ON THE BOUNDARY CONDITIONS ASSIGNMENT FOR A FIVE-EQUATION MORPHODYNAMICAL MODEL

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The knowledge of the correct number of boundary conditions which has to be considered to guaranteed the well-posedenes of a morphodynamical model represents a key points for the practical use of the model itself. The hyperbolic nature, and therefore the characteristic theory, allows to trivially individuate the number of boundary conditions, as far as the three and four equations model are concerned. The present paper aims to individuate the mathematical character of the five-equations two-layer model proposed by Capart and Young (2002), which has been successfully applied to simulate morphodynamical processes as antidune formation and bank erosion.

The model by Capart and Young (2002) subdivides the water column into three virtual layers. In the uppermost there is clear water, in the intermediate water-sediment mixture and in the last one wet solid material. The first two layers flows with different non-null velocities, while the third one is a static one.

From the study of eigenvalues of the coefficient matrix of the PDE system expressed in quasi-linear form it has been found that the nature of this model, in contrast with the three and four equations models – which are always hyperbolic – is hyperbolic only for small values of the clear water Froude number.

When Froude number exceeds unity, the problem becomes hybrid for wide ranges of velocity (α_u) and height (α_h) ratios between the two layers. For example, fig. 1 shows eigenvalues map for Frw= 1.5

The loss of hyperbolicity, which gives an ill-posed initial problem according to Hadamard, must be taken into account for concrete application of the model, and especially for what concerns the choice of the numerical method.

Even when hyperbolicity holds, characteristic slopes strongly depends on values of the two parameters α_u and α_h, thus influencing also the number of the conditions to be specified on the boundary of the flow domain.

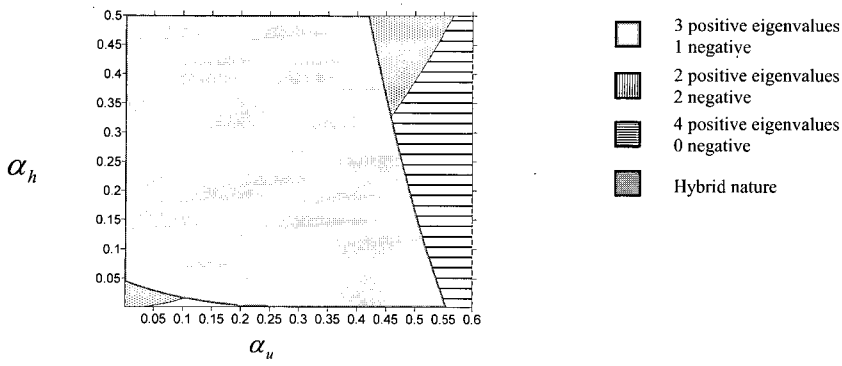

Fig. 1 Eigenvalues map in the (α_u , α_h) plane for Fr_w =1.5.

REFERENCES

Capart, H. and Young, D. (2002) Two-layer shallow water computations of torrential geomorphic flows. Proceedings of Int. Conference "River Flow 2002", Louvain-la-Neuve (Belgium), 3-6 September 2002, Balkema.