DIRECT CONTROL OF THE GRID POINT DISTRIBUTION IN MESHES GENERATED BY ELLIPTIC EQUATIONS

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Solving a partial differential equation defined on an arbitrarily shaped domain is encountered in many physical problems such as fluid mechanics, heat transfer, structures, and all other areas involving field solutions. To obtain an accurate solution, the irregular physical domain is commonly transformed into a rectangular one by employing a numerical grid generation technique. One of the highly developed techniques for generating a boundary-fitted coordinate system is to let the curvilinear coordinates (ξ ,η) be the solution of a system of elliptic differential equations in the physical plane.Although systems of parabolic or hyperbolic differential equations can also be used, only the system of elliptic differential equations allows specified boundary grid distribution along all of the boundaries.

Therefore, a system of Poisson differential equations is often used to perform grid generation,

which method is called Poisson grid generation method.

The major difficulty in the use of Poisson grid generation method is the choice of the control function. The physical grid points may be spaced as desired along the boundaries of the flow region. However, in practice, it is difficult to control the spacing between grid points in the interior of the flow region. To obtain high quality curvilinear grids, various forms of the source terms P and Q in the Poisson differential equations have been devised that contain adjustable parameters and provide some measure of control over the interior grid spacing.

However, the earliest successful development of the Poisson grid generation method was given by Thompson et al.(Thompson, J.F., 1974). The grid distribution thus can be improved by the assigned values of P and Q. The effect of the control functions on the curvilinear coordinates system has been extensively studied by Thompson et al(Thompson, J.F., 1980). Generally speaking, for a boundary with a constant–value, the control function P changes the intersecting angles of the constant curves at that boundary while Q alters the spacing of the constant curves. Based on such a finding, Thompson et al. (Thompson, J.F., 1982) developed functions of exponential form as

$$P_{(\xi,\eta)} = -\sum_{i=1}^{n} a_i \operatorname{sgn}(\xi - \xi i)\exp(-C_i|\xi - \xi_i|) - \sum_{j=1}^{m} b_j \operatorname{sgn}(\xi - \xi_j)\exp\left\{-d_j\left[(\xi - \xi_j)^2 + (\eta - \eta_j)^2\right]^{\frac{1}{2}}\right\}$$

$$Q_{(\xi,\eta)} = -\sum_{i=1}^{n} a_i \operatorname{sgn}(\eta - \eta_i) \exp(-C_i |\eta - \eta_i|)$$
$$- \sum_{j=1}^{m} b_j \operatorname{sgn}(\eta - \eta_j) \exp\left\{ -d_j \left[(\xi - \xi_j)^2 + (\eta - \eta_j)^2 \right]^{\frac{1}{2}} \right\}$$

The explanations of P and Q in detail are given in Ref. (Thompson, J.F., 1982).

However, The forms of these source terms and the values of the adjustable parameters require artful selection and are problem dependent. In addition, numerical instability and convergence could arise and the generated curvilinear grids near boundaries are not usually orthogonal. In the use of Thompson source terms the values of the adjustable parameters are not suitable, and are difficult to be determined. To overcome the disadvantages arising in the use of Thompson source terms, another form for the control functions was proposed by Middlecoff and Thomas (1980). When we performed grid generations, we found that the Thomas' method is better than the Thompson's method; but we also found that in the Thomas' method the constraint that the transverse coordinate curves being locally straight may affect the accuracy of the orthogonality in the neighborhood of the boundary. Therefore, we propose another new form of the control functions for grid generation, with which the resulting interior grid point distribution is controlled entirely by a priori selection of the grid point distribution along the boundaries of the region; and in particular, the transverse lines may be constrained to be orthogonal to the boundary. For a simply -connected region, the sources terms (P and Q) are computed from the Dirichlet boundary values; and multiply -connected regions are treated by segmentation into simply connected subregions. Comparison with Thomas's method, the disadvantage of assumption of boundary lines being locally straight is overcome; and a high quality grid can be generated.

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