of McCormack is probably the most widely used scheme among the second-order space-centered explicit schemes, due to its second-order accuracy and simplicity. The upwind scheme is based on the relation established between the characteristic propagation properties and the differencing such as to apply directional space discretizations in accordance with the physical behavior of the flows. In this paper, the upwind McCormack scheme is introduced to combine the advantage of McCormack scheme, the second-order accuracy and simplicity, and the advantage of the upwind scheme, to be applied to the discontinuous flows. This scheme also has another advantage of treating the source terms effectively.

The upwind McCormack scheme developed in this study is based on the normalized Jacobian which is used to discretize the flow and source term. One of the advantages of this scheme is that there is no non-conservative problem. In the case in which the flux Jacobian is used in the channel with source terms, the Equation $\tilde{J}_{i+1/2}\Delta_{i+1/2}U = \Delta_{i+1/2}F$ cannot be satisfied. As well, another advantage of this scheme is the applicability to the discretization of the source terms. Because no transformation of flux is required, it can be simply applied to discretize both the flux and the source terms without any additional transformation of the original terms.

This model, at first, is applied to a dam-break wave. This test corresponds to a dam-break wave in a wide, frictionless and horizontal channel. The total channel length is 2,000m with a dam placed in the middle. The reservoir depth upstream of the dam is initially fixed at 10m. Two cases were examined corresponding to tail-water depths of 5m and 0.005m. In the case of 5m, the flow remains subcritical throughout the channel. The results of the upwind McCormack scheme show no oscillation through the entire domain, but the McCormack scheme shows oscillation around the discontinuous points. In the case of 0.005m, the McCormack scheme failed to produce any results. The upwind McCormack scheme is again comparable to the Beam and Warming scheme except near the advancing front. The upwind McCormack scheme predicts the position of the front accurately. The upwind McCormack scheme was found to be very robust for these calculations, which may be useful in practical applications.

Next application is steady flow over a bump. A 1D steady flow in a 25-m-long channel with a bump is a classical test problem, which has been used as a benchmark test case for numerical methods at the workshop on dam-break wave simulations. The surface profile in the two models is plotted, which shows much better convergence in the case of upwind McCormack than McCormack which shows big oscillations around the discontinuous points. Results demonstrate the final convergence stage in the upwind McCormack scheme, which depicts good agreements with the analytical solution even near the discontinuous points. The McCormack scheme does not converge to this final stage, due to the big oscillations. The upwind McCormack scheme shows no errors around the bump due to the treatment of the source terms using the normalized Jacobian. From the results of calculation of a flow over a bump evidenced in the two models, it can be said that the upwind McCormack scheme proposed in this paper can present higher accuracy and stability than the McCormack scheme.

Another application is a 1D steady flow in a 500-m-long channel with a breadth variation. The upwind McCormack scheme shows much better results than the McCormack scheme in calculating stage and discharge. Unlike the McCormack scheme, they do not converge, lack oscillations and errors, and show good agreement with the analytical solution.

Last application is conducted a series of laboratory experiments involving a dam-break wave propagating in a flume of rectangular cross section but of varying width. In all cases, the agreement of the upwind McCormack scheme with the laboratory measurements is satisfactory. However, the McCormack scheme shows very unstable results.

In this paper, the upwind McCormack scheme is introduced to combine the advantage of the McCormack scheme, the second-order accuracy and simplicity, and the advantage of the upwind scheme, to be applied to the discontinuous flows. This model is approved through applying to the dam-break flow, and the discontinuous flow case with the analytical solution. As a result of the applications, the upwind McCormack scheme proves that it can overcome the instability problem of the McCormack scheme around the discontinuous flow, while still maintaining the advantage of the McCormack scheme. The upwind McCormack scheme, introduced in this paper, has the simplicity in equations, the second-order accuracy, no oscillations, and applicability to the shock wave in the natural rivers. Resultantly, it is supposed to be easily applied to many kinds of flows in rivers.