

A DIRECT ASSESSMENT OF THE COST AND BENEFITS OF HIGHER ORDER ACCURACY ON THE PREDICTION OF COMPLEX TURBULENT FLOWS

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We develop a computer code based on a fourth-order accurate finite-difference scheme in space and a second-order semi-implicit time integration method, i.e., with $O(\Delta t^2 + \Delta x^4)$ accuracy. The stencil of the fully-conservative, fourth-order accurate scheme on a staggered grid system developed by Morinishi et al. (1998) uses seven points in each direction. It is almost impossible to retain the scheme near boundaries and therefore, in the code it is switched to a second-order scheme near boundaries.

This is compared with results from another code with only $O(\Delta t^2 + \Delta x^2)$ accuracy on several benchmark flows, including the Taylor-Green vortex and lid-driven flows in a square cavity.

The goal is to demonstrate that using a high-order accurate scheme actually leads to a clear superiority even if a low-order scheme is employed near boundaries. Our long-term objective is to develop tools suitable for LES of high-Reynolds-number turbulent flows with complex geometries.

Fig. 1 shows the details of the switching between the second-order and the fourth-order schemes near boundaries. Note that the second-order scheme is introduced only in the direction normal to the boundary and the fourth-order scheme is retained in the other tangential directions.

The time histories of the energy dissipation rate obtained from DNS of the Taylor-Green vortex flow are compared with results of fully-resolved pseudo-spectral computations in Fig. 2. It can be seen that results from the present code are well predicted but that there are discrepancies between the results from the code with $O(\Delta t^2 + \Delta x^2)$ accuracy and the reference spectral simulations.

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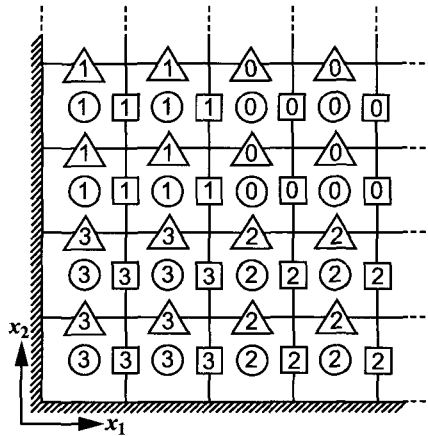


Fig. 1 Order of accuracy in spatial discretization near boundaries: ○, where continuity is evaluated; □, where momentum eq. (x_1 component) is evaluated; △, where momentum eq. (x_2 component) is evaluated; 0, fourth-order accuracy in both x_1 and x_2 directions; 1, second-order accuracy in x_1 direction and fourth-order in x_2 ; 2, fourth-order accuracy in x_1 direction and second-order in x_2 ; 3, second-order accuracy in both x_1 and x_2 directions.

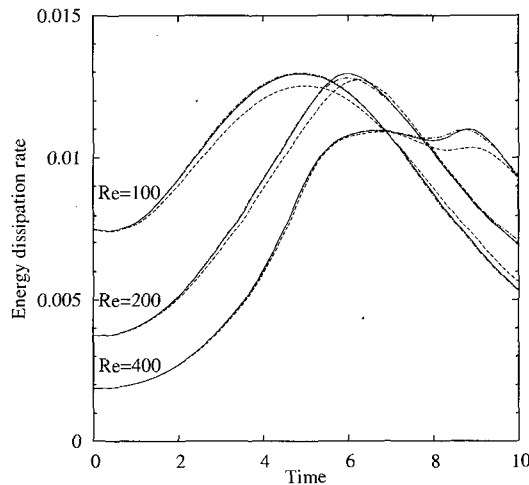


Fig. 2 Time history of energy dissipation rate: solid line, fourth-order accuracy in space; dashed line, second-order accuracy in space; dash-dotted line, pseudo-spectral simulations.

REFERENCES

Morinishi, Y., Lund, T.S., Vasilyev, O.V., Moin, P., 1998. Fully conservative higher order finite difference schemes for incompressible flow. *J. Comput. Phys.*, Vol.143, pp.90-124.