Applying Kalman Filter into a Distributed Hydrological Model for Real-time Updating and Prediction

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Abstract

칼만필터 알고리즘을 분포형 유출모형에 적용하였다. 관측 유량과 상태변수인 유역내 저류량을 갱신하고자 Q-S curve를 도입하였고, 갱신된 저류량과 모형에 의해 모의된 저류량의 비율을 유역 내 각 지점의 수위에 적용하므로써 분포화 된 상태변수를 효율적으로 갱신하였다. 갱신된 상태변수와 상태변수 오차의 시간갱신은 몬테 카를로 시뮬레이션을 이용하여 모의하였다.

Key words: **Kalman filter, distributed hydrological model, updating state variables**

1. Introduction

The Kalman filter (1960) is an optimal recursive data processing algorithm to estimate the state variables for minimizing the error statistically. It combines all available observation data, plus prior knowledge about the system and measuring devices, to produce an estimate of the desired variables in such a manner that the error is minimized statistically (e.g. Maybeck, 1979). Since Hino (1974) initially adapted the Kalman filter theory to a hydrological system, numerous studies have been carried out to use the filter theory in the field of hydrology. While the Kalman filter has been applied to many lumped models for better simulation or more accurate forecasting, it has hardly ever been applied to distributed hydrological models. One of the main reasons is that unlike lumped models, it is complicated to formulate the Kalman filter algorithm in the system structures of distributed models in most cases. A large number of state variables based on a fine grid cell hydrologic system also make it harder to apply the Kalman filter.

In this research, to avoid the computational burden for updating each state variable, several techniques are introduced for applying the Kalman filter into a distributed hydrological model. Q-S curve is used for the observation equation of the filter to update the simulated total storage amount of the basin with discharge observation. Spatially distributed storage amount in the model is reset by multiplying by ratio of the updated total storage amount to the simulated storage amount. For the prediction algorithm, Monte Carlo simulation is carried out to estimate state variable and error variance propagation of the next updating step.

2. Updating State Variables of Distributed Hydrological Model

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2.1 Brief Model Description of CDRMV3

CDRMV3 (Kojima *et al.*, 2003) is one dimensional physically based distributed hydrologic model developed at Flood Disaster Research Laboratory of Disaster Prevention Research Institute, Kyoto University (http://fmd.dpri.kyoto-u.ac.jp/~flood/product/cellModel/cellModel.html). The model solves the Kinematic wave equation using Lax Wendroff scheme on every node in a cell. Discharge and water depth propagate to the next cell according to a predefined routine order determined in accordance with DEM data. An advantage of the CDRMV3 is that the stage-discharge relationship of each cell reflects the topographic and physical characteristics of its own cell. Specified stage-discharge relationship, which incorporates saturated and unsaturated flow mechanism, is included in each cell. Because of the variable slope and roughness coefficient, each cell has its own relationship.

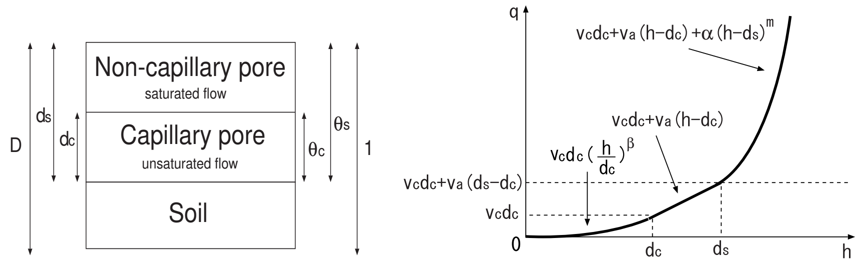


Fig.1 Relationship between unit width discharge and water depth in the CDRMV3

The stage-discharge relationship is expressed by three equations corresponding to the water levels divided into three layers (see Figure 1). When the water depth h is lower than the depth of unsaturated layer ($0 \le h < d_c$), flow is described by Darcy's law with a degree of saturation, (h/d_c) and velocity v_c. If the h increases ($d_c \le h < d_s$), flow from the saturated layer is considered with a different velocity v_a of saturated layer d_s. The velocity of subsurface flow v_a and v_c are calculated by multiplying hydraulic conductivity k_a and k_c by slope i. After the water depth is greater than the soil layer ($d_s \le h$), overland flow is added by using the Manning's equation. According to this mechanism, the equations between discharge per unit width q and water depth h are formulated. More detail on the specified state-discharge relationship and the model structure can be found in Tachikawa *et al.* (2004).

2.2 Updating Spatially Distributed State Variables

The model is applied to the Tamjin dam basin (193km^2) of Korea. The basin has two observations, Tamjin dam station at the outlet of the basin and Bongduk station in the basin. In this research, the state variable to be updated is total storage amount in a basin and its spatial distribution. The parameters of the CDRMV3 are calibrated before applying the Kalman filter and do not change when state variables are updated.

If the total storage amount to be updated is measured directly, we can easily obtain the difference of observation and simulation results. Observed quantities are discharge or river stage rather than distributed storage amount. So, we should think of a relationship between discharge and storage amount. In the CDRMV3, the relationship between the discharge at the outlet and the total amount of storage has a loop shape as shown in Figure 2 with a dash line. The loop shapes would be different for each flood event. However, it is still possible to get a relationship in a specific case like a steady state condition. After reaching the steady state condition with a given constant rainfall on the subject basin, the total storage amount that corresponds to the given rainfall intensity can be acquired by multiplying cell area by water depths of each cell and sum up these entire amounts. The cell size in this study is 250m×250m. Assuming a steady state condition, the Q-S curve for Tamjin dam basin and Bongduk basin are obtained as shown in Figure 2 with solid lines.

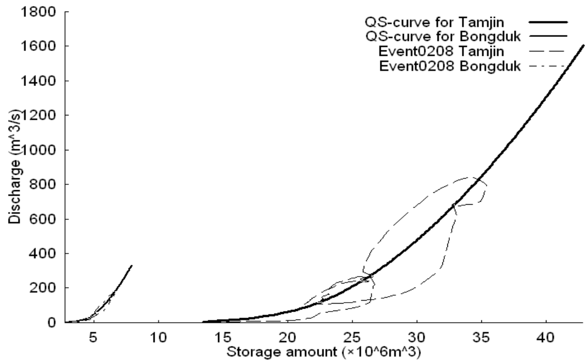


Fig.2 Relationship between discharge and storage amount at Tamjin dam and Bongduk

After observed total storage amount acquired using Q-S curve as shown in Equation 1, the observed storage amount should be distributed to each cell in a subject basin. One efficient way to update each cell's storage amount is using a specific ratio calculated from the observed total storage amount and the simulated storage amount. The calculated ratio is applied to all water depths of each cell in the model, which has the same spatial distribution pattern with the simulation result before updating as shown in Figure 3. This method named as the S-ratio method offers efficient and effective updating skill of state variables considering its spatial distribution pattern.

$$S_O = S_S + H \times (Q_O - Q_S) \quad (1)$$

where, S_O : Storage amount using observed discharge
S_S : Storage amount from simulation result
H : Gradient of Q-S curve according to S_S

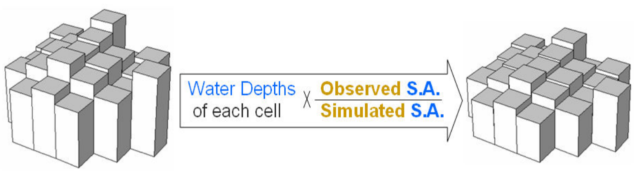
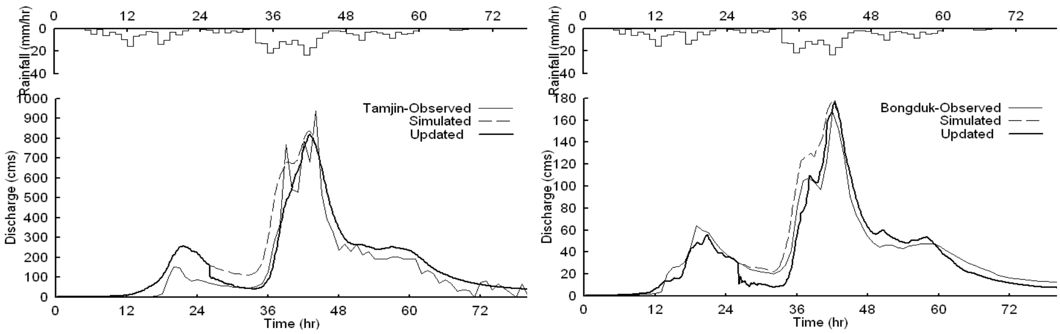


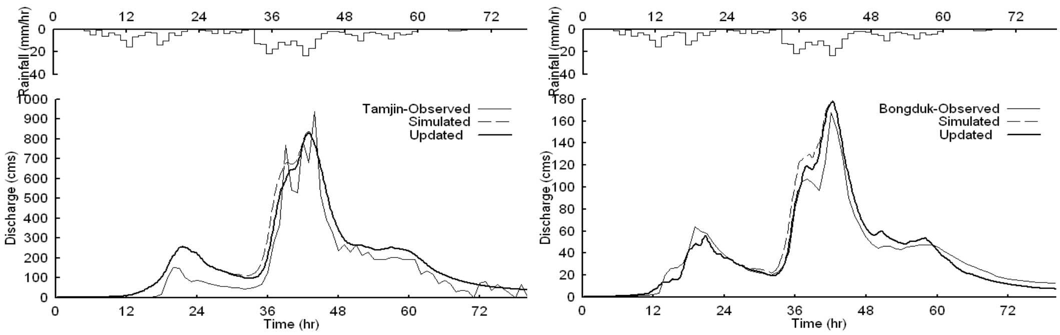
Fig.3 Concept of the S-ratio method

Figure 4 and Figure 5 show updating results by S-ratio at 26th hours. First, S-ratio was calculated using observation at Tamjin dam station and was applied to the whole basin include Bongduk basin. Improved result at Tamjin dam station after the updating is shown in Figure 4-1. Result from the Bongduk station show worse result than offline simulation (Figure 4-b). Second, another S-ratio calculated using observation at Bondgduk station was applied to the whole basin. The updated result at Bongduk station and its influence to the Tamjin dam station are shown in Figure 5-a and b. As the results of Figure 4-b and Figure 5-a, inappropriate S-ratio makes worse update results or too little improvement. When each S-ratio from each basin, Tamjin dam basin and its sub-basin Bongduk, is applied to its own basin, updated results shows reasonable combination of hydrograph, Figure 4-a and Figure 5-b.



a. At Tamjin dam station b. At Bongduk station

Fig.4 Updating results by S-ratio at Tamajin dam station



a. At Tamjin dam station b. At Bongduk station

Fig.5 Updating results by S-ratio at Bongduk station

3. Coupling Kalman Filter into CDRMV3

Kalman filter algorithm (Eq 2~8) is applied to CDRMV3 for updating state variables. Observation equation (Eq. 2) which specifies a relationship between observed discharge data and total storage amount is to get the conversion matrix H in the measurement update algorithm. The matrix H stands for the gradient of the Q-S curve in accordance with simulation results at updating time step. After getting the optimized total storage amount from the measurement update algorithm, every water depth on each computation node is reset using S-ratio method as shown in Figure 3.

In the CDRMV3, a complicated relation exists between the present state variables and the next state variables. So it is impractical to formulate the system matrix F_k, which is essential to update the error variance $P(k+1k)$. Rather than use the conventional concept of the filter, Monte Carlo simulation technique is applied to solve the problem. More detail description for coupling Kalman filter into CDRMV3 is given by Kim *et al* (2005).

Observation equation

$$y_k = H_k x_k + w_k ; \quad w_k \sim N(0, R_k) \tag{2}$$

Measurement update algorithm

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K_k (y_k - H_k \hat{x}(k|k-1)) \tag{3}$$

$$P(k|k) = P(k|k-1) - K_k H_k P(k|k-1) \tag{4}$$

$$K_k = P(k|k-1) H_k^T (H_k P(k|k-1) H_k^T + R_k)^{-1} \tag{5}$$

System equation

$$x_{k+1} = F_k x_k + B_k + v_k ; \quad v_k \sim N(0, Q_k) \tag{6}$$

Time update algorithm

$$\hat{x}(k+1|k) = F_k \hat{x}(k|k) + B_k \tag{7}$$

$$P(k+1|k) = F_k P(k|k) F_k^T + Q_k \tag{8}$$

4. Results and Discussion

The Kalman filter was successfully coupled with the distributed hydrological model, CDRMV3, to update the state variables. The filtered result when there is no observation error in the Kalman filter algorithm is shown in Figure 6. If there is no observation error, which means that the algorithm believes the observation is the true value, the filtered results and observed data should match exactly. There are some discrepancies on the hydrograph rather than exactly match to the observed data. It is considered because ofthe steady state assumption of the Q-S curve. Further research to overcome the steady state assumption on the Q-S curve is going on.

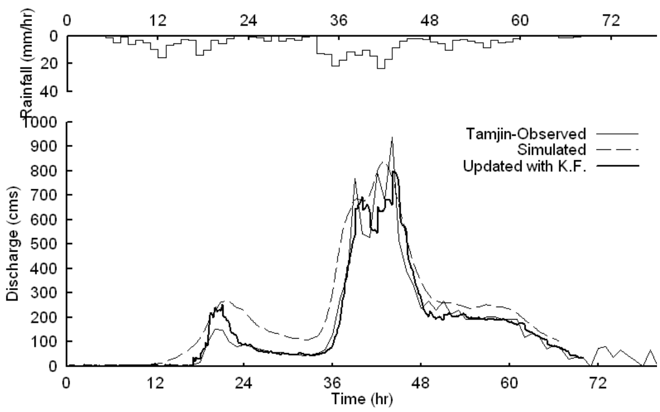


Fig.6 Filtered results at Tamjin dam station

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