2-유체 2상-유동 모델에서 근사 Jacobian 행렬을 이용한 2차원 캐비테이션의 예측

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TWO-DIMENSIONAL CAVITATION PREDICTION BASED ON APPROXIMATE JACOBIAN MATRIX IN TWO-FLUID TWO-PHASE FLOW MODELS

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We developed an upwind numerical formulation based on the eigenvalues of the approximate Jacobian matrix in order to solve the hyperbolic conservation laws governing the two-fluid two-phase flow models. We obtained eight analytic eigenvalues in the two dimensions that can be used for estimate of the wave speeds essential in constructing an upwind numerical method. Two-dimensional underwater cavitation in a flow past structural shapes or by underwater explosion can be solved using this method. We present quantitative prediction of cavitation for the water tunnel wall and airfoils that has both experimental data as well as numerical results by other numerical methods and models.

Key Words: 이상유동(Two-Phase Flow), 이유체모델(Two-Fluid Model), water tunnel, 형상 캐비테이션, eigenvalues, upwind methods

1. Introduction

Cavitation is vaporization phenomena of fluids by depressurization process. Cavitation produces many undesirable problems in hydraulic machines such as generation of intense noise, damage to surface, the loss of performance, etc. Cavitation occurs in the liquids when the local pressure drops below the vapor pressure. For example, a strong rarefaction wave which propagates into a liquid or acceleration of flow on a hydrofoil can produce cavitation regions.

It is very difficult to numerically simulate cavitating flows due to its complex physical process which is still not well understood. Hence, there are many cavitation models that makes use of additional empirical relations good for a single- fluid formulation. This approach is direct and simple because it can utilize the existing single-phase codes. However, since it entirely depends on the cavitation model to be used, its application is quite restrictive.

We address in this paper how to solve the 2-D cavitation problems using the two-fluid two-phase flow model. The one-dimensional approximate Jacobian matrix method that has been proposed by the present authors [1] is extended to two dimensions. Generally the 2-D cavitation prediction using the two-fluid model is notoriously known very difficult due to the mathematical instability. We solve two 2-D shape cavitation problems, i.e., the NACA16012 hydrofoil and the Venturi-type nozzle. The results are compared with the experimental data

2. Governing equations

The two-dimensional compressible two-fluid two-phase flow model can be written as

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$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial x}+H\frac{\partial \alpha_g}{\partial x}+I\frac{\partial \alpha_g}{\partial y}=S \quad (1)$$

where the corresponding vectors are given as

$$U=\begin{pmatrix}\alpha_g\rho_g\\ \alpha\rho_l\\ \alpha_g\rho_g u_g\\ \alpha\rho u_l\\ \alpha_g\rho_g v_g\\ \alpha\rho v_l\\ \alpha_g\rho_g E_g\\ \alpha\rho E_l\end{pmatrix},\ F=\begin{pmatrix}\alpha_g\rho_g u_g\\ \alpha\rho u_l\\ \alpha_g\rho_g u_g^2+\alpha_g p\\ \alpha\rho u_l^2+\alpha p\\ \alpha_g\rho_g u_g v_g\\ \alpha\rho u v_l\\ \alpha_g\rho_g u_g E_g+\alpha_g u_g p\\ \alpha\rho u E_l+\alpha u p\end{pmatrix}$$

$$G=\begin{pmatrix}\alpha_g\rho_g v_g\\ \alpha\rho v_l\\ \alpha_g\rho_g u_g v_g\\ \alpha\rho u v_l\\ \alpha_g\rho_g v_g^2+\alpha_g p\\ \alpha\rho v_l^2+\alpha p\\ \alpha_g\rho_g v_g E_g+\alpha_g v_g p\\ \alpha\rho v E_l+\alpha v p\end{pmatrix},\ H=\begin{pmatrix}0\\0\\-p^i\\p^i\\0\\0\\-p^i u^i\\p^i u^i\end{pmatrix},\ I=\begin{pmatrix}0\\0\\0\\0\\-p^i\\p^i\\-p^i v^i\\p^i v^i\end{pmatrix} \quad (2)$$

The variables α, ρ, u, v, E and p represent the void fraction, the density, the x- and y-velocity, the total energy, and the common pressure, respectively. The interfacial pressure p^i and the interfacial velocities u^i, v^i should be properly modeled. The subscript g and l represent the gas and the liquid phase, respectively. The void fraction obeys $\alpha_g+\alpha_l=1$.

The stiffened-gas equation of state is employed to close the system:

$$p=(\gamma_k-1)\rho_k e_k-\gamma_k p_{\infty,k} \quad (3)$$

where γ_k and e_k is the specific heat ratio and the internal energy of each phase, respectively.

The constants for each phase are given by

$$\begin{cases}\gamma_g=1.4,\ p_{\infty,g}=0\ Pa & \text{for } air\\ \gamma_l=2.8,\ p_{\infty,l}=8.5\times10^8 Pa & \text{for } water\end{cases} \quad (4)$$

To make the two-fluid governing equation system stable, we employ the following two-dimensional interfacial pressure model:

$$p^i=p-\delta\frac{\alpha_g\alpha\rho_g\rho_l}{\alpha_g\rho_l+\alpha\rho_g}[(u_g-u_l)^2+(v_g-v_l)^2] \quad (5)$$

where $\delta\geq 1$ is the damping coefficient.

The interfacial velocities u^i, v^i are modeled as the velocity at the center of mass as follows:

$$u^i=\frac{\alpha_g\rho_g u_g+\alpha\rho u_l}{\alpha_g\rho_g+\alpha\rho_l},\ v^i=\frac{\alpha_g\rho_g v_g+\alpha\rho v_l}{\alpha_g\rho_g+\alpha\rho_l} \quad (6)$$

3. Approximate Jacobian matrix

Yeom & Chang [1] proposed a simple and powerful method to estimate the characteristic wave speeds: First we reduce the equation system to derive a set of analytic eigenvalues. These approximate eigenvalues are then used in the HLL scheme when we numerically integrate the full equation system. Major advantage of the present method is that these eigenvalues do not depend on the stability terms at all.

The governing equation system becomes simpler when the interfacial transfer terms are dropped. The equation system in the rotated frame $(\hat{x},\hat{y})$ becomes an augmented one-dimensional system as follows:

$$\frac{\partial \hat{U}}{\partial t}+\frac{\partial \hat{F}}{\partial \hat{x}}=0 \quad (7)$$

The above equations can be transformed using the primitive variables $W=(\alpha_g,p,u_g,u_l,v_g,v_l,\rho_g,\rho_l)^T$ into

$$\frac{\partial \hat{W}}{\partial t}+(\widehat{A^{-1}B})\frac{\partial \hat{W}}{\partial \hat{x}}=\frac{\partial \hat{W}}{\partial t}+\hat{\Phi}\frac{\partial \hat{W}}{\partial \hat{x}}=0 \quad (8)$$

where $\hat{\Phi}$ is the approximate Jacobian matrix in the rotated frame and has the following form:

$$\hat{\Phi}=\begin{pmatrix}\Phi_{11}&\Phi_{12}&\Phi_{13}&\Phi_{14}&0&0&0&0\\ \Phi_{21}&\Phi_{22}&\Phi_{23}&\Phi_{24}&0&0&0&0\\ \Phi_{31}&\Phi_{32}&\Phi_{33}&0&0&0&0&0\\ \Phi_{41}&\Phi_{42}&0&\Phi_{44}&0&0&0&0\\ 0&0&0&0&\Phi_{55}&0&0&0\\ 0&0&0&0&0&\Phi_{66}&0&0\\ \Phi_{71}&\Phi_{72}&\Phi_{73}&\Phi_{74}&0&0&\Phi_{77}&0\\ \Phi_{81}&\Phi_{82}&\Phi_{83}&\Phi_{84}&0&0&0&\Phi_{88}\end{pmatrix} \quad (9)$$

The eight eigenvalues of this hyperbolic system can be obtained analytically:

$$\lambda_1^c = u_g,\ \lambda_2^c = u_l,\ \lambda_3^c = u_g,\ \lambda_4^c = u_l,\ \lambda_{5,6}^c = u_g \pm a_g,$$

$$\lambda_{7,8}^c = u_l \pm a_l \sqrt{1 - \frac{\alpha_g \gamma_g \gamma_l^2 p_{\infty,l}}{\alpha_g \gamma_g \gamma_l (\gamma_l - 1) p_{\infty,l} + \alpha_l \rho_g a_g^2 \gamma_l + \alpha_g \rho_l a_l^2 \gamma_g}}$$

Among the eight characteristic fields connected to these eigenvalues; the first two are linearly degenerate, the next two are shear waves, and the rest four are genuinely non-linear. If both phasic velocities are set to zero, two eigenvalues λ_5^c, λ_7^c represent the sonic speed of the gas and the liquid phase in the two-phase mixture, respectively.

4. Numerical Method

The equation system is discretized using the finite volume method with numerical flux at the cell interface in two dimensions as

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{A_i} \sum_{s=1}^{4} T_{i,s}^{-1} L_{i,s} F_{i,s}^* - \frac{\Delta t}{A_i} (H, I)_i \cdot \nabla(\alpha_g)_i$$

where $A_i, L_{i,s}$ and $\nabla(\alpha_g)_i$ are the area of cell I, the length of the boundary s in the cell I, and the gradient of void fraction, respectively.

The rotated numerical flux at the cell interface $F_{i,s}^*$ is given using the HLL (Harten, Lax, and van Leer) Riemann solver. The fastest wave speeds used in the HLL scheme are computed by our eigenvalues based on the approximate Jacobian matrix.

5. Results and discussion

5.1 NACA16012

To numerically simulate the shape cavitation phenomena in two dimensions using the current two-fluid model, we first consider the NACA16012 hydrofoil. Franc and Michel[2] performed the experimental study on cavitating flows around NACA16012 foil in the hydrodynamic tunnel. The chord of the symmetrical foil is 10 cm. Fig. 1 shows the experimental photograph of cavitating bubbles attached on the foil surface.

The free stream water consists of 99% pure water and 1% dispersed gas at atmospheric pressure. At inlet, the flow is set to the constant speed, v=6 m/s. At outlet, we use a first-order extrapolation in accordance with wave directions. A CFL=0.4 and 61×15 computational grids are used.

Fig. 2 shows the numerical computation of 2-D cavitation of the NACA16012 hydrofoil. The result reveals that the cavitation zone is well predicted and its position is similar to the experiment: it is located somewhat more backward than the experiment.

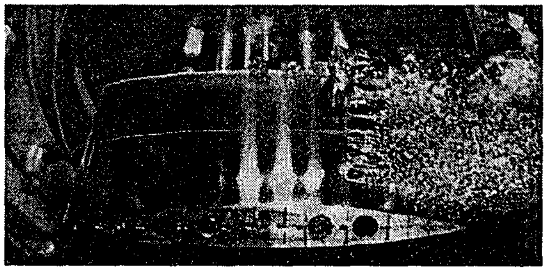

Fig. 1 Cavitating flow experiment of NACA16012 hydrofoil at v=6 m/s by Franc & Michel (1985).

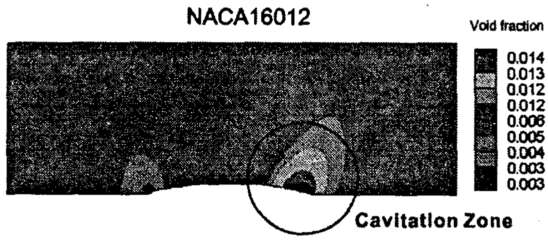

Fig. 2 Numerical simulation of 2-D cavitation of NACA16012 hydrofoil

5.2 Venturi-type nozzle

Next, we consider a cavitation generation in the Venturi-type nozzle which was investigated experimentally by Stutz and Legoupil[3]. They conducted the experiment in a cavitation tunnel. The est section is 520 mm long, 44 mm wide, and 520 mm high. The angle of the convergent part of the nozzle is 18 degree and the angle of diverge nt part is 8 degree. Fig. 3 shows the X-ray image of the cloud cavitation in the Venturi-type nozzle at v=8 m/s.

Numerical computation is made with 220×40 computational grids. A water with 1% dispersed gas is used for free stream flows at atmospheric pressure. A CFL=0.4 is used. The boundary conditions are the same as the previous case.

Fig. 4 shows the numerical simulation of cavitating flows of the Venturi-type nozzle in two dimensions. The cavitation occurs from the nozzle throat, which well explains the experimental result.

Fig. 3 Photograph of the cloud cavitation in a Venturi-type nozzle at v=8 m/s by Stutz & Leogoupil (2003)

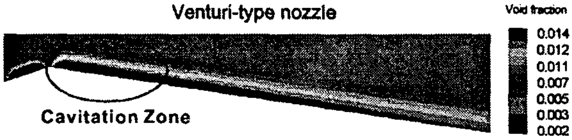

Fig. 4 Numerical simulation of cavitating flows of the Venturi-type nozzle in two dimensions

5. Conclusions

A two-dimensional compressible two-fluid two-phase model has been formulated in this paper. We numerically have solved the 2-D shape cavitation phenomena for the NACA16012 hydrofoil and the Venturi-type nozzle using a new method proposed by the present authors, namely, the approximate Jacobian matrix method. The numerical results show that the our method well predicts the cavitation directly, i.e., without additional empirical cavitation models, so can be used widely.

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