1D AND 2D MODELS OF SOLUTE TRANSPORT USING CONSERVATIVE SEMI-LAGRANGIAN TECHNIQUES

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The development of two-dimensional unstructured adaptative meshes has made possible a better resolution of real problems when special features of the fluid flow or complex geometries are present, but this is a challenge for high order numerical methods. An important effort has been done in improving the quality of the methods for convection dominated flows but they are not efficient in presence of discontinuities in unstructured meshes. A conservative semi-lagrangian technique is proposed in this work. Contrary to classical semi-lagrangian schemes which impose the conservation of the characteristic variable along the characteristic curves, in the proposed semi-lagrangian conservative schemes, the equation is solved imposing the propagation of the conserved variable contained in the full cell. After resolving the propagation of the grid cells, to achieve the conserved variable in the physical grid nodes without rating the conservation property of this method a conservative interpolation is performed

Solute transport modelling is a key issue in water quality studies. In order to develop efficient numerical techniques, the transport equation can be simplified assuming zero diffusion. In that case, it becomes a pure advection equation and the simplest hyperbolic example. Numerical methods based on the characteristic form of the equations are called Semi-lagrangian methods. The pure Semi-lagrangian schemes, although accurate, present a considerable error in conservation in presence of variable advection velocity. We propose a new treatment for this schemes in order to conserve the space integral of the advected variable. For this purpose, the semi-lagrangian technique will be applied to the conservative form of the equation instead of the characteristic form.

Contrary to classical semi-lagrangian schemes which impose the conservation of the characteristic variable along the characteristic curves, in the proposed semi-lagrangian conservative schemes, the equation is solved imposing the propagation of the conserved variable contained in the full cell. After resolving the propagation of the grid cells, to achieve the conserved variable in the physical grid nodes without rating the conservation property of this method a conservative interpolation is performed.

The idea will be presented in 1D and 2D spatial approaches using interpolation techniques able to generate high order without oscillations. The results will be shown using test cases with analytical solution.

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