CHUTE SPILLWAY FLOW MODELING USING VARIOUS CARTESIAN FLUX CONVECTIONS TECHNIQUES ON OVERLAPPING UNSTRUCTURED FINITE VOLUMES

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This paper presents the comparison between the numerical results of some techniques for computation of Cartesian convective fluxes on the boundaries of unstructured overlapping control volumes. The accuracy and efficiency of the techniques are evaluated by comparison of the numerical simulation of super-critical free surface flow in channels with non-parallel side walls. The utilized mathematical model for present computations is the set of shallow water equations which uses water depth and velocity components in horizontal plane in the continuity and equations of motion in horizontal plane. The governing equations are discretized utilizing cell vertex finite volume method on triangular meshes. The numerical oscillations of explicit solution procedure are damped out using either artificial viscosity scheme or upwind averaging fluxes at control volume boundary edges. The algorithm of evaluation of the fluxes at edges and artificial dissipation terms at nodes is adopted for unstructured meshes. Using all utilized techniques, no unwanted dissipation is introduced to the computed results and expected shockwaves are simulated accurately. The accuracy of the computational techniques are assessed by comparison between the numerical results and reported experimental measurements for super-critical flow in chute canals with expanded and contracted walls and using with. Finally, the simulation of sub and supercritical flow from dam reservoir over the geometrically complex chute canal of a real case is used to present the ability of the developed flow solver.

Governing equations are shallow water equation. The Shallow-water equations describe unsteady gradually varied flow in open channels. The equations are obtained from integrating the Navier-Stokes equation over the channel depth “Sanders (2001), Gharmy and Steffer (2002)”. Some assumptions to extract shallow water equations are; hydrostatic pressure distribution, incompressible flow (water), distribution of velocity in vertical direction is uniform. These equations are suitable for solving two-dimensional flow over mild slope beds. Shallow water continuity equations equation is:

$$\frac{\partial(h)}{\partial t}+\frac{\partial(hu_i)}{\partial x_i}=0 \qquad i=1,2 \tag{1}$$

And the equations of motion in two Cartesian horizontal *i* direction are:

$$\frac{\partial}{\partial t}(hu_i)+\frac{\partial}{\partial x_j}(hu_iu_j)=-\frac{1}{2}gh\frac{\partial}{\partial x_j}(h+z)-\frac{\tau_{bi}}{\rho_0} \qquad j=1,2 \tag{2}$$

For numerical solution model, consider the set of governing equations in the conservative form can be written as:

$$\frac{\partial Q}{\partial t}+\frac{\partial(u_j Q_i)}{\partial x_j}=gh\frac{\partial H}{\partial x_i}+S \tag{3}$$

Following formulations present various techniques of Cartesian flux convections. The 1st formulation is obtained using Cartesian convection flux averaging, as:

$$Q^{n+1}=Q^{n}-\frac{\Delta t}{\Omega}\sum_{k}^{N}[\overline{F}_1\Delta x_2-\overline{F}_2\Delta x_1]_k+gh\frac{\Delta t}{\Omega}\sum_{k}^{N}[(\overline{H}\Delta x_2)\delta_{i1}-(\overline{H}\Delta x_1)\delta_{i2}]_k-S\Delta t \tag{4}$$

In the 2nd formulation, the flux of conserved variables transport by Cartesian velocity components at boundary edges of Ω, as:

$$Q^{n+1}=Q^{n}-\frac{\Delta t}{\Omega}\sum_{k}^{N}[\overline{u}_1\overline{Q}\Delta x_2-\overline{u}_2\overline{Q}\Delta x_1]_k+gh\frac{\Delta t}{\Omega}\sum_{k}^{N}[(\overline{H}\Delta x_2)\delta_{i1}-(\overline{H}\Delta x_1)\delta_{i2}]_k-S\Delta t \tag{6}$$

The 3rd formulation is:

$$Q^{n+1}=Q^{n}-\frac{\Delta t}{\Omega}\sum_{k}^{N}[(\overline{Q}\,\Delta S\,U_{normal}]_k+gh\frac{\Delta t}{\Omega}\sum_{k}^{N}[(\overline{H}\Delta x_2)\delta_{i1}-(\overline{H}\Delta x_1)\delta_{i2}]_k-S\Delta t \tag{7}$$

Here ΔS is the length and $\overline{Q}$ is average value of conserved variables at boundary edges of Ω and the normal velocity at boundary edge is $U_{normal}=\overline{U}.\breve{h}$.

In order to evaluate the computational results of above mentioned formulations (for Cartesian flux convection), the results of the model are compared with experimental test measurements of the previous workers on super-critical flow in two canals with contracting and expanding width, "Kruger et al. (1998), Younus et al (1994)".

Also, the ability of the numerical model to simulate a real spillway flow case, which is built for TABARAKABAD Dam, is demonstrated. The computed water surface is compared with the laboratory measurements.

Can be concluded that: Three Cartesian flux convection formulation of finite volume method are examined for simulation of super-critical flow in contracting and expanding canals. Using all three formulations shock waves similar to experimental observations are captured and similar convergence behaviors are observed. However, the CPU time consumption of 3rd formulation (using normal velocity at control volume edges) is approximately 8.7% less than the 1st formulation (using Cartesian velocity components at nodes) for flux convection in finite volume model. Finally the numerical model successfully is applied to the spillway flow in a contracting three dimensional chute canal and the comparison with reported experimental works presents acceptable agreement.

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