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Stability of Switched Linear Systems Using Upper Bounds of Solutions of Lyapunov Matrix Equations

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Abstract - In this paper, we propose a novel stability criterion for switched linear systems. The proposed method employs the results on the upper bound of the solution of LME(Lyapunov Matrix Equation) and on the stability of hybrid system. The former guarantees the existence of Lyapunov-like energy functions and the latter shows that the stability of switched linear systems by using these energy functions. The proposed criterion releases the restriction on the stability of switched linear systems comparing with the existing methods and provides us with easy implementation way for pole assignment.

Key Words : Switched linear systems, Lyapunov matrix equations, stability criterion, pole assignment.

1. Introduction

In recent years, the stability of switched systems has received growing attention. A switched system is defined as a family of subsystems that is converted to another subsystem according to switching signals. Especially, when each subsystem is linear, an overall system is called a switched linear system. The widespread application of such systems is motivated by the fact that high performance control systems can be realized by switching between relatively simple LTI systems. But even if all subsystems are linear, the overall system may not necessarily be linear because the system has discontinuities at each switching instants. This means that general stability criterion and controller design methods for linear systems are no longer acceptable because the stability of switched linear systems can not be guaranteed even when each subsystem is stable [5].

One of representative results on switched linear systems is that if all eigenvalues of the sum of each system matrix and its transpose are less than zero, there exists a common Lyapunov function for such switched linear system [4]. This requirement however is so hard to achieve that applicable systems are very limited. In this paper, we propose a new stability criterion which is less restrictive than the existing one. The proposed method employs the recent results on the upper bound of the solution of Lyapunov matrix equation [9] and on the stability of hybrid systems [10].

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2. Stability of Hybrid Systems

There are several results in the literature using Lyapunov theory to show stability for hybrid systems. Most of them show the stability of hybrid systems through multiple Lyapunov functions have decreasing values at consecutive switching instances and are well defined on each switching interval, which means that time derivatives of each Lyapunov function should be negative definite so as to decrease the energy functions on each interval. In general, this is very severe requirement. On the other hand, some results which need not such requirement are reported in the work by Ye, Michel and Hou [10].

The stability result in [10] is very general in the sense that it can be applied to different types of systems. A discontinuous Lyapunov function is introduced which guarantees stability if the sequence of values of the Lyapunov function at consecutive switching times is non-increasing and the energy between these times is bounded by a continuous function which is zero at the origin. Asymptotic stability is guaranteed by requiring the sequence of values of the Lyapunov function at consecutive switching time to be decreasing. In this case, the energy function does not need to decrease on each switching interval contrary to other traditional stability results.

The stability result mentioned above requires that values of the Lyapunov like functions decrease at each switching instance. However, we propose a modified stability result which requires that maximum values of energy functions decreases in consecutive switching intervals instead of decreasing of values of energy functions at switching instances.

Theorem 1. Let $M \subset I$ be a set of equilibrium points and define metric $d(x,M)$ as the distance between the current state and M . Assume that there exist a energy function $V: X \times R^+ \rightarrow R^+$ and $\phi_1, \phi_2 \in$ class K defined on R^+ such that

$$\phi_1(d(x,M)) \le V(x,t) \le \phi_2(d(x,M)) \quad (1)$$

for all $x \in X, t \in R^+$, which satisfies

i) For all motions of trajectories $x(t,a,t_0) \in S$, $V(x(t,a,t_0),t)$ is continuous except on switching instances $\{t_1, t_2, \cdots\}$, where a, t_0 denote the initial state and initial time, respectively.

ii) $V(x(t_n,a,t_0),t_n)$ is non-increasing for $n=1,2,\cdots$.

iii) There exists a continuous function $g(\cdot)$ such that

$$g(0)=0,$$
$$V(x(t,a,t_0),t) \le g(V(x(t_n,a,t_0),t_n)) \quad (2)$$

for $t \in (t_n, t_{n+1})$, where t_n denotes an instance when each energy function has the maximum value on corresponding switching intervals. Then, the motions of trajectories are uniformly stable.

iv) In addition, there exists $\phi_3 \in K$ such that

$$\frac{1}{t_{n+1}-t_n}\left[V(x(t_{n+1},a,t_0),t_{n+1}) - V(x(t_n,a,t_0),t_n)\right]$$
$$\le -\phi_3(d(x(t_n,a,t_0),M)) \quad .(3)$$

Then, the motions of trajectories are uniformly asymptotically stable (The proof is omitted by limitation of space).

3. Energy Functions

In this section, we build a energy function which satisfy the requirements in Theorem 1. To do this, we use results on the upper bound of the solution of LME (Lyapunov Matrix Equations). Considerable research efforts have been devoted to finding the upper bound of the solution of LME, $A'P + PA + Q = 0$, where A is a given matrix and especially Q is a positive definite matrix.

Theorem 2. Under the assumptions of Proposition 3.1 in [9], suppose that

$$\| X \| \le \| X^{-1} \| .$$

Let $\tilde{P} = X^* P X > 0$,where X^* denotes the conjugate transpose of X and $AX = X\Lambda$ with $\Lambda = diag(\{\lambda_i\})$. Then

$$\|\tilde{P}\| \le \frac{\|Q\|}{2\min \mathrm{Re}[-\lambda(A)]} \mathrm{K}(X)$$

(The proof is omitted by space limitation).

Trivially, we can obtain that if $\mathrm{K}(X)/(2\min \mathrm{Re}[-\lambda(A)]) < 1$, then $\|\tilde{P}\| < \|Q\|$. By using this result, Lyapunov like energy functions can be designed recursively.

4. Stability of Switched Linear Systems

In this section, we show asymptotic stability of switched linear systems under an arbitrary switching signal by combining the results of Section 2 and 3. In general, hybrid systems can be represented by $\{R^+, X, I, S\}$ as shown in Section 2, where metric space X includes continuous and discrete spaces. Another representation of hybrid systems is given in state space form as

$$X = R^n \times N \quad (4)$$

where $x \in R^n$ and $\sigma \in N$ denote continuous variable and discrete variable, respectively. Switched linear systems are a subset of hybrid systems. So, such systems can be represented in hybrid manner as

$$\begin{cases} \dot{x} = F_\sigma x + B_\sigma u , \\ u = K_\sigma x , \\ \sigma(k+1) = h(x, \sigma(k)) , \end{cases} \quad (5)$$

where $\sigma = \{1,2,\cdots\}$ denotes the index of subsystem. Replacing with state feedback, $K_\sigma x$, in the above system yields

$$\begin{cases} \dot{x} = A_\sigma x , \\ \sigma(k+1) = h(x, \sigma(k)) , \end{cases} \quad (6)$$

where $A_\sigma = F_\sigma + B_\sigma K_\sigma = \{A_1, A_2, \cdots\}$. Therefore, all results in Section 2 and 3 are applicable to switched linear systems.

Theorem 3. If the system matrix A_σ of each subsystem satisfies

$$\| X \| \le \| X^{-1} \|$$
$$\frac{\mathrm{K}(X_\sigma)}{2\min \mathrm{Re}[-\lambda(A_\sigma)]} < 1 \quad , \quad (7)$$

then the origin of the family of such linear systems is globally asymptotically stable under an arbitrary switching signal (The proof is omitted by space limitation).

5. Illustrative Example

Consider a switched linear system as follows

$$\begin{cases} \dot{x} = F_\sigma x + B_\sigma u , \\ u = K_\sigma x , \\ \sigma(k+1) = h(x, \sigma(k)) , \end{cases}$$

where $h(\cdot)$ is a mapping that determines which subsystem is activated next. For example, a switched linear system is given as following a family of linear systems.

$$\left\{\begin{matrix}\left(F_1=\begin{bmatrix}1&2\\3&3\end{bmatrix}, B_1=\begin{bmatrix}1\\0\end{bmatrix}\right), \left(F_2=\begin{bmatrix}2&5\\4&3\end{bmatrix}, B_2=\begin{bmatrix}1\\0\end{bmatrix}\right),\\ \left(F_3=\begin{bmatrix}4&3\\-2&1\end{bmatrix}, B_3=\begin{bmatrix}0\\1\end{bmatrix}\right), \left(F_4=\begin{bmatrix}2&3\\7&4\end{bmatrix}, B_4=\begin{bmatrix}0\\1\end{bmatrix}\right)\end{matrix}\right\}.$$

Assign poles of each subsystem as follows

$$p_1=\{-3\pm j4\},\ p_2=\{-2\pm j3\},$$
$$p_3=\{-4\pm j4\},\ p_4=\{-2\pm j5\}\ .$$

Then, the resulting closed-loop subsystems are

$$\left\{\begin{matrix}A_1=\begin{bmatrix}-9&-17.3333\\3&3\end{bmatrix}, A_2=\begin{bmatrix}-7&-8.5\\4&3\end{bmatrix},\\ A_3=\begin{bmatrix}4&3\\-26.6667&-12\end{bmatrix}, A_4=\begin{bmatrix}2&3\\-13.6667&-6\end{bmatrix}\end{matrix}\right\}.$$

The norms of diagonalizing matrices of each subsystem are

$\|X_1\|=1.3854, \|X_1\|=1.3701, \|X_1\|=1.4011, \|X_1\|=1.3416,$

$\|X_1^{-1}\|=3.5212, \|X_1^{-1}\|=2.8544, \|X_1^{-1}\|=5.1956, \|X_1^{-1}\|=2.2361.$

So, the assumption is satisfied. The condition numbers of diagonalizing matrices of each subsystem are

$\mathrm{K}(X_1)=4.8737,\ \mathrm{K}(X_2)=3.9110,\ \mathrm{K}(X_3)=7.2793,\ \mathrm{K}(X_4)=3$

Each condition number dose not exceed $2\min\mathrm{Re}[-\lambda(A_\sigma)]$. Therefore, we can say that the switched linear system under an arbitrary switching signal is asymptotically stable as shown in the below figure.

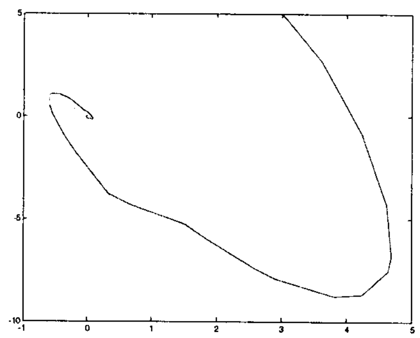

Fig. Converging trajectory under an arbitrary switching

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